Optimal PMU Placement applied in Hybrid State Estimation

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***Abstract*— The Optimal PMU Placement (OPP) in state estimation in real variables is widely researched in the literature. However, the state estimation in complex variables is still poorly investigated. Thus, this paper presents the OPP in state estimation in complex variables through Integer Linear Programming (ILP) and Genetic Algorithm (GA). The proposed methodology was applied to an IEEE 14, 30 and 57 bus systems. The state estimation in complex variables proved to be efficient and has faster convergence than state estimation in real variables.**

***Keywords*— Complex Variables; Genetic Algorithm; ILP; Optimal PMU Placement; State Estimation.**

I. Introduction

Nowadays the operation of the electrical system is becoming an increasingly complex task. This is due to several factors, such as increased energy demand and the inclusion of renewable energy sources. This increase in operational complexity requires an improvement in the state estimation of the electrical system [1].

The Phasor Measurement Unit (PMU) is one of the main equipment that has improved the operation of the electrical system. Several works in the literature have already presented the contributions of the PMU in state estimation [2].

Adding the PMUs in the measurement vector, the accuracy in the state estimation of electrical systems is improved. In addition, due to the rapid sampling rate of the PMUs, it is possible to follow the dynamics of the voltage with enough accuracy [3].

Normally, the state estimation of the electrical system is performed using the WLS method on real variables. However, the state estimation in complex variables has gained considerable prominence due to its easy implementation and rapid convergence [3, 9].

Among the methods of Optimal PMUs Placement (OPP), the Genetic Algorithm (GA) and Integer Linear Programming (ILP) gained prominence [4]. However, these methods were not applied on OPP in the state estimation in complex variables. Thus, the objective of this paper is to perform the OPP in state estimation in complex variables using GA and ILP.

By allocating PMUs optimally, it is possible to increase the accuracy of state estimation and, as a consequence, improve the operation of the electrical system [1].

The rest of this paper is organized as follows. Section II presents and analyzes several recent works on OPP. Section III discourses on Theoretical Reference. Section IV presents the proposed Methodology. Section V presents the Experimental Studies and the Section VI concludes the paper.

II. Related Works

This section presents the works related to the Optimal PMU Placement and State Estimation.

The authors [4], presented a review of different optimization techniques applied to the OPP. The integer linear programming (ILP) showed high computational efficiency and low complexity when applied to the OPP.

The authors in [5] presented a multi-objective probabilistic model for OPP in electrical power systems. The contribution of this work was the minimization of the number of PMUs and the maximization of the system measurement redundancy. A mixed integer linear programming (MILP) was used to achieve these objectives.

In [6], the authors use a binary genetic algorithm to acquire OPP. This paper presents an OPP approach that considers the communication network infrastructure and the PMU implementation costs.

The authors in [7] applied integer linear programming (ILP) in the optimal allocation of PMUs in state estimation. It was noted that the use of PMU decreased the error of magnitude and angle of the voltage sharply.

In [8], an OPP technique has been proposed for complete and partially PMU observability applied in a Hybrid State Estimation. The Hybrid State Estimation has been implemented by integrating SCADA measurements and PMUs in an IEEE 14-bus system and 30-bus system.

The authors in [9] presented a Hybrid State Estimation in complex variables based on Weighted Least-Squares (WLS). The results demonstrated that the hybrid state estimator WLS in complex variables converges faster than the traditional state estimator into real variables. The speed level convergence increases as the number of PMUs increases. The efficiency of WLS estimation in complex variables is as effective as WLS in real variables.

III. Theoretical Referencere

This section presents the basic concepts of OPP, Genetic Algorithm, Integer Linear Programming and WLS State Estimation.

A. Optimal PMU Placement (OPP)

The OPP is referenced on Ohm's impedance law. The current between buses multiplied by the line impedance results in the drop voltage between those buses. The Ohm's impedance law is shown in (1) where I and J are buses, V_I and V_j are voltages from their respective buses, I_{IJ} and Z_{JI} respectively represent the current and impedance between buses I and J [1, 10]. Considering that Z_{JI} is known, it is enough to have two of the remaining variables to calculate the last one.

$$V_I - V_j = I_{IJ} Z_{JI} \quad (1)$$

The principle of operation of the PMU is that when installing the PMU on the I bus, the voltage of this bus and all currents of the buses connected to the I bus are measured. Then, based on this principle and in equation (1) it is possible to monitor the electrical system completely without installing PMU on all buses in the electrical system [1, 10].

B. Integer Linear Programming (ILP)

Linear programming is a problem where the objective function and the constraints are linear functions. There are basically three main classifications, integer, binary and mixed. A problem modeled as an Integer Linear Programming is when all variables are integers. When all variables are binary [0.1] then it is classified as Binary Integer Linear Programming. Finally, if some variables are not discrete, it is a Mixed-integer Linear Programming problem [13, 14].

C. Genetic Algorithm

The genetic algorithm (GA) is an optimization algorithm that is part of evolutionary algorithms. The GA is based on Darwin's theory of evolution, such as crossover, mutation and natural selection. In each generation, the quality of the individuals as solutions is evaluated by a fitness function; individuals are selected based on their fitness, and are modified by crossover and mutation operators to generate a new population of solutions. This process is carried out until the best solution is found or the stop criterion is reached. A general code of the genetic algorithm is presented below [11, 12].

Algorithm : GA

```
Initialization of chromosomes
while not Termination_Criteria() do
    Fitness_Function()
    Selection()
    Crossover()
    Mutation()
    Population_update()
    Elitism()
end while
Output_data()
```

D. WLS State Estimation

There are several ways to solve the state estimation of the electrical system. The WLS method is one of the most used in the electrical sector [15]. The basic principle of the WLS State Estimation is presented in this section.

Based on a set of measurements the following equation is formed [15]:

$$z = h(x) + e \quad (2)$$

where

z is measurement vector;

$h(x)$ are the nonlinear functions of the state vector (x);

e is the measurement error vector.

The Jacobian matrix, $H(x)$ given by

$$H(x) = \frac{\partial h(x)}{\partial x} \quad (3)$$

The Gain matrix, $G(x^k)$ is given by

$$G(x^k) = [H^T(x_k)R^{-1}H(x_k)]^{-1} \quad (4)$$

The error covariance matrix of the estimate x is given by

$$Cov(x) = H^T R^{-1} H \quad (5)$$

Finally, the state vector is obtained

$$[x^{k+1}] = [x^k] + [G(x^k)]^{-1}[H^T R^{-1} H][z - h(x^k)] \quad (6)$$

This is an iterative method and its respective stopping criteria is presented below,

$$\max|\Delta x^k| \le \varepsilon \quad (7)$$

where

ε is the tolerance range.

IV. Methodology

This section presents the proposed methodology for OPP and its respective application in state estimation in complex variables.

Integer Linear Programming (ILP) and Genetic Algorithm (GA) were used to model and solve the Optimal PMU Placement (OPP).

A. GA and ILP applied in OPP

The purpose of OPP is to ensure system observability ensuring the minimum number of PMU. Mathematically the OPP can be formulated as follows [16]:

$$\min = \sum_{k=1}^{n} X_n C_n \quad (8)$$

where

n is the number of buses;

X is a binary decision;

C is the cost.

The constraints of the problem are exemplified by a seven-bus system, Fig. 1, as follows:

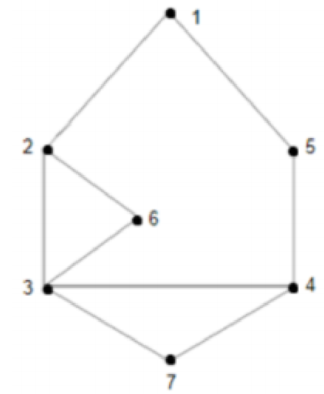

Fig. 1. *Seven Bus System*

$$x1 + x2 + x5 \geq 1 \quad (9)$$

$$x1 + x2 + x3 + x6 \geq 1 \quad (10)$$

$$x2 + x3 + x4 + x6 + x7 \geq 1 \quad (11)$$

$$x3 + x4 + x5 + x7 \geq 1 \quad (12)$$

$$x1 + x4 + x5 \geq 1 \quad (13)$$

$$x2 + x3 + x6 \geq 1 \quad (14)$$

$$x3 + x4 + x7 \geq 1 \quad (15)$$

The parameters of the GA used to solve the OPP were as follows. The initial population of the AG was 100 individuals with a crossover and mutation rate of 0.5 and 0.05 respectively, the stopping criterion was 200 generations.

The individual of the GA is modeled as $1 \times n$, where n represents the number of the electrical system buses. The genes of the GA's individual, X_n, are represented as follows [1].

$$X_n \begin{cases} 1 & \text{PMU installed on bus } n \\ 0 & \text{Otherwise} \end{cases} \quad (16)$$

B. State Estimation in Complex Variables

The state estimation via Weighted Least-Squares (WLS) requires the calculation of the Taylor series expansion of real-valued measurement functions but this is not possible in complex variables. This is the importance of Wirtinger's Calculus applied in state estimation, as it allows defining partial derivatives of a non-analytic function of real or complex value. With the partial derivative, it becomes possible to define the first-order complex Taylor series expansion. In state estimation in power system, the state vector x contains the phasor nodal voltages u: $[x; \bar{x}]$[9]. Fig. 2 presents a flowchart of the complex state estimation [9].

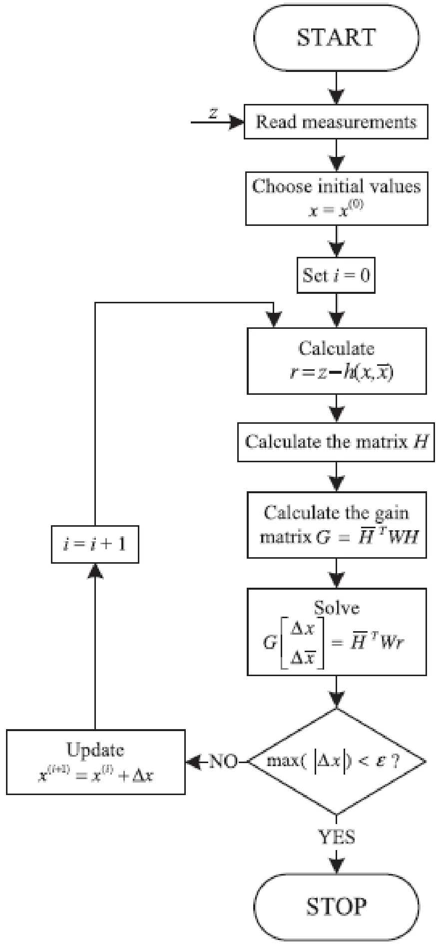

Fig. 2. *Flowchart of the Complex State Estimation [9]*

The measurement functions are composed of complex power flows, complex power injections, phasor voltages and phasor currents. The real and reactive power injection measurements P_i and Q_i are equivalently represented as complex measurements S_i and $\bar{S}_i$ [9]. This section presented the basic concepts of state estimation in complex variables. In [9] this methodology is presented in a more complete and detailed way.

V. EXPERIMENTAL STUDEIS

The OPP with and without $n - 1$ contingency have been tested at three test beds, IEEE 14, 30 and 57 bus system. The $n - 1$ contingency means that the system must remain fully monitored even if a PMU fails. Table I shows OPP without considering $n - 1$ contingency.

TABLE I. OPP UNDER NORMAL CONDITIONS

Base Test	***Number PMUs***	***Buses with PMUs***
IEEE-14Bus	4	2, 8, 10, 13.
IEEE-30Bus	10	3, 5, 8, 10, 11, 12, 19, 23, 26, 30.

Base Test	***Number PMUs***	***Buses with PMUs***
IEEE-57Bus	27	1, 2, 4, 6, 9, 12, 15, 19, 20, 22, 24, 25, 26, 28, 29, 31, 32, 36, 38, 39, 41, 44, 46, 47, 50, 53, 54.

Table I shows the results of the OPP for the IEEE 14, 30 and 57 bus systems under normal operating conditions. To monitor the electrical system completely, 4 PMUs are required for the IEEE 14-bus system, 10 PMUs for the IEEE 30-bus system and 27 PMUs for the IEEE 57-bus system. The Table II shows OPP by considering $n - 1$ contingency. Ensuring the monitoring of the electrical system under n-1 contingency can be seen as a measurement redundancy system, where the buses of the electrical system are monitored at least twice.

TABLE II. OPP UNDER $N - 1$ CONTINGENCY

Base Test	***Number PMUs***	***Buses with PMUs***
IEEE-14Bus	9	1, 2, 3, 6, 7, 8, 9, 10, 13
IEEE-30Bus	21	1, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 18, 19, 21, 23, 25, 26, 29, 30
IEEE-57Bus	33	1, 2, 4, 6, 9, 12, 15, 19, 20, 22, 24, 25, 26, 28, 29, 31, 32, 33, 35, 36, 38, 39, 41, 43, 44, 46, 47, 50, 51, 53, 54, 56, 57.

Table II shows the results of the Optimal PMU Placement for the IEEE 14, 30 and 57 bus systems under $n - 1$ contingency condition. To monitor the electrical system completely under $n - 1$ contingency condition, 9 PMUs are required for the IEEE 14-bus system, 21 PMUs for the IEEE 30-bus system and 33 PMUs for the IEEE 57-bus system. This means that to monitor the electrical system completely at least twice, an increase of 5 PMUs for the IEEE 14-bus system, an increase of 11 PMUs for the IEEE 30-bus system and an increase of 6 PMUs for the IEEE 57-bus system are required.

Table III presents the effects of OPP in the hybrid state estimation in complex variables of the IEEE 14-bus system for voltage magnitude. Table III also presents the real values, acquired by the power-flow study, the estimated values and their respective errors.

TABLE III. HYBRID STATE ESTIMATION IN COMPLEX VARIABLES (VOLTAGE)

Bus Number	***True Value***	***Estimated Value***	***Error***
1	1.06	1.0544	0.0056
2	1.045	1.0449	0.0001
3	1.01	1.0020	0.0080
4	1.013	1.0084	0.0046
5	1.017	1.0195	0.0025
6	1.07	1.0631	0.0069
7	1.046	1.0443	0.0017
8	1.08	1.0798	0.0002
9	1.031	1.0283	0.0027
10	1.03	1.0299	0.0001
11	1.046	1.0412	0.0048
12	1.053	1.0441	0.0089
13	1.047	1.0468	0.0002
14	1.019	1.0131	0.0059

The effects of the OPP, in the buses 2, 8, 10 and 13 of the IEEE 14-bus system, in the hybrid state estimation of the voltage magnitude in complex variables under normal operating conditions is shown in Table III.

Table IV presents the effects of OPP in the hybrid state estimation in real variables of the IEEE 14-bus system for voltage magnitude.

TABLE IV. HYBRID STATE ESTIMATION IN REAL VARIABLES (VOLTAGE)

Bus Number	***True Value***	***Estimated Value***	***Error***
1	1.06	1.0548	0.0052
2	1.045	1.0449	0.0001
3	1.01	1.0025	0.0075
4	1.013	1.0061	0.0069
5	1.017	1.0095	0.0075
6	1.07	1.0634	0.0066
7	1.046	1.0448	0.0012
8	1.08	1.0799	0.0001
9	1.031	1.0276	0.0034
10	1.03	1.0297	0.0003
11	1.046	1.0393	0.0067
12	1.053	1.0465	0.0065
13	1.047	1.0467	0.0003
14	1.019	1.0122	0.0068

Table V presents the effects of OPP in the hybrid state estimation in complex variables of the IEEE 14-bus system for voltage angle under normal operating condition. The effects of the OPP, in the buses 2, 8, 10 and 13 of the IEEE 14-bus system, in the hybrid state estimation of the voltage angle in complex variables is shown in Table V.

TABLE V. HYBRID STATE ESTIMATION IN COMPLEX VARIABLES (VOLTAGE ANGLE)

Bus Number	***True Value***	***Estimated Value***	***Error***
1	0	0	0
2	-4.99	-4.9898	0.0002
3	-12.75	-12.9067	0.1567
4	-10.24	-10.3642	0.1242
5	-8.76	-8.8626	0.1026
6	-14.45	-14.6817	0.2317
7	-13.24	-13.4479	0.2079
8	-13.24	-13.2399	0.0001
9	-14.82	-15.0406	0.2206
10	-15.04	-15.0398	0.0002
11	-14.86	-15.0563	0.1963
12	-15.30	-15.4902	0.1902
13	-15.33	-15.3299	0.0001
14	-16.08	-16.2824	0.2024

Table VI presents the effects of OPP in the hybrid state estimation in real variables of the IEEE 14-bus system for voltage angle. The effects of OPP, in the buses 2, 8, 10 and 13 of the IEEE 14-bus system, in the hybrid state estimation of the voltage angle in real variables is shown in Table VI.

TABLE VI. HYBRID STATE ESTIMATION IN REAL VARIABLES (VOLTAGE ANGLE)

Bus Number	***True Value***	***Estimated Value***	***Error***
1	0	0	0
2	-4.99	-4.9898	0.0002
3	-12.75	-12.9055	0.1555
4	-10.24	-10.3668	0.1268
5	-8.76	-8.8651	0.1051
6	-14.45	-14.6280	0.1780
7	-13.24	-13.4479	0.2079
8	-13.24	-13.2397	0.0003
9	-14.82	-15.0701	0.2501
10	-15.04	-15.0399	0.0001
11	-14.86	-15.0450	0.1850
12	-15.30	-15.4893	0.1893
13	-15.33	-15.3299	0.0001
14	-16.08	-16.2756	0.1956

This section presented the effects of the OPP for the IEEE 14 bus system on state estimation in complex and real variables. As presented in this section, the precision for state estimation in complex variables is as accurate as states estimation in real variables. Regarding the convergence time, state estimation in complex variables is faster than the state estimation in real variables. The convergence speed in state estimation in complex variables is increased as the number of PMUs is increased.

VI. Conclusion

This paper presented a method for Optimal PMU Placement in state estimation in complex variables for IEEE 14, 30, 57 bus systems. The results were compared with the traditional state estimation in real variables. The results showed that the state estimation in complex variables is competitive in relation to accuracy and has a faster convergence compared to state estimation in real variables. In state estimation in complex variables, the speed of convergence increases as the amount of PMU is increased

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