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COMPARISON OF BLENDED REGULA FALSI–BISECTION METHOD USING ARITHMETIC MEAN AND HARMONIC MEAN FOR THE DETERMINATION OF ORBITAL ECCENTRICITY ANOMALY

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Abstract— **In this paper, comparison of blended Regula Falsi–bisection method using arithmetic mean and harmonic mean for the determination of orbital eccentricity anomaly was presented. Notably, in the classical bisection algorithm, the next root of a function is estimated using the arithmetic mean of the lower and the upper guess roots. In this paper, the classical bisection method is blended with the Regula Falsi iteration method. Also, another version of the bisection algorithm that uses harmonic mean of the lower and the upper guess roots is blended with the Rgula Falsi iteration method. The blended algorithm with harmonic mean and with arithmetic mean was implemented in Matlab software. For the blended Regula Falsi–Bisection method using arithmetic mean where eccentricity, e=0.999 , mean anomaly, M (°) = 7 and error tolerance, ϵ is of the order of 10^{-12} , the algorithm converged at the 12th iteration with actual value of eccentricity anomaly, E = 0.912288165 radians (or 52.26 degrees). Similar iteration with the blended Regula Falsi–Bisection method using harmonic converged at the 13th iteration . Also, for the blended Regula Falsi–Bisection method using arithmetic mean where eccentricity, e=0.5 , mean anomaly, M (°) = 7 and error tolerance, ϵ is of the order of 10^{-12} , the algorithm converged at the 9th iteration with actual value of eccentricity anomaly, 0.241991 radians (or 13.86328 degrees).Similar iteration with the blended Regula Falsi–Bisection method using harmonic converged at the 9th iteration with error tolerance of the order of 10^{-13}. In all, the blended Regula Falsi–Bisection method has almost the same convergence performance in both cases where arithmetic mean is used and where the harmonic mean is used.**

Keywords— Regula Falsi Method, Harmonic Mean Bisection Method, Arithmetic Mean, Blended Regula Falsi–Bisection Method, Eccentricity Anomaly, Mean Anomaly, Eccentricity

1. INTRODUCTION

There are numerous numerical iteration schemes in solving different complex equations, each with its features; strength and weaknesses. Among the popular iteration schemes are Newton Raphson method, bisection method, secant method, Regular Falsi method and fixed point iteration method [1,2,3,4,5,6,7,8,9,10]. Over the years, efforts are still made by researchers to assess the efficiency of the iteration schemes and also propose alternative approaches that can be used to improve on the efficiency of the schemes. Among the various iteration schemes, the Regula Falsi and bisection methods are related in that both are bracketing schemes that require that the two guess roots should be such that one of the roots is above the actual root while the second guess root is below the actual root [3,11,12,13,14]. As such, it is quite easy to blend the two iteration schemes into a hybrid scheme with higher convergence efficiency.

Notably, in this paper, two forms of blended Regula Falsi–bisection method are presented for the determination of orbital eccentricity anomaly [16,17,18,19,20,21,22]. Specifically, blended Regula Falsi–bisection method [3] based on arithmetic mean and another blended Regula Falsi–bisection method based harmonic mean are presented. The study utilized the known lower and upper bounds on the solution of circular and elliptical orbital eccentricity anomaly (E) whereby for any given mean anomaly (M) and orbital eccentricity (e), the expected orbital eccentricity anomaly (E) for circular and elliptical orbit is bounded as follows; $M \leq E \leq M + e$ [15,16]. The relevant mathematical expressions and detailed procedure for the blended Regula Falsi–bisection method are presented for both cases of arithmetic mean and harmonic mean. The two approaches are implemented using Matlab software and the results are presented and discussed.

2 METHODOLOGY

The Regular Falsi algorithm adapted for the computation of the eccentricity anomaly (E) is presented in this section. The input parameters required for the Regular Falsi algorithm are the mean anomaly (M) , the eccentricity (e) and the error tolerance (ϵ). The algorithm of blended Regula Falsi-Bisection Methods using harmonic mean is presented as follows:

Step 1: Input $M, e,$ and ϵ

Step 2: k = 0

Step 3:

$$a_{L(k)} = M \tag{1}$$

Step 4:

$$a_{u(k)} = M + e \tag{2}$$

Step 5:

$$f\left(a_{L(k)}\right) = a_{L(k)} - M - e\left(\sin\left(a_{L(k)}\right)\right) \tag{3}$$

Step 6:

$$f\left(a_{u(k)}\right) = a_{u(k)} - M - e\left(\sin\left(a_{u(k)}\right)\right) \tag{4}$$

Step 7:

$$E_{(k)} = \frac{a_{L(k)}[f(a_{u(k)})] - a_{u(k)}[f(a_{L(k)})]}{f(a_{u(k)}) - f(a_{L(k)})} \quad (5)$$

Step 8:

$$f(E_{(k)}) = E_{(k)} - M - e(\sin(E_{(k)})) \quad (6)$$

Step 9:

$If\ f(E_{(k)}) < \epsilon$ Then Goto Step 16

Step 10:

$If\ f(a_{L(k)}) * f(E_{(k)}) < 0$ Then

$$b_{L(k)} = a_{L(k)} \quad (7)$$

$$b_{u(k)} = E_{(k)} \quad (8)$$

Else

$$b_{L(k)} = E_{(k)} \quad (9)$$

$$b_{u(k)} = a_{u(k)} \quad (10)$$

EndIf

Step 11: Harmonic Mean of $b_{L(k)}$ and $b_{u(k)}$

$$M_{H(k)} = \frac{2(b_{L(k)})(b_{u(k)})}{b_{L(k)} + b_{u(k)}} \quad (11)$$

Step 12:

$$f(b_{L(k)}) = b_{L(k)} - M - e(\sin(b_{L(k)})) \quad (12)$$

$$f(b_{u(k)}) = b_{u(k)} - M - e(\sin(b_{u(k)})) \quad (13)$$

$$f(M_{H(k)}) = M_{H(k)} - M - e(\sin(M_{H(k)})) \quad (14)$$

Step 13:

$If\ f(b_{L(k)}) * f(M_{H(k)}) < 0$ Then

$$a_{L(k+1)} = b_{L(k)} \quad (15)$$

$$a_{u(k+1)} = M_{H(k)} \quad (16)$$

Else

$$a_{L(k+1)} = M_{H(k)} \quad (17)$$

$$a_{u(k+1)} = b_{u(k)} \quad (18)$$

EndIf

Step 14:

K = k+1 (19)

Step 15:

Goto Step 5

Step 16:

Output k, $E_{(k)}$

Step 17:

Stop

The algorithm of blended Regula Falsi-Bisection Methods using arithmetic mean is presented as follows:

Step 1: Input $M, e,$ and ϵ

Step 2: k = 0

Step 3:

$a_{L(k)}$ **= M**

Step 4:

$a_{u(k)}$ **= M + e**

Step 5:

$$f(a_{L(k)}) = a_{L(k)} - M - e(\sin(a_{L(k)}))$$

Step 6:

$$f(a_{u(k)}) = a_{u(k)} - M - e(\sin(a_{u(k)}))$$

Step 7:

$$E_{(k)} = \frac{a_{L(k)}[f(a_{u(k)})] - a_{u(k)}[f(a_{L(k)})]}{f(a_{u(k)}) - f(a_{L(k)})}$$

Step 8:

$$f(E_{(k)}) = E_{(k)} - M - e(\sin(E_{(k)}))$$

Step 9:

$If\ f(E_{(k)}) < \epsilon$ Then Goto Step 16

Step 10:

$If\ f(a_{L(k)}) * f(E_{(k)}) < 0$ Then

$$b_{L(k)} = a_{L(k)}$$

$$b_{u(k)} = E_{(k)}$$

Else

$$b_{L(k)} = E_{(k)}$$

$$b_{u(k)} = a_{u(k)}$$

EndIf

Step 11: Arithmetic Mean of $b_{L(k)}$ and $b_{u(k)}$

$$M_{A(k)} = \frac{b_{L(k)} + b_{u(k)}}{2} \quad (20)$$

Step 12:

$$f(b_{L(k)}) = b_{L(k)} - M - e(\sin(b_{L(k)}))$$

$$f(b_{u(k)}) = b_{u(k)} - M - e(\sin(b_{u(k)}))$$

$$f(M_{A(k)}) = M_{A(k)} - M - e(\sin(M_{A(k)})) \quad (21)$$

Step 13:

$If\ f(b_{L(k)}) * f(M_{A(k)}) < 0$ Then

$$a_{L(k+1)} = b_{L(k)}$$

$$a_{u(k+1)} = M_{A(k)} \quad (22)$$

Else

$$a_{L(k+1)} = M_{A(k)} \quad (23)$$

$$a_{u(k+1)} = b_{u(k)}$$

EndIf

Step 14:

K = k+1

Step 15:

Goto Step 5

Step 16:

Output k, $E_{(k)}$

Step 17:

Stop

3 RESULTS AND DISCUSSION

Matlab program was used to implement the two algorithms for different values of eccentricity and mean anomaly. The results are shown in Table 1 for the blended Regula Falsi–Bisection methods using arithmetic mean where e=0.999 , M (°) = 7 and ϵ of the order of 10^{-12} . According to the results in Table 1, about 12 iterations were required to attain the error tolerance of the order of 10^{-12}; at the convergence cycle, the actual value of E is 0.912288165 radians (or 52.26 degrees).

The results are shown in Table 2 for the blended Regula Falsi–Bisection methods using harmonic mean where e=0.999 , M (°) = 7 and ϵ of the order of 10^{-12} . According to the results in Table 2, about 13 iterations were required to attain the error tolerance of the order of 10^{-12}; at the convergence cycle, the actual value of E is 0.912288 radians (or 52.26 degrees).

The results are shown in Table 3 for the blended Regula Falsi–Bisection methods using arithmetic mean where e= 0.5 , M (°) = 7 and ϵ of the order of 10^{-12} . According to the results in Table 3, about 9 iterations were required to attain the error tolerance of the order of 10^{-12}; at the convergence cycle, the actual value of E is 0.241991 radians (or 13.86328 degrees).

The results are shown in Table 4 for the blended Regula Falsi–Bisection methods using harmonic mean where e=0.5 , M (°) = 7 and ϵ of the order of 10^{-12} . According to the results in Table 4, about 9 iterations were required to attain the error tolerance of the order of 10^{-13} ; at the convergence cycle, the actual value of E is 0.241991 radians (or 13.86328 degrees).

In all, the blended Regula Falsi–Bisection methods has almost the same convergence performance in both cases where arithmetic mean is used and where the harmonic mean is used.

Table 1 Results of Blended Regula Falsi–Bisection Methods using arithmetic mean where e= 0.999 and M = 7° .

Cycle (k)	$a_{L(k)}$ (radians)	$a_{u(k)}$ (radians)	$E_{(k)}$ (radians)	Harmonic Mean (radians)	$E_{(k)}$ (degree)	F($E_{(k)}$)
0	0.122173	1.121173	0.672423	0.896798	38.52201	-7.20E-02
1	0.896798	1.121173	0.909436	1.015304	52.10008	-1.11E-03
2	0.909436	1.121173	0.911767	1.01647	52.23364	-2.02E-04
3	0.911767	1.121173	0.912193	1.016683	52.25804	-3.69E-05
4	0.912193	1.121173	0.912271	1.016722	52.26249	-6.73E-06
5	0.912271	1.121173	0.912285	1.016729	52.2633	-1.23E-06
6	0.912285	1.121173	0.912288	1.01673	52.26345	-2.24E-07
7	0.912288	1.121173	0.912288	1.016731	52.26348	-4.08E-08
8	0.912288	1.121173	0.912288	1.016731	52.26348	-7.44E-09
9	0.912288	1.121173	0.912288	1.016731	52.26348	-1.36E-09
10	0.912288	1.121173	0.912288	1.016731	52.26348	-2.47E-10
11	0.912288	1.121173	0.912288	1.016731	52.26348	-4.51E-11
12	0.912288	1.121173	0.912288	1.016731	52.26348	-8.22E-12
13	0.912288	1.121173	0.912288	1.016731	52.26348	-1.50E-12

Table 2 Results of Blended Regula Falsi–Bisection Methods using harmonic mean where e = 0.999 and M = 7° .

Cycle (k)	$a_{L(k)}$ (radians)	$a_{u(k)}$ (radians)	$E_{(k)}$ (radians)	Harmonic Mean (radians)	$E_{(k)}$ (degree)	F($E_{(k)}$)
0	0.122173	1.121173	0.672423	0.840660444	38.52201	-7.20E-02
1	0.84066	1.121173	0.898608	0.997628004	51.47977	-5.24E-03
2	0.898608	1.121173	0.909772	1.004470168	52.11934	-9.75E-04
3	0.909772	1.121173	0.911829	1.005722464	52.23716	-1.78E-04
4	0.911829	1.121173	0.912204	1.005950934	52.25868	-3.26E-05
5	0.912204	1.121173	0.912273	1.005992592	52.26261	-5.94E-06
6	0.912273	1.121173	0.912285	1.006000187	52.26333	-1.08E-06
7	0.912285	1.121173	0.912288	1.006001572	52.26346	-1.97E-07
8	0.912288	1.121173	0.912288	1.006001824	52.26348	-3.60E-08
9	0.912288	1.121173	0.912288	1.006001870	52.26348	-6.56E-09
10	0.912288	1.121173	0.912288	1.006001879	52.26348	-1.20E-09
11	0.912288	1.121173	0.912288	1.00600188	52.26348	-2.18E-10
12	0.912288	1.121173	0.912288	1.006001881	52.26348	-3.97E-11
13	0.912288	1.121173	0.912288	1.006001881	52.26348	-7.25E-12

Table 3 Results of Blended Regula Falsi–Bisection Methods using arithmetic mean where e= 0.5 and M = 7°.

Cycle (k)	$a_{L(k)}$ (radians)	$a_{u(k)}$ (radians)	$E_{(k)}$ (radians)	Arithmetic Mean (radians)	$E_{(k)}$ (degree)	F($E(k)$)
0	0.122173	0.622173	0.23521	0.428692	13.47481	-3.49E-03
1	0.428692	0.622173	0.255	0.438586	14.60851	6.70E-03
2	0.255	0.622173	0.242807	0.43249	13.91003	4.20E-04
3	0.242807	0.622173	0.242042	0.432108	13.86618	2.61E-05
4	0.242042	0.622173	0.241994	0.432084	13.86346	1.62E-06
5	0.241994	0.622173	0.241991	0.432082	13.86329	1.01E-07
6	0.241991	0.622173	0.241991	0.432082	13.86328	6.28E-09
7	0.241991	0.622173	0.241991	0.432082	13.86328	3.90E-10
8	0.241991	0.622173	0.241991	0.432082	13.86328	2.43E-11
9	0.241991	0.622173	0.241991	0.432082	13.86328	1.51E-12
10	0.241991	0.622173	0.241991	0.432082	13.86328	9.38E-14

Table 4 Results of Blended Regula Falsi–Bisection Methods using harmonic mean where e = 0.5 and M =

Cycle (k)	$a_{L(k)}$ (radians)	$a_{u(k)}$ (radians)	$E_{(k)}$ (radians)	Harmonic Mean (radians)	$E_{(k)}$ (degree)	f($E_{(k)}$)
0	0.122173	0.622173	0.23521	0.34136779	13.47481	-3.49E-03
1	0.341368	0.622173	0.248585	0.355237668	14.24104	3.40E-03
2	0.248585	0.622173	0.242403	0.348879996	13.88687	2.12E-04
3	0.242403	0.622173	0.242017	0.348479741	13.86474	1.32E-05
4	0.242017	0.622173	0.241993	0.348454837	13.86337	8.20E-07
5	0.241993	0.622173	0.241991	0.348453288	13.86328	5.10E-08
6	0.241991	0.622173	0.241991	0.348453192	13.86328	3.17E-09
7	0.241991	0.622173	0.241991	0.348453186	13.86328	1.97E-10
8	0.241991	0.622173	0.241991	0.348453186	13.86328	1.22E-11
9	0.241991	0.622173	0.241991	0.348453186	13.86328	7.61E-13
10	0.241991	0.622173	0.241991	0.348453186	13.86328	4.73E-14

4. CONCLUSION

A modified version of Regular Falsi numerical iteration method is presented where the classical Regula Falsi is blended with the bisection iteration algorithm. Furthermore, in the classical bisection algorithm, the next root estimate of a function is determined from the arithmetic mean of the lower and the upper guess roots. In this paper, another version of the bisection method that uses harmonic mean instead of arithmetic mean is presented. Some numerical simulation examples were performed in Mtalab software. In all, the blended Regula Falsi–Bisection methods have almost the same convergence performance in both cases where arithmetic mean is used and where the harmonic mean is used.

REFERENCES

1. Azure, I., Aloliga, G., & Doabil, L. (2019). Comparative study of numerical methods for solving non-linear equations using manual computation. *Math. Lett*, *5*(4), 41-46.
2. Sahni, M., Shah, D., & Sahni, R. (2019). A new modified accelerated iterative scheme using amalgamation of fixed point and NR method. *Journal of Interdisciplinary Mathematics*, *22*(5), 679-688.
3. Sabharwal, C. L. (2019). Blended Root Finding Algorithm Outperforms Bisection and Regula Falsi Algorithms. *Mathematics*, *7*(11), 1118.
4. Sharma, S. K. (2017). A Comparative Analysis of Rate of Convergence For Linear And Quadratic Approximations in NR Method. *World Journal of Research and Review (WJRR) ISSN*, 2455-3956.
5. Martín-Caraballo, A. M., & Tenorio-Villalón, Á. F. (2015). Teaching numerical methods for non-linear equations with Geogebra-based activities. *International Electronic Journal of Mathematics Education*, *10*(2), 53-65.
6. Jamil, N. (2013). A comparison of iterative methods for the solution of non-linear systems of equations. *Int J Emerg Sci*, *3*(2), 119-130.
7. Bhuyan, M. H. (2012). Comparative Study of Different Root Location Methods Using MATLAB Program. *Green University Review*, *3*(2), 30-35.
8. Chun, S., & Kwasinski, A. (2011). Analysis of classical root-finding methods applied to digital maximum power point tracking for

sustainable photovoltaic energy generation. *IEEE transactions on power electronics*, *26*(12), 3730-3743.
9. Kashmar, A. H. (2018). Numerical Analysis Course.
10. Herceg, D., & Herceg, Đ. (2008). Numerical mathematics with GeoGebra in high school. *Teaching Mathematics and Computer Science*, *6*(2), 363-378.
11. Sulaiman, I. M., Mamat, M., Waziri, M. Y., Fadhilah, A., & Kamfa, K. U. (2016). Regula Falsi method for solving fuzzy nonlinear equation. *Far East Journal of Mathematical Sciences*, *100*(6), 873.
12. Sulaiman, I. M., Mamat, M., Waziri, M. Y., Fadhilah, A., & Kamfa, K. U. (2016). Regula Falsi method for solving fuzzy nonlinear equation. *Far East Journal of Mathematical Sciences*, *100*(6), 873.
13. Ehiwario, J. C., & Aghamie, S. O. (2014). Comparative study of bisection, Newton-Raphson and secant methods of root-finding problems. *IOSR Journal of Engineering*, *4*(04), 01-07.
14. Hager, W. W. (1989). A derivative-based bracketing scheme for univariate minimization and the conjugate gradient method. *Computers & Mathematics with Applications*, *18*(9), 779-795.
15. R.E. Deakin (2017) Solutions of Kepler's Equation Bonbeach VIC, 3196, Australia, December 2017
16. Smith, G. R. (1979). A simple, efficient starting value for the iterative solution of Kepler's equation. *Celestial mechanics*, *19*, 163-166.
17. Tokis, J. N. (2014). A Solution of Kepler's Equation. *International Journal of Astronomy and Astrophysics*, *4*(04), 683.
18. Danby, J. M. A., & Burkardt, T. M. (1983). The solution of Kepler's equation, I. *Celestial Mechanics*, *31*(2), 95-107.
19. Odell, A. W., & Gooding, R. H. (1986). Procedures for solving Kepler's equation. *Celestial mechanics*, *38*(4), 307-334.
20. Davis, J. J., Mortari, D., & Bruccoleri, C. (2010). Sequential solution to Kepler's equation. *Celestial Mechanics and Dynamical Astronomy*, *108*(1), 59-72.
21. Palacios, M. (2002). Kepler equation and accelerated Newton method. *Journal of Computational and Applied Mathematics*, *138*(2), 335-346.
22. Fukushima, T. (1996). A method solving Kepler's equation without transcendental function evaluations. *Celestial Mechanics and Dynamical Astronomy*, *66*(3), 309-319.