Journal of Multidisciplinary Engineering Science and Technology (JMEST)
ISSN: 2458-9403
Vol. 8 Issue 5, May - 2021

# The Azimuth-only Estimation of the Course of a Uniform Straight Line Target By Means of the Tangent Median Relationship

**Tao YU**
China Academy of Management Science
Beijing, China
tyt0803@163.com

***Abstract*—In two-dimensional plane, for a target moving uniformly in a straight line, if a single stationary station can obtain the azimuth information of the target three times in a row at equal spacing in the moving path of the target, the flight trajectory of the target can be regarded as the baseline of the one-dimensional double-base array. Therefore, by using the relationship among the course angle, the lead angle and the arrival angle, the definite solution of the target course angle can be obtained from the median tangent relationship.**

***Keywords—course angle; single fixed station; DF; mobile radiation source; passive location;***

## I. INTRODUCTION

The estimation of course angle has a wide range of applications[1-5]. The theory of azimuth-only target motion analysis proves that a stationary single station can estimate the course of a moving target with uniform speed by at least three azimuth-only measurements on a two-dimensional plane[6-9]. However, in fact, the existing analysis process contains time parameters, so the information required to obtain does not seem to be purely directional.

The author focuses on the azimuth-only estimation of the course angle of a moving target with a fixed single station. Existing research results show that in the plane rectangular coordinate system, assumed that a single stationary station can obtain the azimuth of the target with equal spacing in the moving path of the target by only three successive direction finding, the analytical expression of target course angle can be derived by using the equation of the moving trajectory line of the target and the azimuth line equation between the station and the target in the case of has nothing to do with the detection of time parameters[10].

In this paper, the azimuth-only estimation of target course is studied again by using the tangent median relation among three arrival angles of a one-dimensional double-base array.

## II. TANGENT MEDIAN RELATION

For the one-dimensional equidistant double-base array shown in Fig.1, the following identity can be obtained by projecting the radial distances of the three stations onto the X and Y axes respectively

$$r_2 \sin\theta_2 = r_1 \sin\theta_1 - d \tag{1}$$

$$r_2 \sin\theta_2 = r_3 \sin\theta_3 + d \tag{2}$$

$$r_2 \cos\theta_2 = r_1 \cos\theta_1 \tag{3}$$

$$r_2 \cos\theta_2 = r_3 \cos\theta_3 \tag{4}$$

If we add Equations (1) and (2), we can get

$$2r_2 \sin\theta_2 = r_1 \sin\theta_1 + r_3 \sin\theta_3 \tag{5}$$

Then substitute Equations (3) and (4) into the above equation to eliminate radial distances $r_1$ and $r_3$, and get

$$2r_2 \sin\theta_2 = \frac{r_2 \sin\theta_1 \cos\theta_2}{\cos\theta_1} + \frac{r_2 \sin\theta_3 \cos\theta_2}{\cos\theta_3} \tag{6}$$

Thus it can be proved that there exists the following tangential median relation among the arrival angles of the three stations

$$2tg\theta_2 = tg\theta_1 + tg\theta_3 \tag{7}$$

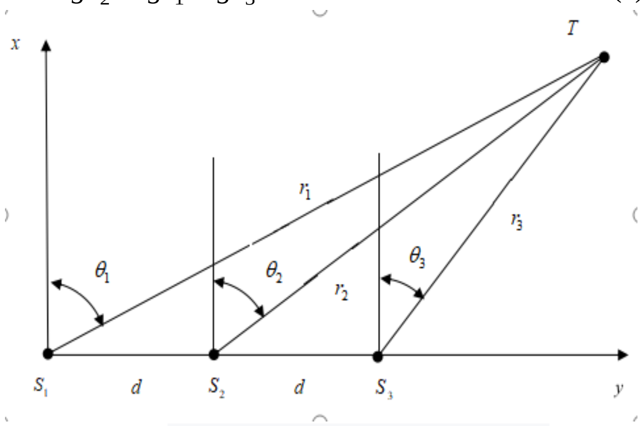


Fig. 1. *One-dimensional double-base array*

## III. EQUATIONS FOR SOLUTIONS

As shown in Fig. 2, a fixed single station is taken as the origin of the Cartesian coordinate system. It is assumed that the target moves at uniform speed along the line AB, and the stationary single station is assumed to be able to obtain the azimuth information of the target three times in a row with equal spacing on the target's moving path. At this point, the flight path of the target from position 1, through position 2 and to position 3 can be regarded as the baseline of the one-dimensional double-base array. And by the trigonometric relationship, we have

$$\theta_i = 90^0 - \beta_i \quad (\beta_i < 90^0) \quad (i = 1-3) \tag{8}$$

$$\theta_i = \beta_i - 90^0 \quad (\beta_i > 90^0) \quad (i = 1-3) \tag{9}$$

$$\alpha = \varphi_i + \beta_i \quad (i = 1-3) \tag{10}$$

Where : $\theta$ is the arrival angle relative to the fixed detection station $S$ ; $\beta$ the lead angle of the target; $\alpha$ the course angle of the target; $\varphi$ the azimuth angle of the target measured by the fixed detection station.

By substituting Equation (10) into Equation (8) or Equation (9), it can be obtained

$$\theta_i = 90^0 - \alpha + \varphi_i \quad (\beta_i < 90^0) \tag{11}$$

$$\theta_i = \alpha - \varphi_i - 90^0 \quad (\beta_i > 90^0) \tag{12}$$

Substitute Equation (11) or (12) into Equation (7), the equation containing only unknown course angle can be obtained

$$\begin{aligned} &2tg\left(90^0 - \alpha + \varphi_2\right) \\ &= tg\left(90^0 - \alpha + \varphi_1\right) + tg\left(90^0 - \alpha + \varphi_3\right) \quad (\beta_i < 90^0) \end{aligned} \tag{14}$$

$$\begin{aligned} &2tg\left(\alpha - \varphi_2 - 90^0\right) \\ &= tg\left(\alpha - \varphi_1 - 90^0\right) + tg\left(\alpha - \varphi_3 - 90^0\right) \quad (\beta_i > 90^0) \end{aligned} \tag{15}$$

Because for any angle $\phi$

$$tg(-\phi) = -tg\phi$$

Therefore, in actual calculation, Equation (14) or (15) can only be used.

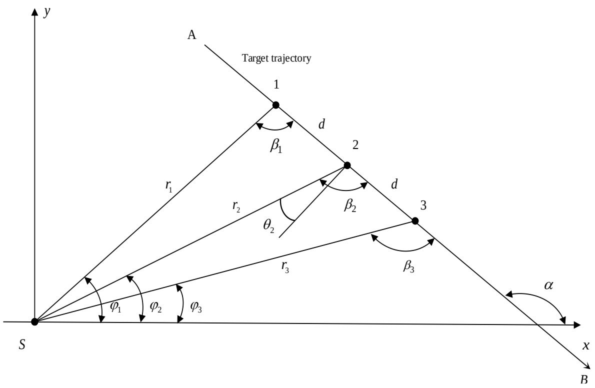


Fig. 2. *Static single station azimuth-only tracking diagram*

## IV. ANALOG CALCULATION

The equations are solved directly by using the fsolve() function of MATLAB.

In order to verify the accuracy of the course angle formula, the theoretical value is used to replace the measured value to carry out the mathematical simulation. Preestablish the distance $r_1$ between the target and the detection station, the azimuth angle $\varphi_1$, the moving distance $d$ of the target. And make the lead angle of the target end change continuously in the specified area $\left(90^0 - \varphi_1 + 5^0 < \beta < 180^0 - \varphi_1\right)$.

Then, from the trigonometric relationship, the remaining radial distances $r_2$ and $r_3$ , and the azimuthal angles $\varphi_2$ and $\varphi_3$ can be solved in turn. The leading angle $\beta_2$ and $\beta_3$ at the target are calculated from the relationship between internal and external angles, and the theoretical value of the course angle $\alpha$ of the target is obtained directly. Then, the Matlab fsolve() function is directly used to solve the equation (14/15), and the course angle of the target can be solved. Then, the relative calculation error is obtained by comparing with the corresponding theoretical value. The parameter values taken are : (1) The equidistant moving distance of the target: $d = 10km$; (2) Radial distance of the detection station: $r_1 = 300km$; (3) Initial azimuth of the station: $\varphi_1 = 75^0$.

Fig.3. shows the relative calculation error of course angle. The simulation results show that: within the limited area : $\left[90^0 - \varphi_1 + 5^0 < \beta < 180^0 - \varphi_1\right]$, the derived course angle calculation formula is correct. By increasing or decreasing the moving distance of the target, radial distance and initial azimuth, the effect on the relative calculation error is very limited.

At present, it seems difficult to give the correct value of the course angle outside the restricted area.

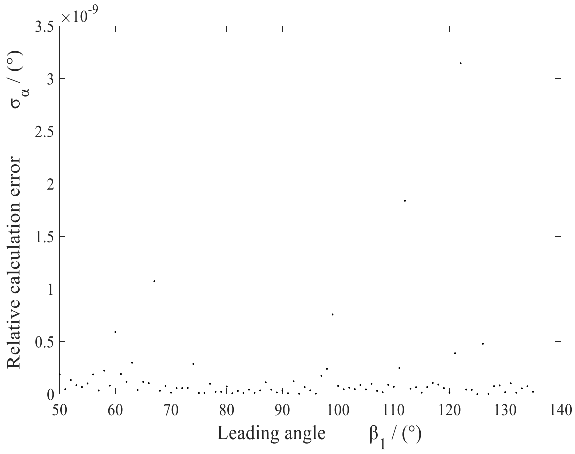


Fig. 3. *Relative calculation error of course angle*

## V. ERROR ANALYSIS

According to error estimation and synthesis theory, the total measurement error of course angle generated by direction finding is

$$\sigma = \sigma_\varphi \sqrt{\sum_{i=1}^{3} \left(\frac{\partial \alpha}{\partial \varphi_i}\right)^2} \tag{16}$$

Where: $\sigma_\varphi$ is the root mean square value of the direction finding angle error of the detection station.

By differentiating $\varphi_i$ on both sides of Equation (15), we obtain

$$\frac{\partial\alpha}{\partial\varphi_1}=\frac{\sec^2\left(90^0-\alpha+\varphi_1\right)}{\left[\sec^2\left(90^0-\alpha+\varphi_1\right)+\sec^2\left(90^0-\alpha+\varphi_3\right)-2\sec^2\left(90^0-\alpha+\varphi_2\right)\right]}$$

$$\frac{\partial\alpha}{\partial\varphi_2}=\frac{2\sec^2\left(90^0-\alpha+\varphi_2\right)}{\left[2\sec^2\left(90^0-\alpha+\varphi_2\right)-\sec^2\left(90^0-\alpha+\varphi_1\right)-\sec^2\left(90^0-\alpha+\varphi_3\right)\right]}$$

$$\frac{\partial\alpha}{\partial\varphi_3}=\frac{\sec^2\left(90^0-\alpha+\varphi_3\right)}{\left[\sec^2\left(90^0-\alpha+\varphi_1\right)+\sec^2\left(90^0-\alpha+\varphi_3\right)-2\sec^2\left(90^0-\alpha+\varphi_2\right)\right]}$$

Because for any angle $\phi$, there is

$$\sec(-\phi)=\sec\phi$$

Therefore, the partial differential result obtained from Equation (15) is also applicable to Equation (14).

Using the same parameter value as in the simulation calculation, take the root mean square value of the direction finding angle error of the detection station as $\sigma_\varphi=0.5^0\pi/180^0$.

Figure 4 shows the measurement errors of course angle at different starting azimuth angles. Figure 5 shows the measurement errors of course angle at different moving distances. Figure 6 shows the measurement errors of course angle at different radial distances.

Additional note: in order to make the drawing process simple, the theoretical value of course angle is directly used to replace the solution value in the calculation process of measurement error, which will not have a great impact on the expression of error curve.

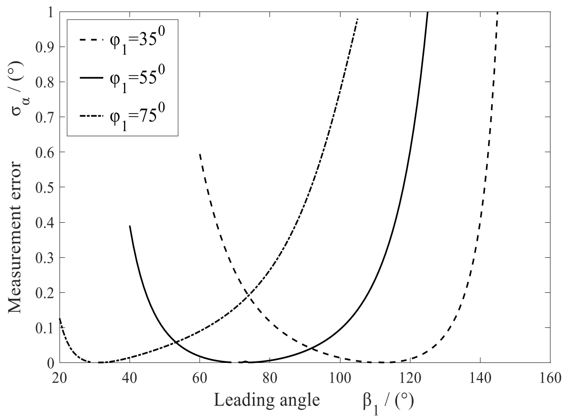


Fig. 4. *Measurement error of course angle at different starting azimuth angles*

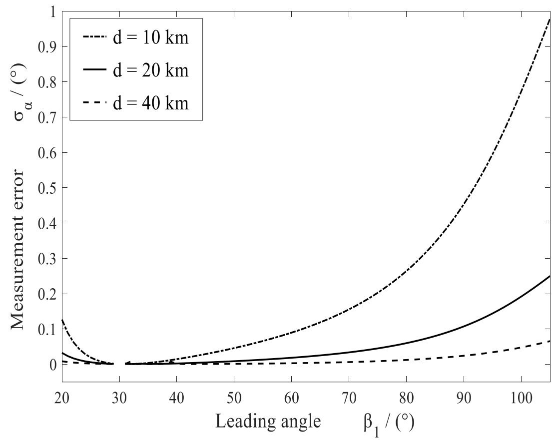


Fig. 5. *Measurement error of course angle at different moving distances*

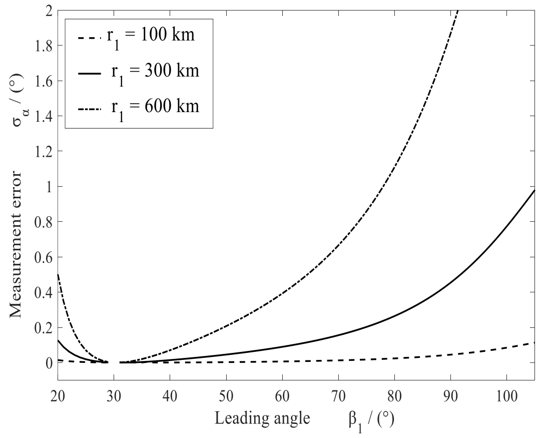


Fig. 6. *Measurement error of course angle at different radial distances*

## VI. Conclusion

In this paper, the tangent median relation is used to study the problem of determining the direction of motion of a moving target only by single station direction finding. Finally, the analytical formula is purely related to the azimuth measurement angle. However, when the target moving speed is unknown, it is still difficult to accurately obtain the target azimuth in the equidistant distance of the target moving path, so the current research results may not be applicable to engineering.

## References

[1] XIONG Lu,LU Yishi,XIA Xin,GAO Letian,YU Zhuoping. Heading Angle Estimation of Low-Speed Automated Electric Vehicle Based on Loosely Coupled Global Navigation Satellite System/Inertial Measurement Unit Integration[J]. Journal of Tongji University(Natural Science),2020, 48(4):545-551

[2]CHENG Jian-hua,WANG Nuo,SHANG Xiu-neng. Research on Course Angle Estimation Method of Integrated Navigation System Based on Improved UKF[J].Navigation Positioning and Timing,2020,7(3):112-119

[3]LIU Yong,CHEN Ben-jun,CHENG Yue-bing,LI Shao-long. Base on IMU module heading angle error correction algorithm[J].Electronic Design Engineering,2019, 27(2):80-84

[4]ZHANG Zhigang,ZHU Qiming,HE Jie,WANG Hui,YUE Binbin,DING Fan. Vehicle heading angle measure technology based on RTK-GNSS and MEMS gyroscope[J].Journal of South China Agricultural University,2019, 40(5):34-37

[5]PAN Chang. Method on Heading Angle Based on Based on Bessel's Formula for Inverse Solution of Geodetic Problem[J].Ship Electronic Engineering, 2019, 39(6):61-62,137

[6]Liu Zhong, Zhou Feng, etc. Bearings-only Target Motion Analysis[M]. Beijing: National Defense Industry Press, 2009.

[7]SHI Zhang-song,LIU Zhong.The Analysis of the Observability on the Single Platform Bearings-only Target Tracking System[J]. FIRE CONTROL & COMMAND CONTROL,2007,32(2):26-29,33.

[8]TANG Koulin,Liu Yun,Zhao Chundong. Study on Location Method of Single Station Azimuth Measurement[J].FIRE CONTROL AND COMMAND CONTROL,2009,34(12):112-116.

[9]Yu Tao. A measuring method of course angle for mobile target with fixed station[J].Aerospace Electronic Warfare,2011, 27(6):49-50,61

[10]YU Tao. Bearing-only Estimation for Target Heading Under Uniform Linear Motion Conditions Based on Equispaced Measurement[J]. MODERN NAVIGATION,2014, (6):446-449