

Modeling of Dielectric Constant for the Application of Ground-Penetrating Radar in Clayey Soil

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Abstract— Ground-Penetrating Radar (GPR) is widely used in many applications including detection of buried objects such as pipes in soil and for subsurface site characterization. The propagation of radar waves in soil is dependent on the soil's electromagnetic properties such as the dielectric constant and electrical conductivity. Thus, knowledge of the electromagnetic properties of the soil or adequate estimation of such properties is essential for efficient and reliable GPR field surveys. This paper presents models for predicting the dielectric constant of clayey soil. The models were developed using data from field experiments in clayey soil. The models correlate the dielectric constant as a function of the volumetric moisture content of clayey soil. The paper also presents inverse models to predict the volumetric moisture content from the measured dielectric constant. Polynomial, square, and linear equations are provided for both the forward and inverse models to ensure that the models are applicable for quick estimation of dielectric constants in the field as well as for a more in-depth prediction of dielectric constant for clayey soils during detailed analysis.

Keywords— ground-penetrating radar (GPR); dielectric-constant; modeling; clayey-soil; pipelines

I. INTRODUCTION

Accurate location of underground utilities and other subsurface/buried objects is essential for the management and operation of infrastructure assets all over the world. Locating subsurface utilities is also necessary for safe excavation near such utilities, either for rehabilitation and maintenance or for new construction. Though several nondestructive testing techniques exist for subsurface prospecting, ground-penetrating radar (GPR) has emerged as one of the leading and effective technologies for locating subsurface utilities. The advantages of GPR over other techniques have been reviewed in [1], while its effectiveness as a subsurface prospecting technology has been demonstrated in recent studies [1,2,3].

GPR is a very versatile technique that has been successfully employed in many infrastructure monitoring and geophysical survey applications. Some of these applications include buried pipe/utility and

container detection [1,2,4,5,6,7], locating of reinforcing steel in concrete [8,9,10], detection of defects in concrete or fiber-reinforced-polymer bridge decks [11,12], measurement of pavement thickness [13], snow thickness measurement and/or subglacial topography detailing [14,15], irrigation and soil water content monitoring [16,17], detection of cracks in concrete or pipe wall [18,19], and underground/concrete void detection [20,21]. The GPR technique has also been used for archaeological investigations [22,23], detection of landmines [24], and detection of tree roots [25,26,27].

The ground-penetrating radar technique operates by transmitting high frequency electromagnetic pulses (10 MHz to 4 GHz typical) from an antenna into a material and measuring the reflected signal response from subsurface features/interfaces. The receiving antenna records the reflected signal amplitude, phase, and elapsed time of the received signal. The recording of successive reflections from different depths produces a signal trace, which helps in locating subsurface features and objects. The amplitude of the received reflected or echo signal depends on the reflection coefficient and the incident signal amplitude at the interface as well as the signal attenuation properties of the overlaying material [1,2]. The reflection coefficient directly correlates with the difference/contrast between the electromagnetic wave impedance of the two media at an interface.

The electromagnetic wave impedance is estimated from the dielectric constant of the two materials at the interface [1,2]. Thus, larger difference between dielectric constants of materials at an interface results in higher potential of the underlying material being detected with GPR (provided the electromagnetic signal can penetrate the overlaying layers). This phenomenon of dielectric contrast is demonstrated in recent studies. Although GPR generally penetrates deeper and performs very well in dry sand mediums, the buried 30 cm pipes at 40 cm depth in a recent study could not be located [28] due to similarities in the dielectric constants between the pipe material and the surrounding medium. Contrastingly, another study showed that 30.48 cm diameter pipes buried with 122 cm of soil cover was easily located in clayey soil (a much more difficult soil type for GPR surveys), producing clear reflected signals due to higher dielectric contrast created using externally bonded material on the pipes as discussed in the study [1,2].

The contrast discussed in the above studies underscores the need to understand the electromagnetic properties, such as the dielectric constant, of the test medium and the buried object of the test for successful GPR surveys. The dielectric properties for a test site are often not available, requiring the survey team to assume these parameters or model them from known data. Existing models for estimating dielectric constant are available for concrete or sandy soils [29,30]. These models are not applicable to clayey soils. Also, the calibrations provided for organic soils in some of the existing models may not be applicable to clayey soils due to the high variability between soils from different regions.

This paper provides alternate models for estimating the dielectric properties of the clayey soils found at a study area in West Virginia, U.S.A. This experimental study involved the development of forward and inverse dielectric models, and comparison of the proposed models with existing published models for soil surveys. For each of the model types (forward and inverse models), polynomial, square, and linear equations are provided, with different levels of accuracy to aid in quick estimation of dielectric constant during field work as well as for subsequent in-depth analysis.

II. SOIL DIELECTRIC PROPERTIES FOR GPR SURVEY

GPR signal attenuation, penetration depth, and amplitude of received signals are all affected by the dielectric properties of the media under investigation. Dielectric properties of the test media are therefore required to estimate the penetration depth of the signal, set the appropriate time range in the GPR scanning system, and estimate the likelihood of identifying the object of interest. The dielectric constant is also useful for accurately estimating the depth at which objects are buried from interpretation of the GPR data. Effective planning and implementation of successful GPR surveys therefore requires adequate knowledge of soil properties such as dielectric constant and electrical conductivity. It is often difficult or impractical to measure dielectric constant of the soil, and to the depth of interest, at the test site during GPR surveys.

A common approach that is often used in estimating soil dielectric constant for GPR surveys is to rely on published dielectric constants for soils that are similar to the type of soil anticipated at the survey site. Table 1 provides a summary of some of the published dielectric constant data, together with the corresponding electrical conductivity [31,32,33,34]. Due to the high variability in the electromagnetic properties of soils from one location to the other, the published dielectric data are often provided in ranges as evident in Table 1. This leads to speculation and guesswork as to what values will be more appropriate for the test site at hand.

For certain GPR applications, material properties provided in Table 1, and the assumption of low-loss condition can be combined to estimate the GPR wave velocity, attenuation, and wavelength in a material [31]. For a more reliable and/or detailed characterization of

most porous material (e.g., soils), the material is considered as a mixture of its various components (mixture of solid particles/grains, air, and water/pore fluid). An appropriate mixture model is then selected to estimate the effective dielectric permittivity (also known as dielectric constant) of the mixture.

A. Forward Dielectric Models

There are several empirical relations for estimating the effective relative dielectric permittivity of mixtures [29,30]; most of these relations rely on the volumetric water content and porosity of the mixture to calculate the dielectric constant. This is because the dielectric permittivity of water is significantly higher than that of most dry soils, as such the dielectric permittivity of soils is primarily controlled by the presence of water in the pore space.

One of the widely used basic dielectric models (ϵ - θ relationships) is the Topp equation [30], which requires selection of appropriate polynomial coefficients for accurate results. A general form of the formula for mineral soils is provided below.

$$\epsilon_r' = 3.03 + 9.3\theta + 146\theta^2 - 76.7\theta^3 \quad (1)$$

where ϵ_r' is the dielectric constant and θ is the volumetric water content of the soil.

Equation 1 is applicable only to sandy soils. For clay or organic rich soils, Equation 1 is considered inappropriate, and a site-specific calibration may be required [31,35,36]. A calibration of Equation 1 for organic soils was also provided by appropriate selection of polynomial coefficients as given by [30]:

$$\epsilon_r' = 1.74 - 0.34\theta + 135\theta^2 - 55.3\theta^3 \quad (2)$$

Equation 2 is considered to be applicable "for soils with higher water content or organic matter" [37]:

Many other ϵ - θ relationship models have been developed by various researchers, some of which are reviewed by [37,38]. Some of the ϵ - θ models are presented in Equations 3 through 10.

The following model was developed from an experiment conducted on mineral soils [39]:

$$\epsilon_r' = 2.87 - 11.1\theta + 276\theta^2 - 272\theta^3 \quad (3)$$

And for organic soils:

$$\epsilon_r' = 0.97 + 10.9\theta + 87.4\theta^2 - 28.0\theta^3 \quad (4)$$

The original equation for organic soils in the literature by [39] had coefficient of the second term to be negative (-10.9 θ), this however did not conform to the data and plots presented in the same paper. It was realized after investigation that, changing the coefficient to positive (+10.9 θ) as presented in Equation 4 made the equation conform to the plot presented in the original literature. Detailed comparison between the original (erroneous) equation and the corrected equation (Equation 4) has been provided in a previous publication [1].

Table 1. Relative dielectric constants and conductivity of common subsurface materials

Material	Relative permittivity, ϵ_r				Conductivity (mS/m)			
	[31]	[32]	[33]	[34]	[31]	[32]	[33]	[34]
Air	1	1	1	1	0	0	0	0
Clay – dry	2 – 20		2 – 6	5 – 40	1 – 100	1	100 – 1E3	2 – 1000
Clay – wet	15 – 40	12	5 – 40		100 – 1000	100	100 – 1E3	
Concrete – dry	4 – 10		4-10		1 – 10		1 – 10	
Concrete – wet	10 – 20	7	10 – 20		10 – 100	0.1	10 – 100	
Freshwater /Distilled	78 – 88	81	81	80	0.1 – 10	0.1 – 10	0.001 – 10	0.01 – 0.5
Seawater	81 – 88	81	81	80	4000	4000	100000	30000
Freshwater ice	3	4	4	3 – 4	1 – 1E-6	1	0.1 – 1	0.01
Seawater ice	4 – 8		4 – 8		10 – 100		10 – 100	
Permafrost	2 – 8	8	4 – 8		0.1 – 10	1	0.01 – 10	
Granite – dry	5 – 8		5	4 – 6	1E-3 – 1E-5		1E-5 – 1E-3	0.01 – 1
Granite – wet	5 – 15	5	7		1 – 10	1E-5	1 – 10	
Limestone – dry	4 – 8		7	4 – 8	1E-3 – 1E-7		1E-5 – 1E-3	0.5 – 2
Limestone – wet	6 – 15		8		10 – 100		10 – 100	
Sandstone – dry	4 – 7		2 – 5		1E-3 – 1E-7		1E-3 – 0.01	
Sandstone – wet	5 – 15	6	5 – 10		0.01 – 0.001	40	0.1 – 10	
Shale – dry			4 – 9				1 – 10	
Shale – saturated	6 – 9		9 – 16		10 – 100		1 – 100	
Sand – dry	3 – 6	9	2 – 6	3 – 5	0.0001 – 1	1	1E-4 – 1	0.01
Sand – wet	10 – 30		10 – 30	20 – 30	0.1 – 10		1 – 10	0.1 – 1.0
Sand – coastal, dry	5 – 10				0.01 – 1			
Soil – sandy, dry	4 – 6	2.5	4 – 10		0.1 – 100	0.14	0.1 – 10	
Soil – sandy, wet	15 – 30	25	10 – 30		10 – 100	7	10 – 100	
Soil – loamy, dry	4 – 6	2.5	4 – 10		0.1 – 1	0.11	0.1 – 1	
Soil – loamy, wet	10 – 20	19	10 – 30		10 – 100	21	10 – 100	
Soil – clayey, dry	4 – 6	2.4	4 – 10		0.1 – 100	0.3	10 – 100	
Soil – clayey, wet	10 – 15	15	10 – 30		100 – 1000	50	1 – 1E3	
Soil – average	16				5			
Asphalt		6	2 – 12			1	1 – 100	
Basalt		8				10		
Copper		1				5800		
Silicon		12				1		
Snow		1.4	6 – 12			1E-4	1E-3 – 0.01	
Iron		1				1E9		
Wood (dry)		3				3		
Sand		8				3		
Shales				5 – 15				1 – 100
Silts				5 – 30				1 – 100
Coal, dry			3.5				1 – 10	
Coal, wet			8				1 – 100	
Salt, dry			4 – 7	5 – 6			0.1 – 10	0.01 – 1

A simple square root averaging model for ϵ - θ relationship as presented in Equation 5 was developed by [40]. The author also showed that this relation (Equation 5) conforms to the Topp equation for mineral soils (Equation 1).

$$\theta = 0.1181\sqrt{\epsilon_r'} - 0.1841 \quad (5)$$

$$\therefore \epsilon_r' = \left(\frac{\theta + 0.1841}{0.1181} \right)^2 \quad (6)$$

Another ϵ - θ relationship model was also developed from experiments conducted on organic soil floor samples [41]. This model is similar to Equation 5:

$$\theta = 0.136\sqrt{\epsilon_r'} - 0.119 \quad (7)$$

$$\therefore \epsilon_r' = \left(\frac{\theta + 0.119}{0.136} \right)^2 \quad (8)$$

Another form of this ε - θ relationship is proposed by [42] as provided by [37]:

$$\theta = 0.1138\sqrt{\varepsilon'_r} - 0.1758 \quad (9)$$

$$\therefore \varepsilon'_r = \left(\frac{\theta + 0.1758}{0.1138} \right)^2 \quad (10)$$

The most popular mixing model formulation for GPR applications is the Complex Refractive Index Model (CRIM). This estimates the effective dielectric permittivity of a mixture by considering the properties of its constituents. This model is simple and has been found to be robust and accurate [31]. The general form of the CRIM formula is written as:

$$\varepsilon_{mix}^e = \left(\sum_{i=1}^n f_i \sqrt{\varepsilon_i} \right)^2 \quad (11)$$

where:

ε_{mix}^e = relative complex effective permittivity of the mixture

f_i = volume fraction of the i^{th} component

ε_i = relative complex effective permittivity of the i^{th} component

A three-component model is appropriate for soils in most cases, which results in formula given below:

$$\varepsilon_{mix}^e = \left[(\phi S \sqrt{\varepsilon_w}) + ((1 - \phi) \sqrt{\varepsilon_s}) + (\phi(1 - S) \sqrt{\varepsilon_g}) \right]^2 \quad (12)$$

where:

ε_{mix}^e = complex relative effective permittivity of the mixture

ϕ = porosity of the mixture

S = degree of saturation

$\varepsilon_s, \varepsilon_w, \varepsilon_g$ = relative permittivities of solids, water, and gas/air components respectively.

Thus, using Equation 12 and known properties of the constituents of a given soil, the relative dielectric permittivity can be computed. This can greatly help in analyzing and interpreting GPR data. However, Equation 12 is not very helpful for practical field applications of GPR surveys since relative permittivity, degree of saturation, and porosity values must be known for the different components of the soil under consideration.

Other widely recognized models include the semiempirical mixing dielectric model and the generalized refractive mixing dielectric model [43].

B. Inverse Dielectric Models

Similar to the models used to estimate dielectric constant from volumetric water content measurements, a number of inverse models exist for computing volumetric water content from dielectric data. These relations are useful in estimating the water content of the soil from GPR survey data if the depth to target or dielectric constant of the soil is known.

The following inverse ε - θ relation for mineral soils, was provided based on the same experimental data used to derive Equation 1 [30]:

$$\theta = -5.3 \times 10^{-2} + 2.92 \times 10^{-2} \varepsilon'_r - 5.5 \times 10^{-4} \varepsilon_r'^2 + 4.3 \times 10^{-6} \varepsilon_r'^3 \quad (13)$$

Another inverse ε - θ relation was proposed from experimental data for mineral soils [39] as given by:

$$\theta = -0.0728 + 0.0448 \varepsilon'_r - 0.00195 \varepsilon_r'^2 + 0.0000361 \varepsilon_r'^3 \quad (14)$$

And for organic soil material, the following modification was proposed [39]:

$$\theta = -0.0233 + 0.0285 \varepsilon'_r - 0.000431 \varepsilon_r'^2 + 0.00000304 \varepsilon_r'^3 \quad (15)$$

Yet another model was provided based on experiment conducted on a wide range of soil textures as [44]:

$$\theta = -0.0286 + 0.02435 \varepsilon'_r - 0.0003421 \varepsilon_r'^2 + 0.00000237 \varepsilon_r'^3 \quad (16)$$

Other models provided by [40,41,42] have already been presented in Equations 5, 7, and 9 respectively.

III. EXPERIMENTAL PROGRAM IN THIS STUDY

Soil moisture content and dielectric constants were measured using GS3 sensors manufactured by Decagon Devices, Inc. (METER Group) as part of a study on the application of GPR in detecting buried non-metallic pipes in clayey soil. The sensors enabled measurement and determination of volumetric water content, electrical conductivity, dielectric constant, and temperature of the soil. A total of five sensors were installed at three different depths in trenches beside buried pipes to measure soil properties during this study. Two of the GS3 sensors were buried at a depth of 122 cm (4 ft), two more sensors were buried at a depth of 61 cm (2 ft), and the final sensor was used to measure soil properties at various locations on the ground surface. The test site for this study is located in North Central West Virginia and consists of soil categorized as Zoar silt loam [45].

Figure 1a shows one of the soil sensors and Figure 1b shows the five-channel data logger, with the sixth channel (communication port) used to connect the data logger to a laptop computer for data acquisition. Figure 2a shows one of the trenches showing placement of pipes, Figure 2b shows the soil sensors with data wires protected from damage using 2.54 cm (1") diameter non-metallic conduit, and Figure 2c shows the soil sensor in a 61 cm (2 ft) deep trench placed near the 7.62 cm (3") diameter pipes. Trench depths were measured to the top of buried pipes. Figure 3 shows the site after backfilling and grass seeding to restore the field to its original condition. The GS3 sensors use a capacitance/frequency domain technology, with a measurement frequency of 70 MHz for dielectric constant measurements. Specifications for the GS3 sensor, including accuracy and resolution are given in Table 2.



(a)



(b)

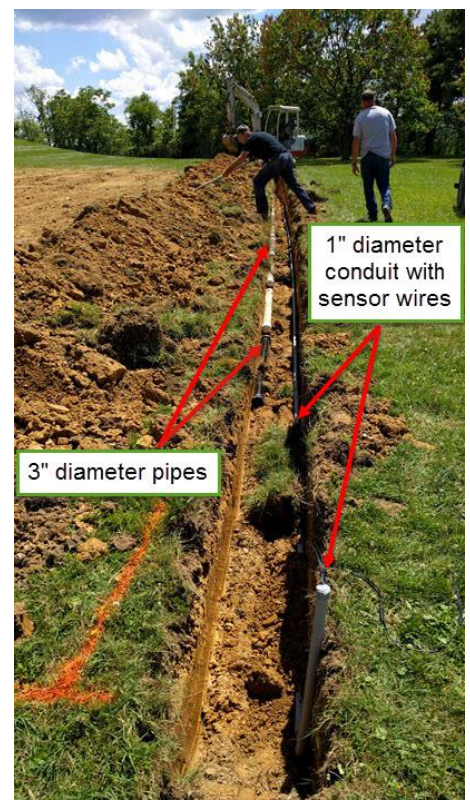
Figure 1. Sensor for measuring dielectric constant, electrical conductivity, temperature, and volumetric water content of soil. (a) Soil moisture and dielectric permittivity sensor. (b) Data logger.



(a)



(b)



(c)

Figure 2. Pictures showing the trench of buried pipes and GS3 soil sensors. (a) Picture of a trench showing placement of pipes. (b) Soil sensors with data wires running through conduits to protect the wire. (c) 7.62 cm (3") diameter pipes and soil sensors at 61 cm (2 ft) depth.

Table 2. GS3 sensor specifications*.

Parameter	Volumetric Water Content, VWC or θ (m^3/m^3)	Dielectric Constant, ϵ_r	Electrical Conductivity (mS/m)	Temperature ($^{\circ}\text{C}$)
Range	0.0 to 1.0	1 to 80	0 to 2500	-40 to +60
Accuracy	± 0.03	± 1 from 1 to 40 $\pm 15\%$ from 40 to 80	$\pm 5\%$ from 0 to 500 $\pm 10\%$ from 500 to 2500	± 1
Resolution	0.002 from 0.0 to 0.4 0.001 from 0.4 to 1.0	0.1 from 1 to 20 <0.75 from 20 to 80	0.1	0.1

*From soil moisture sensor specifications provided by Decagon Devices, Inc. (METER Group)



Figure 3. Site after soil backfilling and grass seeding.

IV. RESULTS, ANALYSIS, AND DISCUSSION

A. Test Data

Dielectric soil properties were measured periodically using the buried GS3 soil sensors under different soil moisture conditions over a 24-month period during a GPR study. Some of this data is summarized and provided in Table 3. Prolonged rainy spells were experienced during the testing period, thus most of the test was conducted in wet soil conditions. Data for soil VWC below 0.208 (not shown in Table 3) was therefore obtained by taking a block of soil from the field and allowing it to dry under normal laboratory conditions while the dielectric properties were continuously measured using the GS3 soil sensor.

B. Dielectric Modelling

The dielectric constant was plotted against the volumetric water content as shown in Figure 4. A third order polynomial function (Equation 17) was fitted to the experimental $\epsilon' - \theta$ data in this study. This function modelled the data accurately (passing through all the field data points and overlapping the original line connecting the data points) with an R^2 value of 1.000. Though Equation 17 is an excellent fit for the data, it should be noted that this model was developed based on volumetric water content values between 0.119 and 0.489; thus, the model might not work properly for data outside this range. This limitation is evident in the slightly negative ϵ' that will be produced by this model if the θ value is 0.

$$\epsilon'_r = -0.0267 + 61.576\theta - 127.88\theta^2 + 247.25\theta^3 \quad (17)$$

The field data (and Equation 17) was compared to the forward dielectric models presented in Equations 1 through 10 in Section 2 as shown in Figure 5. Equation 2, which is the Topp [30] model for organic soils is the closest to the data in this study, especially at higher VWC, while Equations 1, 6, and 10 are closer to the data at lower VWC (θ).

Equation 3, which is for mineral soils, under-predicts the dielectric constant by an average of 9.4% (minimum 0.7% and maximum 16.7%) when volumetric water content is below 0.185 and over-predicts by an average of 18.8% (minimum 0.5% and maximum 27.8%) when volumetric water content is above 0.185. Equations 1, 6, and 10 over-predict the dielectric constant by an average of 18 – 22%, while the remaining models (Equations 2, 4, and 8) under-predict by an average of 14 – 34%.

To facilitate quick estimation of dielectric constant from volumetric water content measurements in the field, two other functions that are simpler than Equation 17 were fitted to the data. These are a quadratic and a linear function (Equations 18 and 19), with R^2 values of 0.996 and 0.979 respectively. These two equations proposed in this study are slightly less accurate as evident in the R^2 value but are convenient for quick computations in the field during GPR surveys. In addition, all three models/functions presented in this section are only applicable for VWC (θ) between 0.119 and 0.489. The three models are summarized in Table 4 and plotted as shown in Figure 6.

$$\epsilon'_r = \left(\frac{\theta + 0.1953}{0.1312} \right)^2 \quad (18)$$

$$\epsilon'_r = 58.482\theta - 2.332 \quad (19)$$

C. Inverse Dielectric Modelling

The discussions in the previous sections have shown the important role that reliable estimation of dielectric constant can play in GPR surveys. Conversely, there is a need to estimate soil properties such as volumetric water content from GPR data or dielectric constant measurements for other applications such as agriculture. Dielectric constant can be estimated from GPR data if the depth to the target object is known. The depth to buried objects can also be estimated using other techniques including

hyperbolic fitting, and the Common Mid-Point (CMP) survey approach [1].

The volumetric water content data measured in this study is plotted against the dielectric constant as shown in Figure 7.

Similar to the dielectric model in Section 4.2, a third order polynomial function (Equation 20) was fitted to the inverse $\epsilon_r - \theta$ data. This function also modelled the

data accurately (passing through all the experimental data points and overlapping the original line connecting the data points) with an R^2 value of 1.000 as shown in Figure 7. It should be noted that this model was also developed based on dielectric constant values between 6.02 and 28.86, and volumetric water content values between 0.119 and 0.489; thus, the model might not work properly for data outside this range.

Table 3. Measured soil dielectric properties.

Depth (cm (ft))	Volumetric Water Content, θ (m^3/m^3)	Conductivity (mS/m)	Dielectric Constant, ϵ_r'
0 (0)	0.339	14.88	15.90
	0.399	10.91	19.84
	0.394	18.41	19.52
	0.346	15.24	16.28
	0.388	18.10	19.04
	0.297	9.89	13.56
	0.285	7.82	12.92
	0.308	12.26	14.13
	0.307	9.63	14.08
	0.235	10.33	10.58
	0.465	15.35	25.74
	0.275	8.80	12.46
	0.382	19.96	18.79
	0.479	25.19	27.56
61 (2)	0.366	15.00	17.57
	0.396	17.36	19.67
	0.410	16.37	20.72
	0.419	16.02	21.39
	0.416	15.76	21.16
	0.406	14.55	20.39
	0.417	16.37	21.29
	0.395	14.24	19.57
	0.388	14.33	19.05
	0.345	11.56	16.25
	0.482	18.82	27.79
	0.411	16.13	20.74
	0.361	12.23	17.28
	0.489	21.54	28.86
122 (4)	0.458	21.39	25.04
	0.463	20.28	25.50
	0.476	22.33	27.06
	0.470	20.28	26.35
	0.474	21.16	26.75
	0.461	19.01	25.28
	0.464	18.81	25.62
	0.456	17.45	24.75
	0.456	17.64	24.77
	0.442	16.02	23.44
	0.473	18.81	26.66
	0.465	16.72	25.74
	0.456	13.10	24.79
	0.479	14.76	27.36

Table 4. Forward dielectric models derived from experimental data.

Model	R^2	Equation Number
$\epsilon_r' = -0.0267 + 61.576\theta - 127.88\theta^2 + 247.25\theta^3$	1.000	17
$\epsilon_r' = \left(\frac{\theta + 0.1953}{0.1312} \right)^2$	0.996	18
$\epsilon_r' = 58.482\theta - 2.332$	0.979	19

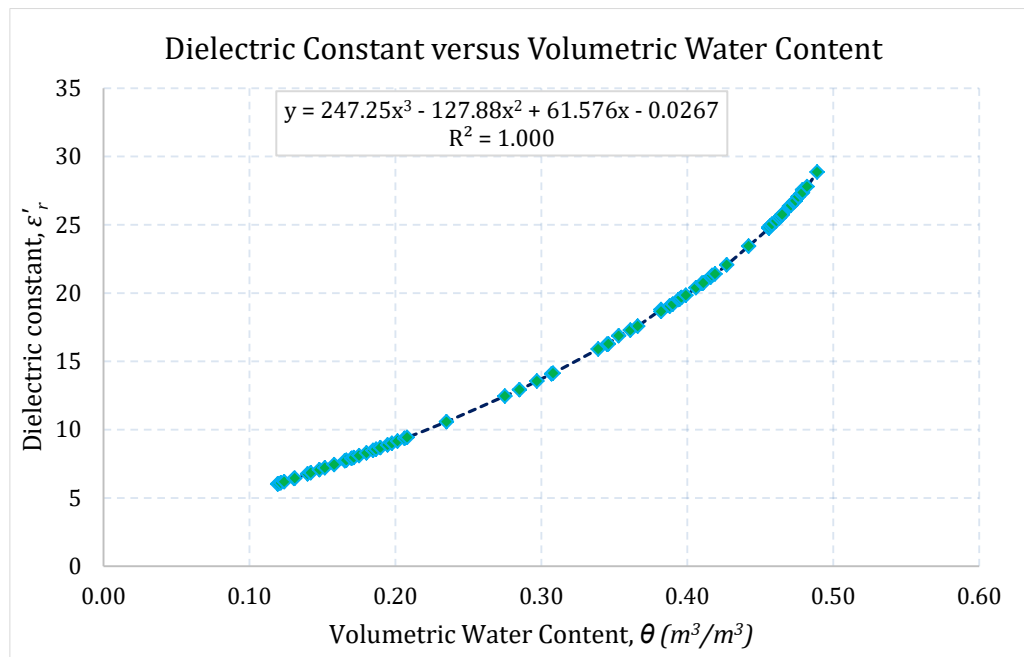


Figure 4. Experimental dielectric constant versus VWC.

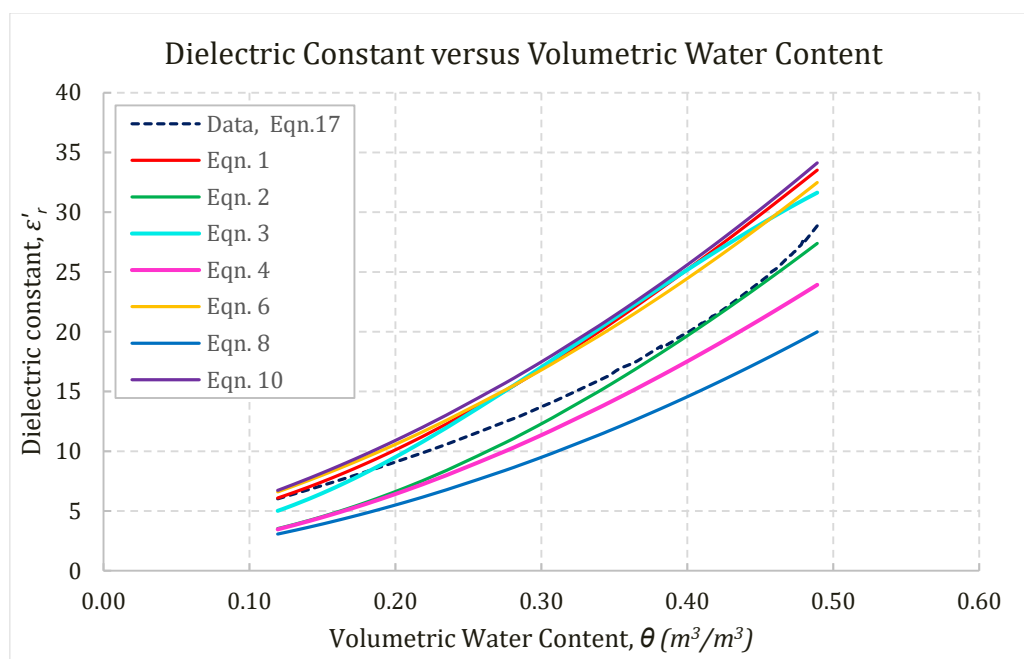


Figure 5. Comparison between forward dielectric models and experimental data.

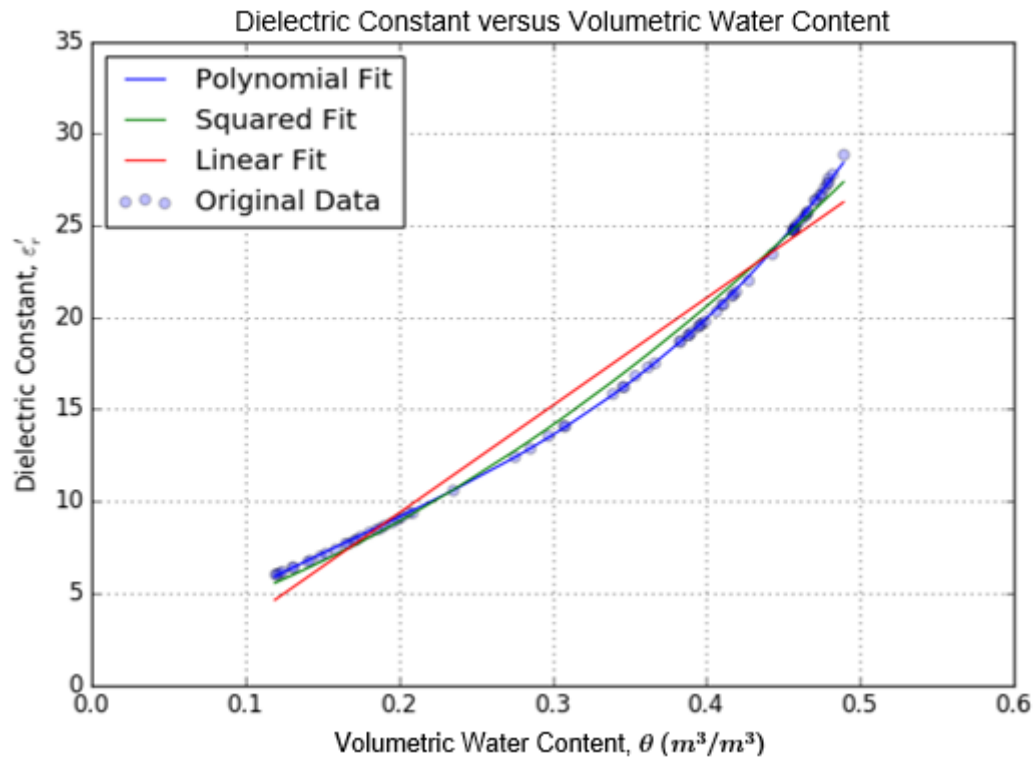


Figure 6. Comparison between the three forward dielectric models and experimental data.

$$\theta = -0.0737 + 0.0364\varepsilon_r' - 0.00074\varepsilon_r'^2 + 0.00000547\varepsilon_r'^3 \quad (20)$$

The field data (and Equation 20) was compared to the inverse dielectric models presented in Section 2.2 as shown in Figure 8. Equation 15, which is a model for organic soils provides the closest fit to the data. This equation only over-predicts by an average of 2.2%, with maximum over-prediction of 11.7% and maximum under-prediction of 1.97% compared to the experimental data. Equation 7 over-predicts by an average of 36% (maximum of 80%), while Equation 9 under-predicts by an average of 15%. All the other remaining models under-predict by an average of 13%.

Similar to the forward dielectric model, two other functions simpler than Equation 20 were fitted to the

data to make it easier for quick estimation of volumetric water content (VWC) from dielectric constant measurements in the field. These include a radical (square root) and a linear function (Equations 21 and 22), with R^2 values of 0.996 and 0.979 respectively. Similar to Equations 18 and 19 from the forward model, the two other equations proposed in this inverse model (Equations 21 and 22) are slightly less accurate but are convenient for quick computations in the field. All three models/functions presented in this section may only be applicable for dielectric constant values between 6.02 and 28.86, and VWC between 0.119 and 0.489. The three models are summarized in Table 5 and plotted in Figure 9.

$$\theta = 0.1312\sqrt{\varepsilon_r'} - 0.1953 \quad (21)$$

$$\theta = 0.0167\varepsilon_r' + 0.0460 \quad (22)$$

Table 5. Inverse dielectric models derived from experimental data.

Model	R^2	Equation Number
$\theta = -0.0737 + 0.0364\varepsilon_r' - 0.00074\varepsilon_r'^2 + 0.00000547\varepsilon_r'^3$	1.000	20
$\theta = 0.1312\sqrt{\varepsilon_r'} - 0.1953$	0.996	21
$\theta = 0.0167\varepsilon_r' + 0.0460$	0.979	22

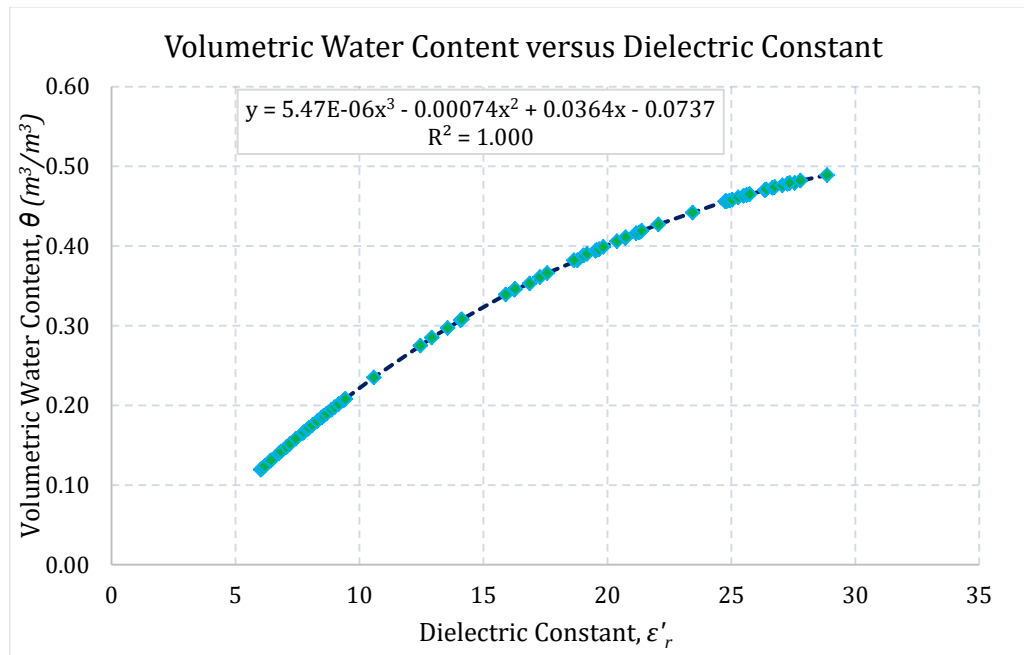


Figure 7. Experimental VWC versus dielectric constant.

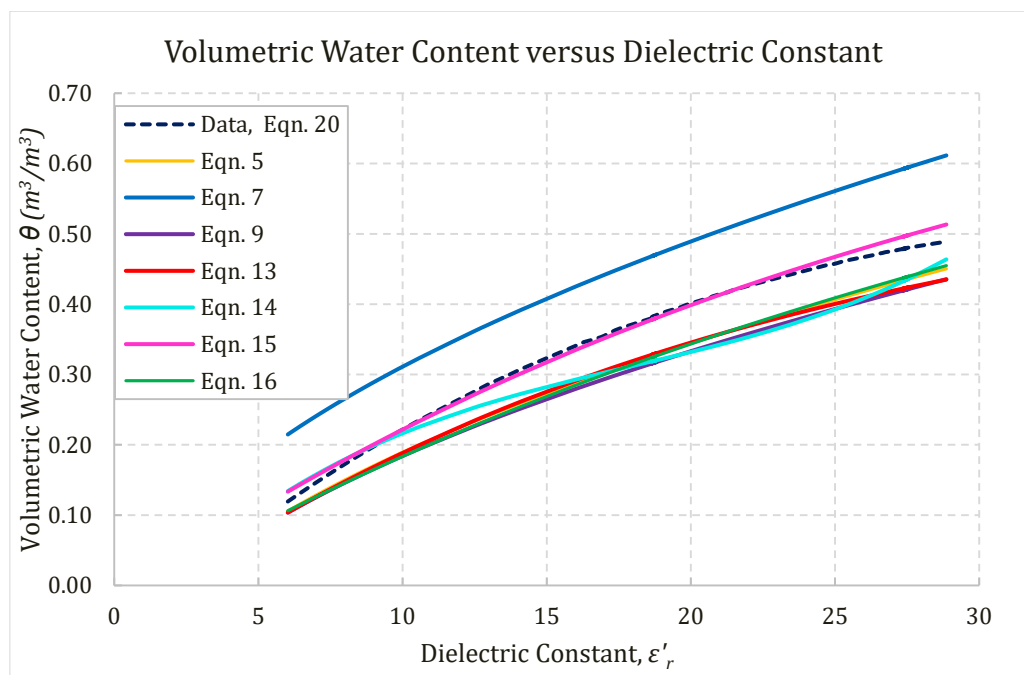


Figure 8. Comparison between inverse dielectric models and experimental data.

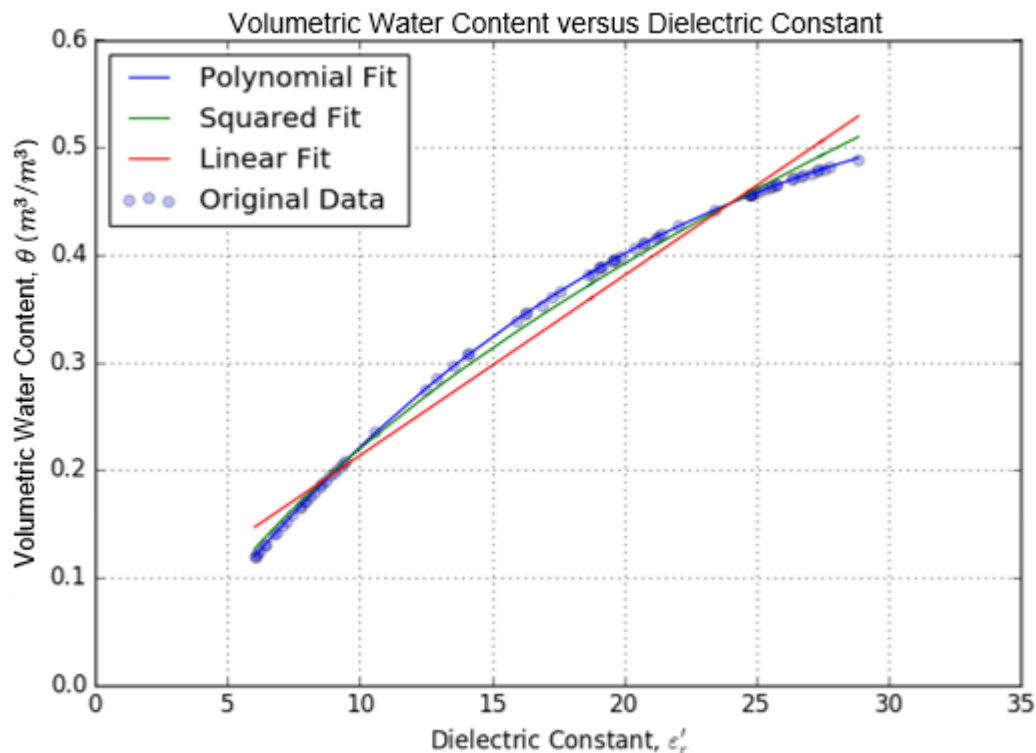


Figure 9. Comparison between the three inverse dielectric models and experimental data.

Finally, the ϵ - θ data and model from this study (Equation 17) was compared to secondary data from previous research, as compiled by [38]. The plot provided by [38] is overlaid with the polynomial model from this study (Equation 17), represented by the thick red curve at the center of the plot as shown in Figure 10. This figure shows the dielectric model proposed in this study fits right withing the existing models from previous studies. Equation numbers in parentheses represent equations found in [38], while the numbers in square brackets represent equations as they appear in this paper.

Based on all the comparisons, the previously published forward and inverse models closest to the experimental data from this study are Equations 2 and 15, which are models for organic soils presented by [30] and [39] respectively. This paper has also proposed Equations 17 and 20 which provide the best fitting forward and inverse models for the experimental data acquired during this study. In addition, simplified square and linear models have been proposed in this paper for quick computations in the field.

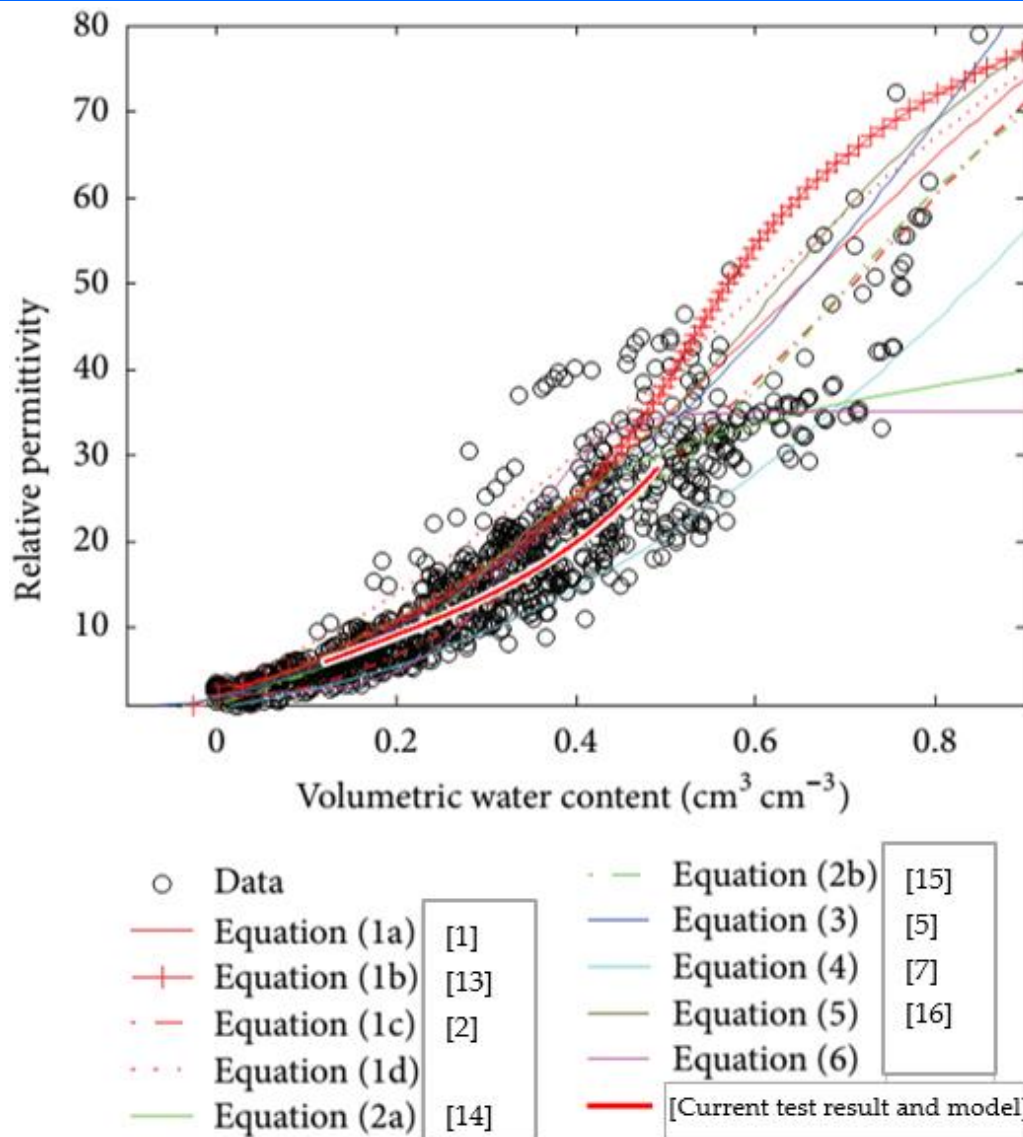


Figure 10. Comparison between experimental data/model from this study and secondary data and models [Reprinted/Adapted from Ref.38].

V. CONCLUSIONS

Dielectric models for estimating soil dielectric constant from volumetric water content measurements and vice versa have been developed in this study. Some of the popular existing models have also been evaluated and compared to the experimental data obtained in this study. From all the models evaluated, the two of the ones for organic soils [30,39] were found to be the closest to the experimental data in this study for the forward and inverse dielectric models respectively. This paper has also proposed the best fitting forward and inverse polynomial models for the experimental data acquired during this study. In addition, simplified square and linear models have been proposed in this paper for quick estimation of dielectric constant or volumetric water content for organic soil in the field.

A comparison of the proposed forward and inverse dielectric models with the existing models shows that the models proposed here fit right in the middle of these existing models. This comparison validates the

usefulness of the proposed model for organic and clayey soils.

Additionally, the simplified models presented here will make it easier for quick estimation of dielectric properties of soil during field testing to evaluate the likely velocity and penetration depth of the GPR system. The full polynomial form of the proposed models can then be used to refine the estimated dielectric properties or predict the volumetric water content of the soil during detailed data processing and analysis after GPR field data is acquired.

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