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Analysis of Systems Design Strategies Based on Control Theory

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***Abstract*—The formulation of the analog system design process was carried out based on the application of control theory. This approach generalizes the design process and produces different design trajectories inside the same optimization procedure. The problem of constructing an optimal speed algorithm is defined as the problem of minimizing a functional in control theory. The governing equations for the proposed design methodology have been developed, which provide a set of different design strategies within this approach. An analysis of the evaluation of the number of operations of some new strategies is given. The numerical results of designing some electronic circuits demonstrate the effectiveness of the proposed approach. These examples show that the traditional design strategy is not time-optimal, and the potential computer time gain of the optimal strategy over the traditional one increases with system size and complexity.**

Keywords—System design; optimal control theory; control vector; circuit optimization; Lyapunov function

I. INTRODUCTION

One of the main problems of a large system design is the excessive computer time that is necessary to achieve the final point of the design process. The traditional design strategy for the topology defined analog system includes two main parts as a rule: a model of the system and a parametric optimization procedure that achieves the objective function optimum point. In this case, optimization is carried out in the space of independent parameters, and dependent parameters are determined as a result of the analysis of the system model. Besides the traditionally used ideas of sparse matrix techniques and decomposition techniques [1-5] some another ways were proposed to reduce the total computer design time. To overcome these problems some special methods were developed. For example, a method that determines initial point of the optimization process by centering [6], geometric programming methods [7] that guarantee the convergence to a global minimum, but, on the other hand, this require a special formulation of the design equations to which additional difficulties accompany. Another approach based on the idea of a space mapping method [8-9] may also provide a completely satisfactory solution with the necessary accuracy.

Some alternative stochastic search algorithms, especially evolutionary computation algorithms, can solve the problem of finding the global minimum and have been developed in recent years. An analysis of various stochastic algorithms for system optimization allowed select some groups: simulated annealing method [10-12], evolutionary computing techniques that produce some different approaches as evolutionary algorithms [13-16] particle swarm optimization (PSO) method, GA, differential evolution, genetic programming. A PSO technique [17-19] is one of the evolutionary algorithms that competes with genetic algorithms. This method has been successfully used to solve electromagnetic problems and to optimize microwave systems.

However, the design idea can be changed by connecting both parts, the system model and the parametric optimization procedure, provided that all system parameters are independent. In this case the objective function of the optimization procedure includes additional penalty functions that simulate the model of the physical system. On the other hand it is possible to re-determine the total design problem, to generalize it, to obtain a set of the different design strategies. A general formulation of the circuit optimization problem was developed on a heuristic level some decades ago [20]. With this approach, we can ignore the Kirchhoff laws for all or part of the chain in the optimization process. The practical aspects of this idea were developed in works [21]-[22] in an extreme case where all the equations of the circuit were not solved during the optimization process.

From the computer time viewpoint the optimal design strategy can be defined as the strategy that achieves the optimum point of the objective function of the design process for the minimum CPU time. The generalized approach for the analog system design on the basis of control theory formulation was elaborated in some previous works, for example [23].

The idea of using control theory was developed on a continuous form in [24]. In this paper, this idea is presented in a discrete form more familiar to system design problems. This approach generalizes the design problem and can reduce the overall system design time by finding the best optimization strategy.

II. Problem Formulation

The design process for any analog system design can be defined as the problem of the objective function $C(X)$ minimization for $X \in R^N$ with the system of constraints. It is assumed that the minimum of the objective function corresponds to the achievement of all design goals, and the system of constraints is a mathematical model of a physical system. It is supposed also that the topology of the physical system is done and the model can be described as the system of nonlinear algebraic equations:

$$g_j(X) = 0 \,,\; j = 1,2,\ldots,M \,, \qquad (1)$$

where M is the number of dependent variables. The vector X can be separated in two parts: $X = (X', X'')$, where the vector $X' \in R^K$ is the vector of independent variables, K is the number of independent variables and the vector $X'' \in R^M$ is the vector of dependent variables (N=K+M). This separation is very conditional, because any variable can be defined as independent or dependent parameter.

The parametric optimization process for minimizing the objective function $C(X)$ for a two-step procedure is generally determined by the following vector equation:

$$X^{s+1} = X^s + t_s \cdot H^s \,, \qquad (2)$$

with constraints (1), where s is the iterations number, t_s is the iteration parameter, $t_s \in R^1$, H is the operator that defines the direction of the objective function $C(X)$ decreasing. The vector H is the function of $C(X)$. This is a typical formulation for the constrained optimization problem. This problem is transformed to the unconstrained optimization problem for K variables if the system (1) is solved for M dependent components of the vector X. In this case, the design problem is defined as the traditional design strategy (TDS) in the space of independent variables R^K:

$$X'^{s+1} = X'^s + t_s \cdot H^s \,, \qquad (3)$$

with the system (1) which is solved at each step of the optimization procedure.

The specific character of the design process at least for the electronic systems consists in the fact that it is not necessary to fulfill the conditions (1) for all steps of the optimization process. The fulfillment of these conditions is quite sufficient for the end point of the design process.

The problem (1), (3) can be re-defined in the form when there is no difference between independent and dependent variables. This is the main idea for the penalty function method application. In this case the vector function H is the function of the objective function $C(X)$ and the additional penalty function $\psi(X)$. The structure of the penalty function must include all equations of system (1) and can be defined, for example, in the following form:

$$\psi(X^s) = \frac{1}{\delta}\sum_{j=1}^{M} g_j^2(X^s), \qquad (4)$$

where δ is the adjusting parameter.

In this case we define the design problem as the unconstrained optimization in the space R^N without any additional system but for the other type of the objective function $F(X)$. This function is defined for example as an additive function:

$$F(X) = C(X) + \psi(X) \qquad (5)$$

In this case, it is necessary to achieve the minimum of the initial objective function $C(X)$ and to comply with system (1) at the end point of the optimization process by minimizing the function $F(X)$. This is a modified traditional design strategy (MTDS) and it produces another design trajectory line in the space R^N. On the other hand, the idea of using an additional penalty function can be generalized if the penalty function is composed only as part of system (1), and the other part of this system is specified as constraints. In this case the penalty function includes first Z items only, $\psi(X^s) = \frac{1}{\delta}\sum_{i=1}^{Z} g_i^2(X^s)$, where $Z \in [0, M]$ and M-Z equations make up one modification of the system (1):

$$g_j(X) = 0 \,,\; j = Z+1, Z+2, \ldots, M \qquad (6)$$

It is clear that each new value of the parameter Z produces a new design strategy and a new trajectory line. This idea can be generalized more in case when the penalty function $\psi(X)$ includes Z arbitrary equations from the system (1). The total number of different design strategies in this case is equal to 2^M if the parameter Z can have all values of the region $[0, M]$. These strategies form the structural basis of the set of different design strategies. All these strategies exist inside the same optimization procedure. The optimization procedure is realized in the space R^{K+Z}.

The number of the dependent variables M increases with the system complexity increasing and the number of different design strategies increases exponentially. These strategies have various computer times because they have different operation number. It is appropriate in this case to define the problem of the optimal design strategy search that has a minimal computer time. Here and further the optimality of the design process is defined as the computer time minimization.

III. Operations Number Evaluation

It is very useful to evaluate the operations number to compare the different kinds of design strategies. Let us define the operator *H* by means of the gradient method of optimization. In case when the number of independent parameters is variable and equal to *K+Z* the following two systems are used:

$$\frac{dx_i}{dt} = -b \cdot \frac{d\ F(X)}{d\ x_i},\ j = 1,2,...,K+Z \quad (7)$$

$$g_j(X) = 0\ ,\ j = Z+1, Z+2,..., M \quad (8)$$

where $F(X) = C(X) + \frac{1}{\delta}\sum_{j=1}^{Z} g_j^2(X)$.

In this case the total operations number No for the solution of the systems (7)-(8) can be evaluated as:

$$N_0 = L\{K+Z+(1+K+Z)\{C+(P+1)Z + S[(M-Z)^3 + (M-Z)^2(1+P) + (M-Z)P]\}\} \quad (9)$$

when the Newton's method is used for the solution of the system (6). Formula (9) gives the operations number for the traditional design strategy when Z=0 and for the modified traditional design strategy when *Z=M*. Sometimes the necessary operation number *C* for the cost function *C(X)* calculation do not has dependency from the independent parameters number *K+Z*, but for the majority of electronic systems is in proportion to the sum *K+Z* (*C*=c(*K+Z*)). Formula (9) in this case is transformed into following expression:

$$N_0(Z) = L\{K+Z+(1+K+Z)\{c(K+Z)+(P+1)Z + S[(M-Z)^3 + (M-Z)^2(1+P) + (M-Z)P]\}\} \quad (10)$$

Analysis of the number of operations *N* as a function of *Z* by formula (10) gives the conditions for the minimum computer time. In the case when system (6) is linear, this general design strategy has practically no preference in computer time, as shown in [1]. Formula (10) gives the optimum point Z_{opt} that is within the region (*0, M*) for the nonlinear system (6).

In more general case, when the system's model can be separate on two parts as linear and nonlinear we have the following two systems:

a) nonlinear part is given by

$$g_j(X) = 0\ ,\ j = 1,2,...,\ \rho(M-Y) \quad (11)$$

b) linear part is given by

$$A\ X\ =\ B \quad (12)$$

where $\rho \in [0,1]$; *A* and *B* are matrices of the order $(1-\rho)\cdot(M-Z)$. In this case, the formula for estimating the number of operations has the following form:

$$N_0(Y,Z) = L\ \{K+Y+Z+(1+K+Y+Z) \cdot \{C+(M+1)Z+[M(1-\rho)-Z]^3 + M(1-\rho) - Z + (P+1)Y + S\cdot[(M\cdot\rho-Y)^3 + (M\cdot\rho - Y)^2 \cdot (P+1) + (M\cdot\rho - Y)P]\}\} \quad (13)$$

An analysis of this formula shows that for most practical problems it is true that the optimum point of the function $N_0(Y,Z)$ is inside the given area. This optimal point exists for various optimization methods, both for the gradient method and for the Newton method or the Davidon-Fletcher-Powell method.

Optimization of the dimension of the space of independent variables leads to a reduction in the total number of operations and, consequently, to a reduction in the total computing time when designing an electronic system. An analysis of the design process of various electronic systems shows that the optimal dimensions of the space of independent parameters can reduce the total CPU time from 100 to n*1000 times. This optimal space dimension has dependency from electronic system size and topology. In this paper, the problem of finding the optimal space dimension is solved by a more general approach based on the theory of optimal control. The total computer time serves as an objective function of finding the optimal strategy.

IV. Control Theory Application

The design process of any analog system can be defined in discrete form as the problem of minimizing the generalized objective function $F(X,U)$ by means of the system (14) with constraints (15):

$$x_i^{s+1} = x_i^s + t_s \cdot f_i(X,U),\ i = 1,2,...,N\ , \quad (14)$$

$$(1-u_j)g_j(X) = 0\ ,\ j = 1,2,...,M\ . \quad (15)$$

The functions $f_i(X,U)$ are defined by the optimization method and give the direction of the generalized cost function $F(X,U)$ decreasing. *U* is the vector of the special control functions $U = (u_1, u_2, ..., u_m)$, where $u_j \in \Omega$; $\Omega = \{0;1\}$. The functions $f_i(X,U)$ for example for the gradient method are defined as:

$$f_i(X,U) = -\frac{\delta}{\delta x_i}F(X,U),\ i = 1,2,...,K$$

$$f_i(X,U) = -u_{i-K}\frac{\delta}{\delta x_i}F(X,U) + \frac{(1-u_{i-K})}{t_s}\{-x_i^s + \eta_i(X)\},\ i = K+1, K+2,...,N \quad (16)$$

where the operator $\delta/\delta x_i$ hear and below means $\frac{\delta}{\delta x_i}\varphi(X) = \frac{\partial\varphi(X)}{\partial x_i} + \sum_{p=K+1}^{K+M}\frac{\partial\varphi(X)}{\partial x_p}\frac{\partial x_p}{\partial x_i}$, x_i^s is equal to $x_i(t-dt)$; $\eta_i(X)$ is the implicit function ($x_i = \eta_i(X)$) that is determined by the system (15). The generalized objective function $F(X,U)$ is defined as:

$$F(X,U) = C(X) + \psi(X,U) \quad (17)$$

and now has a dependence on the vector *U*. The penalty function $\psi(X,U)$ is defined as:

$$\psi(X,U)=\frac{1}{\delta}\sum_{j=1}^{M}u_j\cdot g_j^2(X)\cdot \quad (18)$$

This formulation of the problem of the design process allows you to redistribute the cost of computer time between solving problems (14)-(18). The control vector *U* is the main tool for the redistribution process in this case. Practically an infinite number of the different design strategies are produced because the vector *U* depends on the optimization procedure current step. The problem of finding an optimal design strategy is now formulated as a typical problem of minimizing a functional in control theory. The functional that needs to minimize is the total CPU time *T* of the design process. This functional depends directly on the operations number and on the design strategy that has been realized. The main difficulty of this definition lies in the unknown optimal dependencies of all control functions u_j.

The idea of setting the system design problem as a problem of minimizing the control theory functional does not depend on the optimization method and can be incorporated into any optimization procedure. In this work, the gradient method is used, but any optimization method can be used, as shown in [23].

Now the process for analog circuit design is formulated as a dynamic controllable system. The minimal-time design process can be defined as the dynamic system with the minimal transition time in this case.

V. Lyapunov Function of Design Process

The presence of many different design strategies leads to the need to define some function or functions that could characterize the properties and distinctive features of each individual strategy. It is required to determine a special criterion that allows implementing an optimal or quasi-optimal algorithm by analyzing according to this criterion.

We need to define a special criterion that permits to realize the optimal or quasi-optimal algorithm by means of the optimal switch points searching. A Lyapunov function of dynamic system serves as a very informative object to any system analysis in the control theory. It is proposed to use the Lyapunov function of the design process to identify the optimal structure of the algorithm.

There is freedom in choosing the Lyapunov function due to the ambiguity of the form of this function. We can use any form of the Lyapunov function. Let us define the Lyapunov function of the design process (14)-(18) by the following expression:

$$V(X)=\sum_i (x_i-a_i)^2 \quad (19)$$

where a_i is the stationary value of the coordinate x_i, in other words the set of all the coefficients a_i is the main objective of the design process. The function (19) satisfies all of the conditions of the standard Lyapunov function definition for the variables $y_i=x_i-a_i$. Inconvenience of the formula (19) is an unknown point $A=(a_1,a_2,...,a_N)$, because this point can be reached at the end of the design process only. We can use this form of the Lyapunov function if we already found the design solution someway. On the other hand, it is very important to control the stability of the design process during the optimization procedure. In this case we need to construct other form of the Lyapunov function that doesn't depend on the unknown stationary point. Let us define two new forms of the Lyapunov function by formulas:

$$V(X,U)=[F(X,U)]^r \quad (20)$$

$$V(X,U)=\sum_i\left(\frac{\partial F(X,U)}{\partial x_i}\right)^2 \quad (21)$$

where *F(X,U)* is the generalized cost function of the design process. The formula (20) can be used when the general objective function is non-negative and has zero value at the stationary point *A*. Other formula can be used always because all derivatives $\partial F/\partial x_i$ are equal to zero in the stationary point *A*.

We can define now the design process as a transition process for controllable dynamic system that can provide the stationary point (optimal point of the design procedure) during some time. The problem of the time-optimal design algorithm construction can be formulated now as the problem of the transition process searching with the minimal transition time. There is a well-known idea [25-26] to minimize the transition time by means of a special selection of the right part of the main system of equations, in our case, these are functions $f_i(X,U)$. It is necessary to change the functions $f_i(X,U)$ by means of the control vector *U* election to obtain the maximum speed of the Lyapunov function decreasing (the maximum absolute value of the Lyapunov function time derivative *dV/dt*). Normally the time derivative of Lyapunov function is non-positive for the stable processes. In this case we can compare the different design strategies by means of the function *V(t)* behavior.

VI. Numerical Results

The optimization procedure was analyzed for some non-linear circuits.

A. Example 1

Let us design the nonlinear circuit shown in Fig. 1. Consider a simple non-linear voltage divider circuit. A non-linear element has the following dependency: $y_n=a+b(V_1-V_2)^2$. The admittances y_1, y_2, y_3 are positive and compose a set of independent circuit parameters (*K*=3).

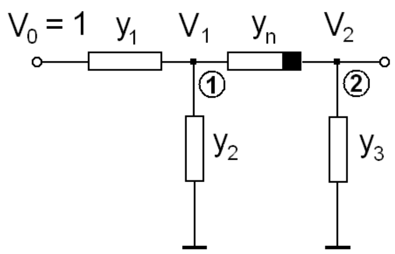

Fig. 1. *Two-node nonlinear passive circuit.*

The node voltages V_1, V_2 are the dependent parameters (M=2). Vector X consists of the following five components: (x_1, x_2, x_3, x_4, x_5), where $x_1^2=y_1$, $x_2^2=y_2$, $x_3^2=y_3$, $x_4=V_1$, $x_5=V_2$. By defining the components x_1, x_2, x_3 using the above formulas, we can automatically obtain positive values of the conductance, which eliminates the issue of positive definiteness for each conductance and allows us to perform optimization in the full space of the values of these variables without any restrictions.

The model of this circuit includes two equations corresponding to Kirchhoff's laws. The objective function $C(X)$ is determined by the formula $C(X)=(x_5-m_1)^2+((x_4-x_5)-m_2)^2$, where m_1 and m_2 are predetermined values of the divider voltages. This circuit is characterised by two (M=2) dependent parameters (x_4, x_5), and three (K=3) independent parameters (x_1, x_2, x_3). The control vector has the next structure: $U=(u_1,u_2)$. The structural basis of the various strategies includes four strategies with the following control vectors: (00), (01), (10), and (11). The mathematical model of the circuit is determined by the following equations:

$$g_1(X) \equiv (1-x_4)x_1^2-(x_4-x_5)(a+b(x_4-x_5)^2)-x_4x_2^2=0$$

(22)

$$g_2(X) \equiv (x_4-x_5)(a+b(x_4-x_5)^2)-x_5x_2^2=0$$

It is from the solution of system (15) that the values of the dependent variables x_4, x_5 can be determined and then the value of the objective function $C(X)$ can be calculated. In the case of the transformation of the two dependent variables x_4, x_5 (or at least one of them) into independent ones, it is necessary to form a generalized objective function $F(X,U)$ according to the following formula:

$$F(X,U)=C(X)+\left(u_1g_1^2(X)+u_2g_2^2(X)\right)/\delta \qquad (23)$$

Consider the optimization problem for the circuit shown in Fig. 1. Let a=1, b=1, m_1=0.25, and m_2=0.35. We define the accuracy of the design process as 10^{-7}. This means that we need to reduce the $V(t)$ function to this value. In this case, the solution of the design problem gives the following results: x_1=1.807, x_2=1.233, x_3=1.255, x_4=0.600, x_5=0.250. These results were obtained using four different strategies defined by four control vector structures (00), (01), (10), and (11). All strategies have the same final result, but different number of iterations and CPU time. The number of iterations and the total computing time for these strategies are shown in Table 1 with an accuracy of $\varepsilon=10^{-7}$. This accuracy corresponds to the value of the Lyapunov function calculated by formula (20) with the parameter r=0.5.

TABLE 1. ITERATIONS NUMBER AND CPU TIME FOR FOUR STRATEGIES OF STRUCTURAL BASIS.

N	Control vector	Iterations number	Total CPU time (s)
1	(00)	6881	0.371
2	(01)	4832	0.232
3	(10)	7266	0.342
4	(11)	1102	0.057

We see that TDS with control vector (00) has more CPU time than other strategies. The best strategy is MTDS with the control vector (11) having a time gain of 6.5 times compared to TDS.

Projections of four trajectories corresponding to four strategies are shown in Fig. 2.

As we can see, all trajectories have the same start S and final F points, but they have very different iterations number and CPU time. It can be seen that all strategies are divided into two groups. The first group includes TDS with control vector (00) and strategy (10). To the second - MTDS, with a control vector (11) and a strategy (01).

The Lyapunov function dependence on the number of iterations is shown in Fig. 3.

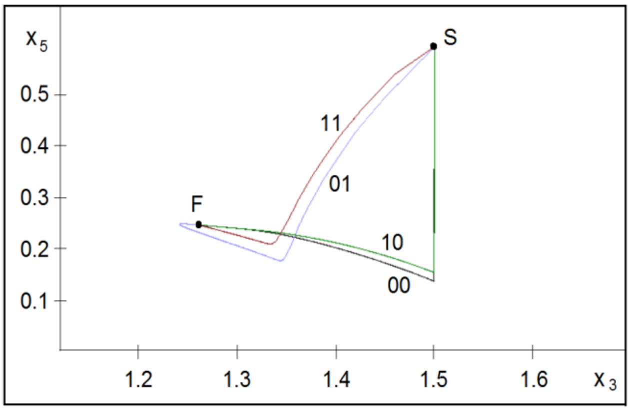

Fig. 2. *Projections $x_3 - x_5$ of trajectories with the control vector (00), (01), (10) and (11) in the phase space.*

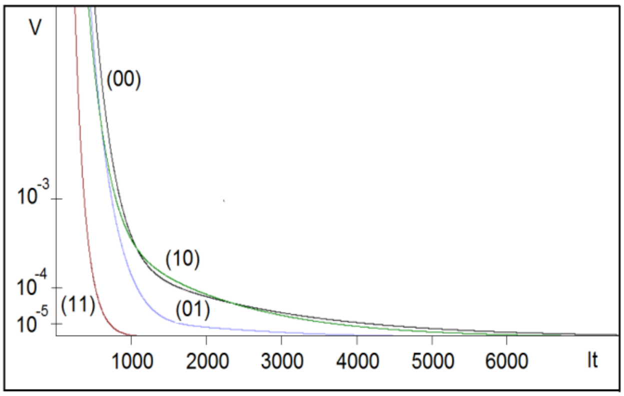

Fig. 3. *Dependence of the Lyapunov function for all strategies of the structural basis.*

It can be seen from this figure that the strategy with a higher Lyapunov function decrease rate has a smaller number of iterations. This idea can serve as the basis for the formation of some hypothesis regarding the relationship between the behaviour of the Lyapunov function of an arbitrary design strategy and the processor time corresponding to this strategy.

B. Example 2

The last example corresponds to the single-stage transistor amplifier in Fig. 4.

The conductivities y_1, y_2, y_3 are positive and compose the set of non-dependent parameters of the circuit (K=3). The Ebers-Moll static model of the transistor has been used. Nodal voltages V_1, V_2, V_3 for nodes 1, 2 and 3 are the dependent parameters (M=3). Let's define a vector of variables $X \in R^6$, including six components (x_1, x_2, x_3, x_4, x_5, x_6): $x_1^2=y_1$, $x_2^2=y_2$, $x_3^2=y_3$, $x_4=V_1$, $x_5=V_2$, $x_6=V_3$. A static Ebers-Moll model of transistor was used [27].

The objective function $C(X)$ of the optimization process was determined as the sum of the squares of the differences between the previously specified and current values of the nodal voltages and corresponds to formula (24):

$$C(X)=\sum_{i=1}^{M}\left(x_{K+i}-V_{i0}\right)^2, \qquad (24)$$

where V_{10},V_{20},V_{30} are the before-defined values of nodal voltages.

The circuit model is defined by Kirchhoff's laws as:

$$\begin{aligned} g_1(X) &\equiv I_B-(E_0-x_4)x_1^2=0\\ g_2(X) &\equiv I_E-x_2^2x_5=0 \qquad (25)\\ g_3(X) &\equiv I_C-(E_1-x_6)x_3^2=0 \end{aligned}$$

where I_B, I_E, I_C – are the base, emitter and collector currents, respectively. This system serves as a system of constraints for minimizing the objective function $C(X)$. The control vector U consists of three components, U=(u_1,u_2,u_3).

Using the generalized approach, we transform system (25) into system (26).

$$(1-u_j)g_j(X)=0,\ j=1,2,3. \qquad (26)$$

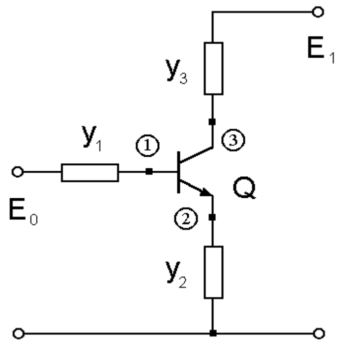

Fig. 4. *Single-stage transistor amplifier.*

The generalized objective function F is now defined as follows:

$$F(X,U)=C(X)+\frac{1}{\varepsilon}\sum_{j=1}^{3}u_j g_j^2(X) \qquad (27)$$

The results of the analysis for a full structural basis of the design strategies are shown in Table 2 and Table 3 for the precision ε=10^{-2} and ε=10^{-3} respectively.

Table 3 shows that there are three strategies (011), (101) and (111) that solve the design problem in significantly less computer time than the other five. The best strategy (111) gives a time gain of 32827 times compared to the TDS with the control vector (000). Strategies (011) and (101) have a gain in computer time of 2000 or more. This comparison was made with a rather low accuracy of solving the problem (ε=10^{-2}). In the case of increasing the accuracy of solving the problem to ε=10^{-3}, only these three strategies solve the problem, since other strategies do not allow achieving such accuracy of the solution, as can be seen from Table 4. Experiments were carried out with an increase in the possible number of iterations up to 10^8 (in this case, the computer time was more than an hour for each strategy), and yet the remaining five strategies did not reach the established accuracy of the solution ε=10^{-3}.

TABLE 2. DATA OF THE FULL STRUCTURAL BASIS OF OPTIMIZATION STRATEGIES FOR ε =10^{-2} FOR THE CIRCUIT SHOWN IN FIG.4.

N	Control vector	Iterations number	Total CPU time (s)
1	(000)	6026364	196.966
2	(001)	324881	12.952
3	(010)	441166	12.317
4	(011)	3092	0.099
5	(100)	648640	20.536
6	(101)	3168	0.089
7	(110)	470755	14.842
8	(111)	308	0.006

TABLE 3. DATA OF THE FULL STRUCTURAL BASIS OF OPTIMIZATION STRATEGIES FOR ε =10^{-3} FOR THE CIRCUIT SHOWN IN FIG.4.

N	Control vector	Iterations number	Total CPU time (sec)
1	(000)	-	-
2	(001)	-	-
3	(010)	-	-
4	(011)	3838	0.123
5	(100)	-	-
6	(101)	4197	0.118
7	(110)	-	-
8	(111)	383	0.007

Studies have shown that the ultimate accuracy of the solution for the strategy (011) is $7.7 \cdot 10^{-5}$ and is achieved in 89000 iterations. Two strategies (101) and (111) allow us to solve the problem with an accuracy of $1.1 \cdot 10^{-8}$. In this case the number of iterations for strategy (101) is 2203105, and for strategy (111) it is 220533. Apparently, the results for the other five possible strategies are related to the problem of getting into local minima of the objective function, from which these strategies are not able to get out. At the same time, strategies (101) and (111) make it possible to find the global minimum of the objective function.

Projections of eight trajectories corresponding to the design strategies are shown in Fig. 5. All strategies have the same initial S and final F points. Again, we can state that all strategies are divided into two groups. The first group includes CDS with control vector (000) and strategies (010), (100) and (110). To the second - MTDS, with the control vector (111) and strategies (001), (011) and (101). It can also be seen that the strategies of the first group have a much shorter execution time and higher accuracy than the strategies of the second group.

The behavior of the functions *V(t)* during the design process is shown in Fig. 6.

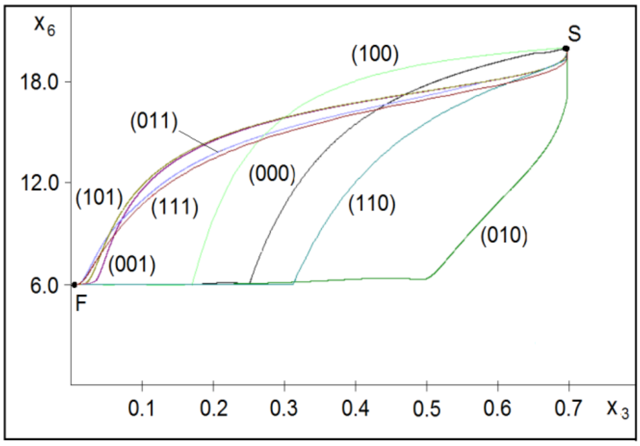

Fig. 5. *Projections $x_3 - x_6$ of trajectories in the phase space for eight strategies.*

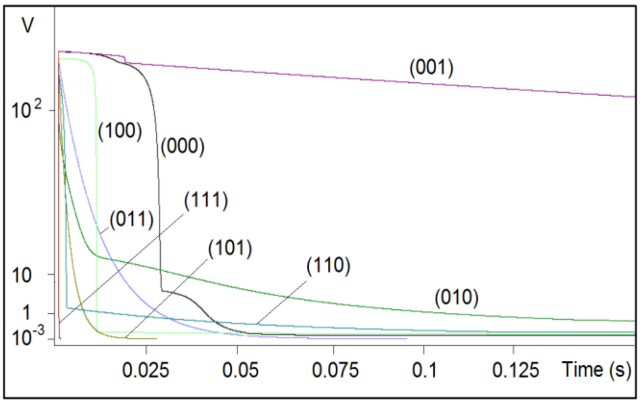

Fig. 6. *Dependence of the Lyapunov function for all design strategies of the structural basis.*

We see that the TDS with the control vector (000) has the largest processor time of all strategies of the structural basis and, at the same time, the minimum rate of decrease of the Lyapunov function. The Lyapunov function was calculated by formula (20) for *r*=0.5. As we can see from Fig. 9 the function *V(t)* give an exhaustive explanation for the design process characteristics. First of all we can conclude that the speed of decreasing of the Lyapunov function is inversely proportional to the design time.

We see that all the curves are very well ordered both in terms of the time required to design the circuit and in terms of the rate of decrease of the Lyapunov function. There is a correlation between the behaviour of the Lyapunov function and computing time. A strategy that has a shorter execution time, at the same time, has a faster decrease in the function *V(t)*. We can analyze the behaviour of the function *V(t)* for the initial time interval by parallel computing for different strategies, and based on this analysis, we can predict the strategies that have the minimum total machine time to develop.

V. Conclusion

The problem of the minimal-time design algorithm construction can be solved adequately on the basis of the control theory. The design process in this case is formulated as the controllable dynamic system. The Lyapunov function and its time derivative include the sufficient information to select more perspective design strategies from infinite set of the different design strategies that exist into the general design methodology. The Lyapunov function *V(t)* was proposed to compare the different design strategies and to predict the strategy that has a minimal design time. The successful solution of this problem allows choosing promising strategies for designing electronic circuits with minimal computing time.

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