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Quantum Oscillator: One-Line Solution;

Infinite-Span Wavefunctions are Not States - No Quantum Tunnelling (Paused-Time QM is Not Testable by Run-Time Experiments)

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***Abstract*—In Quantum Mechanics, if the state of a particle is represented by a wavefunction, although it is necessary for a wavefunction of a Harmonic Oscillator to be an eigenstate of the Hamiltonian, it is not sufficient; maximum span of a wavefunction must also be bound by the energy of the state. Quantum Mechanics representation of states of a Harmonic Oscillator as infinite-span solutions to the wave equation is incorrect. Only a subset of the eigenspace of Hamiltonian can represent states of an Oscillator. Only the solutions to the wave equation under the strict constrain that the eigenstates are of limited span bound by the energy of the states can be realistic states of an Oscillator; the rest of the solutions cannot be states. Finding the states of a Quantum Oscillator must be a constrained eigen problem. Since both position and momentum of any Oscillator are strictly bandwidth limited by the energy, position and momentum cannot be a Fourier Transform Pair. Both Step-Up and Step-Down operators are real, and hence, the use of dedicated complex operator mechanics for solving wave equations of Quantum Oscillators is out of the context, unrealistic, meaningless, and misleading. Span-unconstrained solution to the wave equation of a Quantum Oscillator is straight forward, and there exists a simple one-line solution. Eigenfunctions of the square momentum operator corresponding to specific eigenvalues dependent on square positions are also unconstrained solutions of the wave equation of a Quantum Oscillator. If there exists a span-unconstrained ground state solution, its first derivative is also a solution under certain condition. This information provides the Step-Up and Step-Down operators. Step-Up and Step-Down operators are inverse of each other; their product is a constant. The product of Step-Up and Step-Down operators or the product operator shares the same eigenspace as of the Hamiltonian. One-line solution provides both eigenvalues and eigenstates. If a momentum of any Oscillator is zero at any position, no particle can be beyond that position. All the higher eigenstates contain nulls. If the square of the span-unconstrained eigenstate is assumed to be the probability of particle being at certain location, the probability of particle being at a null is nil. As a result, if a particle is in between two neighborhood nulls, particle will be trapped between those two nulls, and hence the higher span-unconstrained eigenstates cannot represent probability distributions. For an eigenstate to be a probability distribution, particle must have the ability to be at any position in the entire span of the eigenstate; this is not possible for higher eigenstates containing nulls. Only the span-unconstrained ground state can represent a probability distribution since it is positive and free of nulls. Quantum is an entity that is no longer divisible. Quantum-half cannot exist by the very definition of the quantum. If there exists a Quantum-half, then the Quantum-half itself should be the Quantum. Quantum-half is an oxymoron. Ground-state energy cannot be a quantum-half; it must be one quantum. The appearance of quantum-half and spin-half is a result of deBroglie wavelength error. No particle has the energy required to be at deBroglie wavelength. When the correct wavelength is used, quantum-half and spin-half disappear. Momentum does not generate waves, it is the motion of charge, chomentum that generates radiation waves. A charge particle is left behind by the very radiation it generated since these radiation waves travel at the speed of light. Radiation waves cannot represent states of a particle since the particle and radiation are mutually detached. Motion of electrically neutral particles, momentum, does not generate waves. Heisenberg Uncertainty has nothing to do with why position and momentum of a Quantum Oscillator cannot both be simultaneously zero. It is an inherent property of Oscillators itself that prevents position and momentum from being zero simultaneously. The energy of an Oscillator must be a non-zero constant. If the position and the momentum are zero simultaneously, the energy will be zero and hence Oscillator has no existence. Heisenberg Uncertainty is only about the bandwidth compromise between wavefunctions in position domain and momentum domains if they are a Fourier Transform pair. Heisenberg Uncertainty Principle cannot prevent the simultaneous measurability of observables. If two operators have a shared eigenspace, those**

observables are simultaneously measurable; both observables are certain on average simultaneously irrespective of the Heisenberg Uncertainty Principle. If the position and the momentum are a Fourier Transform pair, they must have a shared eigenspace making them simultaneously measurable; otherwise, they will not be a Fourier Transform pair. The claim that the Operators must commute for them to have a shared eigenspace is false, a result of a mathematical mishap. Commutation of operators is neither necessary nor sufficient for them to have a shared eigenspace. Non-commuting observables share an eigenspace if the commutation of operators is a constant as it is the case with position and momentum operators. Although any state of an Oscillator is an eigenstate of Hamiltonian, any eigenstate of Hamiltonian is not a state of an Oscillator. Realistic states of an Oscillator are only a small subset of the whole eigenspace of Hamiltonian. Underline assumption of any Harmonic Oscillator is that the displacements are small, which must be upheld by the solution. It cannot be violated and hence displacement must be finite, cannot be infinite. Any Oscillator must have strictly limited span of position and a strictly limited span of momentum that are determined by the energy. Position and momentum of a Harmonic Oscillator cannot be random at any time since it is a symphony. An infinite span eigenfunction of Hamiltonian cannot be a state of an Oscillator of finite energy, and hence Quantum Tunneling is not possible. The phenomenon of Quantum Tunneling is simply bogus. Neither the electrons in an Atom nor the Atoms in a lattice have restoration forces that are essential for them to be Harmonic Oscillators. No particle can be at multiple places simultaneously except in psychotic voodoo-physics. Quantum Mechanics is a paused-time theory. Momentum has no existence if the time is paused. For the action hero of Quantum Mechanics, the almighty wavefunction to appear from nowhere and carry out its job, the time must be paused, which is only possible on paper and impossible in reality. Notwithstanding many bogus experimental claims, Quantum Mechanics is not experimentally testable in reality since time cannot be paused. Paused-time Quantum Mechanics cannot be tested by run-time experiments. Ensemble average is not a paused-time average. Quantum Mechanics is a mathematical farce wrapped in Double-Slit and Stern-Gerlach experimental blunders, just like religious doctrines wrapped in fake miracles, mysterious artifacts, and rituals, dictated down under the claim that it is blasphemous or a heresy to express any opinion against them; they both only exist in misguided human fantasy, not in reality.

Keywords—Quantum Mechanics; Quantum State; Harmonic Oscillator; Eigenstate; Particle Waves; Schrodinger Equation; Heisenerg Uncertainty; Quantum Oscillator; Wavefunction, Wave Equation

I. INTRODUCTION

For those who are not interested in detail derivation, in other words for lazy people, here is the natural one-line solution to the span-unconstrained wave equation of a Quantum Harmonic Oscillator.

Theorem: One-Line Solution

If $\exists\ \psi_n(y)$ satisfying the condition,

$\partial^2\psi_n(y)/\partial y^2=[y^2-(2n+1)]\psi_n(y),\ \forall n,\ n=0, 1, 2, \dots$

then, $\psi_n(y)$ is an unconstrained solution to the wave equation of a Quantum Oscillator,

$-\partial^2\psi_n(y)/\partial y^2+y^2\psi_n(y)=\varepsilon_n\psi_n(y)$ with $\varepsilon_n=2n+1$,

where, $y=(m\omega_o/\hbar)^{1/2}x$, $\varepsilon_n=2E_n/\hbar\omega_o$, $\hbar=h/2\pi$, h is the Plank constant, E_n is the energy of the Oscillator, x is the displacement of mass m from equilibrium position, $\omega_o=(k/m)^{1/2}$, k is the restoration force constant.

Lemma: States as Eigenvectors of $\mathbf{P}^2$

If the momentum operator of the Quantum Oscillator is **P**, and $\psi_n(y)$ is an eigenvector of $\mathbf{P}^2$ with eigenvalue $-[y^2-(2n+1)]$, where,

$\mathbf{P}^2\psi_n(y)=-[y^2-(2n+1)]\psi_n(y)$

then, $\psi_n(y)$ is an unconstrained solution to the Quantum Oscillator with energy $\varepsilon_n=2n+1$,

$-\partial^2\psi_n(y)/\partial y^2+y^2\psi_n(y)=\varepsilon_n\psi_n(y),\ \forall n,\ n=0, 1, 2, \dots$

where, $\mathbf{P}=j\partial/\partial y$, $\mathbf{P}^2=-\partial^2/\partial y^2$.

Lemma: Hermite Lurking in $\mathbf{P}^2$

Hermite of order n is an eigenfunction $\psi_n(y)$ of the momentum operator $\mathbf{P}^2$ with eigenvalue $-[y^2-(2n+1)]$ given by $\Psi_n(y)=g_n(y)\exp((-1/2)y^2)$,

$\mathbf{P}^2\psi_n(y)=-[y^2-(2n+1)]\psi_n(y)$

where, $g_n(y)$ is the Hermite polynomial of order n.

Now, the solution to the Quantum Oscillator is equivalent to finding the eigenvectors of $\mathbf{P}^2$ corresponding to the eigenvalues $-[y^2-(2n+1)]\ \forall n$, n=0, 1, 2, … As we are going to see later, Hermite of any order is an eigenvector of operator $\mathbf{P}^2$ with the eigenvalue $-[y^2-(2n+1)]\ \forall n$, n=0, 1, 2, …

Lemma: Harmony in Confinement

In span-unconstrained solution, the span of $\psi_n(y)$ is infinite. No Harmonic Oscillator of finite energy can be at a state $\psi_n(y)$ of infinite span.

Hermite of any order satisfies the above condition,

$\partial^2\psi_n(y)/\partial y^2=[y^2-(2n+1)]\psi_n(y),\ \forall n,\ n=0, 1, 2, \dots$

The solution $\psi_n(y)$ to the wave equation is of infinite span. However, the solutions of infinite span to the wave equation do not represent states of a Quantum Oscillator. Irrespective of the size of a particle, position span as well as the momentum span of any oscillator are strictly limited by the energy of the particle and hence acceptable solutions to the wave equation must abide by those span constraints enforced by the energy level.

No particle has the energy required to be at any location described by an eigenstate or a wavefunction of infinite span in position domain. No particle has the energy required to be at any momentum described by a wavefunction of infinite span in the momentum domain. This is one of the major problems associated with the Quantum Mechanics representation of Harmonic Oscillators.

Lemma: Lack of Energy

Wavefunctions of infinite span cannot represent states of a Quantum Oscillator since both the maximum span of the position and the maximum span of the momentum of a Quantum Harmonic Oscillator are strictly limited by the energy of the Oscillator, which is finite.

Forgotten Property: Forget Me Not

Since wave equation $-\partial^2\psi_n(y)/\partial y^2+y^2\psi_n(y)=\varepsilon_n\psi_n(y)$ was a result of the assumption that the displacement y is small, wave function $\psi_n(y)$ applies realistically only for small displacement y.

Any solution $\psi_n(y)$ to the wave equation with y that violates the small y assumption is not a realistic solution to the wave equation that represent a Quantum Harmonic Oscillator. Wavefunction $\psi_n(y)$ with infinite span y is definitely not a state of a Quantum Harmonic Oscillator. The oversight of this fact is a violation of reality and the consequences are detrimental. Quantum Mechanics has violated this fact openly and shamelessly. Rules of mathematics do not seem to matter in Quantum Mechanics; they violate anything and everything as they please claiming that Quantum Mechanics do not have to abide by the rules.

You cannot make an assumption, forget about it, and then inadvertently violate it later on just like you never made such an assumption. If you make an assumption, you have to make sure it is going to hold through out to the end. If y is assumed to be small displacements in the formulation of the problem, then, whatever the solution will be restricted to small displacement y. Infinite span wavefunctions are out of the question; they have no connection to the wave equations of Harmonic Oscillators.

Since the position span and momentum span of a Harmonic Oscillator are strictly limited by the energy of an Oscillator, only the span-constrained solutions to the wave equation can represent a Quantum Harmonic Oscillator. In addition, if you want a wavefunction in position domain to represent a probability distribution of particle being at any location, then the wavefunction must also be positive and non-zero withing the span of the wavefunction.

Theorem: Within My Energy Limit

If $\exists\ \psi_n(y)$ satisfying the wave equation of a Quantum Oscillator,
$-\partial^2\psi_n(y)/\partial y^2+y^2\psi_n(y)=\varepsilon_n\psi_n(y)$,
under the strict condition that, $y^2\leq \varepsilon_n$,
$\psi_n(y)>0\ \forall n$, within the range $-(\varepsilon_n)^{1/2}\leq y \leq (\varepsilon_n)^{1/2}$
$\psi_n(y)=0$, otherwise,
then, $\psi_n(y)$ represents a state of a Quantum Oscillator.

The constrain $\psi_n(y)>0$ allows the representation of the wavefunction $\psi_n(y)$ itself as a probability distribution; no squaring is necessary.

There are no known close form solutions for this constrained wave equation. Solutions have to be obtained numerically. Against the nature and the reality, if one wishes to make the incorrect assumption that the wavefunction represents the probability of particle being at certain location, then, the wavefunction $\psi_n(y)$, $\forall n$ itself can represent a probability distribution if $\psi_n(y)$ is normalized within the range $-(\varepsilon_n)^{1/2}\leq y\leq(\varepsilon_n)^{1/2}$; no squaring is required. The representation of wavefunction $\psi_n(y)$ as a probability of a particle being at position y is still invalid and unnatural since the nature does not normalize wavefunctions. In addition, time has to be paused for this probabilistic representation; pausing the time can only be in done on paper, not in reality.

Lemma: Momentum Limit

For a realistic representation of the wavefunction of a Quantum Oscillator in the momentum domain, the wavefunction in the momentum domain, $\psi_n(p)$ must be such that,
$\psi_n(p)>0$, $\forall p$ within $-(\hbar m\omega_o\varepsilon_n)^{1/2}\leq p \leq (\hbar m\omega_o\varepsilon_n)^{1/2}$
$\psi_n(p)=0$, otherwise.

The span of the momentum p of any Harmonic Oscillator is strictly limited by the constrain, $p^2\leq\hbar m\omega_o\varepsilon_n$.

Corollary: Limited Admission

Any state of a Harmonic Oscillator is an eigenstate of the Hamiltonian. However, any eigenstate of the Hamiltonian is not a state of a Harmonic Oscillator.

Quantum Dilemma: Access Denied

For Quantum observables to come into existence with all of their glory and random characteristics for us to observe, the time has to be paused. Since time cannot be paused in reality, what exists for us to observe is the run-time on-average quantum observables. Run-time on-average observables are not Quantum observables, they are classical observables. As a result, Quantum observables are hypothetical and have no real existence; Quantum observables are not measurable.

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For those who are derivation-savvy and interested in not just the one-line solution but also its derivation and the Big Picture, here is the full story, starting with a brief introduction.

In Quantum Mechanics, a particle with momentum p is incorrectly assumed to behave as a wave of deBroglie wavelength λ,

$$\lambda=h/p \tag{1.1}$$

where, p is the momentum of the particle and h is the Plank constant.

One-dimensional plane wave equation of wavelength λ and angular frequency ω is given by,

$$\zeta(x,t)=\exp(j2\pi x/\lambda)\exp(-j\omega t) \tag{1.2}$$

Moving particles do not behave as waves, and the energy of a moving particle is mechanical energy. Mechanical energy is not quantized. If you make incorrect assumption that a moving particle of momentum p behaves as a wave of deBroglie wavelength λ=h/p, and also equally invalid assumption that the mechanical energy is quantized, E=ħω, then, the wavefunction of a free moving particle $\zeta_n(x,t)$ is given by,

$$\zeta(x,t)=\exp(jpx/\hbar)\exp(-jEt/\hbar) \tag{1.3}$$

where ħ=h/2π. E is the energy of the particle and x denotes the displacement from the equilibrium position, or the position of the particle at any time t in the case of a Harmonic Oscillator.

At any time, t,

$$\zeta(x)=\exp(jpx/\hbar) \tag{1.4}$$

$$\psi(t)=\exp(-jEt/\hbar) \tag{1.5}$$

$$\zeta(x,t)=\zeta(x)\psi(t) \tag{1.6}$$

If we differentiate the wave equation ζ(x,t) with respect to x, we have,

$$\mathbf{P}\zeta(x,t)=p\zeta(x,t) \tag{1.7}$$

where,

$$\mathbf{P}=-j\hbar\partial/\partial x \tag{1.8}$$

P is the momentum operator.

The position is the independent variable and hence the position operator **X** is assumed to be position itself,

$$\mathbf{X}=x\mathbf{I} \tag{1.9}$$

where, **I**, an identity operator.

Differentiating ζ(x,t) with respect to time t, we have,

$$j\hbar\partial\zeta(x,t)/\partial t=E\zeta(x,t) \tag{1.10}$$

This is the time dependent Schrodinger equation. If the Hamiltonian of the particle is **H**,

$$\mathbf{H}\psi(x)=E\psi(x) \tag{1.11}$$

where,

$$\mathbf{H}=(1/2m)\mathbf{P}^2+V(x) \tag{1.12}$$

V(x) is the potential energy of the particle at position x.

The time dependent wavefunction of a particle is given by,

$$\psi(x,t)=\psi(x)\psi(t) \tag{1.13}$$

When V(x)=0, particle is a free-moving and hence,

$$\psi(x)=\zeta(x) \tag{1.14}$$

If ψ(x) differs from ζ(x), it is because particle is not free-moving, or in other word, V(x)≠0. Quantum Mechanics representation of square wavefunction, $|\psi(x)|^2$ as a probability distribution fails when V(x)=0 since it is not square differentiable. In fact, as we are going to see later, neither ψ(x) nor $|\psi(x)|^2$ can represent a probability distribution irrespective of what the potential energy V(x) is.

a) **Wave Function:**

When V(x)≠0, ψ(x) describes the wavefunction of a particle. For a free-moving particle V(x)=0 and hence ψ(x)=ζ(x); ζ(x) sinusoidal and cannot be normalized for the entire range of x. As a result, wavefunction of a free-moving particle does not represent a probability distribution of a particle being at a certain location. Although the square of ζ(x) is positive and can be normalized for a range of wavelength, such normalization does not represent a probability distribution. In order for a function to be a probability distribution, function must be positive for the entire range and the area of the function for the entire range must be unity. Wave function or its square of a free moving particle cannot represent a probability distribution.

b) **Fourier Transform Pair:**

Position and momentum of a particle at any time must be unique. No particle can be at infinitely many positions and infinitely many momentums at the same time. The momentum cannot change without the change of time. Momentum has no existence without change of position. Momentum has no existence if the time is paused. For position and momentum to be random variables time has to be paused. Quantum observables have no existence unless the time is paused. Time can only be paused on paper, not in reality. Position of a particle cannot change without change of time. Change of position and momentum of a particle are time dependent. Momentum exists only in run-time, not at paused-time.

Position and momentum are mutually dependent since the position is determined by the momentum and the momentum is determined by the change of position. When position and momentum are mutually dependent, they cannot be a Fourier Transform pair. If position and momentum cannot be measured simultaneously, they cannot be a Fourier Transform pair. Fourier Transform simply has no business in the affairs of position and the momentum of a particle.

For a given momentum, no particle (mass) can be at infinitely many positions simultaneously. Similarly, for a given position, no particle (mass) can be at infinitely many momentums simultaneously. In order to make that invalid assumption, time has to be paused, which is not possible in reality. As a result, position and momentum cannot be a Fourier Transform pair. The claim in Quantum Mechanics that the position and the momentum of a particle are a Fourier Transform pair is no different from the religious claim that a creator entity created universe; both are blind human fantasies without substance.

However, if one makes the incorrect assumption that a particle (mass) can be at infinitely many locations and at infinitely many momentums at the same time when the time is paused, then, the wave function in the position domain ψ(x) and the wave function in the momentum domain ψ(p) are a Fourier

Transform pair given by,

$$\psi(p)=\int \psi(x)\exp(-jpx/\hbar)dx \quad (1.2.1)$$

$$\psi(x)=\int \psi(p)\exp(jpx/\hbar)d(p/h) \quad (1.2.2)$$

c) **Uncertainty Principle:**

In Quantum Mechanics, function $\exp(-jpx/\hbar)$ is a Fourier Transform kernel only by an invalid bogus assumption. If position x and momentum p pair is a Fourier Transform pair, the width Δx of the wave function ψ(x) in position domain and the width Δp/h of the wavefunction in the momentum domain are bound by [5],

$$\Delta x\, \Delta p/h \geq 1 \quad (1.3.1)$$

$$\Delta x\, \Delta p \geq h \quad (1.3.2)$$

This is the so-called Heisenberg uncertainty principle. It only says that the bandwidth of a wave function cannot be bound both in the position domain and the momentum domain if position and momentum are a Fourier Transform pair. It does not say anything about simultaneous observability or measurability. It is not related to the Commutation of operators. The link between Heisenberg Uncertainty Principle and commutation of operators in Quantum Mechanics is a result of a mathematical error or a mathematical mishap. Heisenberg Uncertainty Principle has nothing to do with commutation of operators.

On the other hand, both position and momentum spans of a Harmonic Oscillator must be bound simultaneously even though the position span and the momentum span of a Fourier transform pair cannot both be bound simultaneously. This prevents the position and the momentum of a Quantum Oscillator from being a Fourier Transform pair. A pair with predefined spans cannot be a Fourier Transform pair. Very definition of a Harmonic Oscillator precludes the position and the momentum pair of a Harmonic Oscillator from being a Fourier Transform pair. No strict span bound pair of functions can be a Fourier Transform pair. Without strict span bounds of both position and momentum, there will be no Quantum Harmonic Oscillator.

Corollary: My Hands are Tied

Position and Momentum pair of a Quantum Oscillator is strictly span bound. A strictly span bound pair cannot be a Fourier Transform pair.

d) **Simultaneous Observability:**

Simultaneous observability has nothing to do with the bandwidth bounds of a Fourier Transform pair or Heisenberg Uncertainty Principle. Simultaneous measurability is related to the shared eigenstates of operators of observables. General Uncertainty principle or Heisenberg Uncertainty Principle only says that a wave function cannot be both position-span limited, and momentum-span limited simultaneously. Heisenberg Uncertainty does not and cannot prevent both position operator and the momentum operator from sharing an eigenspace. If position and momentum pair is a Fourier Transform pair, they must have a shared eigenspace, otherwise, they would not be a Fourier Transform pair. If they have a shared eigenspace, they must be simultaneously measurable.

Property: An Open Book

If two observables are a Fourier Transform pair, their operators must have a common eigenspace, otherwise, they would not be a Fourier Transform pair. Any two observables with common eigenspace must be simultaneously measurable.

The width of a wavefunction ψ(x) is determined by the potential V(x); it is not an observer-controlled quantity. If V(x)=0, then the span of ψ(x) is not finite and in addition ψ(x) is not square integrable. Under the assumption that position x and momentum p pair is a Fourier Transform pair, if the system contains a ψ(x) of narrower width Δx, then the width Δp of the wave function in the momentum domain will be wider and vice versa. The widths Δx and Δp are not under the observer control. The widths Δx and Δp are observer independent.

Since position operator and momentum operator have shared eigenspace, they are simultaneously measurable independent of observers or measurement instruments. If two observables have a shared eigenspace, the measurement of one observable does not alter the state of the other. Position and momentum of a particle do have a shared eigenspace.

Lemma: Relatives Only

For any Harmonic Oscillator, Δx and Δp must be linearly related, not inversely. If Δx and Δp are inversely related they cannot represent a Harmonic Oscillator of any kind. Heisenberg Uncertainty Principle cannot hold for a Harmonic Oscillator.

Corollary: Direct Opposite

For any Harmonic Oscillator, change of |x| and change of |p| are linearly related with a negative gradient.

Corollary: Unavailable for Choosing

For a Harmonic Oscillator, Δx and Δp are determined by the energy of the Oscillator; they are not left to be determined by a Fourier Transform pair.

e) **Measurement of Both Position and Momentum are Not Required:**

If position and momentum are a Fourier Transform Pair, the information content of the wavefunction in the position domain will be the same as the information content of the wavefunction in the momentum domain. It is not required to measure position and momentum separately. If you can obtain the position, you can also obtain the momentum from the position measurement.

If the position and the momentum is a Fourier Transform pair, position and the momentum must be measurable simultaneously, otherwise they cannot be a Fourier Transform pair. Commutation of operators of observable is not necessary for the simultaneous measurability of observables. Non-commutation of observables does not prevent the simultaneous measurability of observables since non-commuting operators can have shared eigenspace. Although the position and the momentum operators do not commute, the position and the momentum operators have shared eigenspace making them measurable simultaneously.

Lemma: No Commuting Required for Sharing

For two operators to have a shared eigenspace, commutation of operators is neither necessary nor sufficient. Non-commuting operators can have shared eigenspace. Non-Commuting observables are simultaneously measurable as long as they contain a shared eigenspace.

f) **Uncertainty Principle Cannot Prevent Simultaneous Measurability of Observables:**

There is no way for the Heisenberg Uncertainty Principle to prevent simultaneous measurability of position and momentum. What is required for the simultaneous measurability of two observables is a common eigenspace for both operators of the observables. Ability have a shared eigenstate for two observables has nothing to do with the Heisenberg Uncertainty Principle. In spite of what Heisenberg Uncertainty principle says, if both position and the momentum have a shared eigen space, both position and momentum can be measured simultaneously and on average they represent the precise position and the momentum. Simultaneous measurability does not require for their operators to commute.

Since the width of the wavefunction in position domain Δx and the width of the wave function in momentum domain Δp are determined by the potential energy $V(x)$ the particle is in, Uncertainty Principle only says that if a particle is in an environment that Δx is narrow, the spread of Δp will be wider and vice versa. Uncertainty Principle cannot prevent us from observing or measuring both the position and the momentum simultaneously. Since the position and the momentum are assumed to be a Fourier Transform Pair, the simultaneous measurability is guaranteed. Any Fourier Transform pair must be simultaneously measurable.

g) **Simultaneous Measurability is Guaranteed if the Position and the Momentum are a Fourier Transform Pair:**

If the position x and the momentum p are a Fourier Transform Pair, they are mutually independent and must exit simultaneously at the same eigenstate. If the position and the momentum do not have a common eigenspace, they cannot be a Fourier Transform Pair. If the position and the momentum are at the same eigenstate, then, the position x and the momentum p must be measurable simultaneously. The simultaneous measurability is guaranteed when the invalid assumption that the position and the momentum are a Fourier Transform Pair is being made.

At any state $\psi(x)$, by definition, the probability of observing a particle at x is $\psi^*(x)\psi(x)$, where $\psi^*(x)\psi(x)\geq 0$ and the probability of observing the same particle at momentum p is $\psi^*(p)\psi(p)$, where $\psi^*(p)\psi(p)\geq 0$. Spread of $\psi(x)$ and $\psi(p)$ says nothing about the simultaneous observability or measurability. Precision in both observables can be achieved on average simultaneously. Later, we see why an eigenstate $\psi(x)$ or its square $|\psi(x)|^2$ of a particle with nulls cannot be a probability distribution of particle being at location x.

Lemma: Simultaneity in FT Pair, Guaranteed

Simultaneous measurability is an inherent property of any Fourier Transform pair. If two observables are not simultaneously measurable, then, they cannot be a Fourier Transform pair.

II. QUANTUM HARMONIC MOTION

Consider a Harmonic Oscillation of a particle of mass m and the restoration force constant k. If the displacement of the particle from its equilibrium position is x, then, the Hamiltonian of the Harmonic Oscillator H is given by,

$$\mathbf{H}=(1/2m)\mathbf{P}^2+(1/2)kx^2 \qquad (2.1)$$

where, position operator **X** and momentum operator **P** are given by,

$$\mathbf{P}=-j\hbar\partial/\partial x \qquad (2.2)$$

$$\mathbf{X}=x. \qquad (2.3)$$

If $\omega_o^2=k/m$, we have,

$$\mathbf{H}=(1/2m)\mathbf{P}^2+(1/2)m\omega_o^2x^2 \qquad (2.4)$$

ω_o is the fundamental angular frequency of the Harmonic Oscillator, which is a constant. It is the minimum angular frequency of the oscillating particle. It can oscillate at integer multiples of the fundamental frequency, $n\omega_o$ at higher energy levels, but not at fractional ω_o/n, where n is an integer.

If the particle is at state $\psi_n(x)$, then, the energy E_n of the particle is given by,

$$\mathbf{H}\psi_n(x)=E_n\psi_n(x) \qquad (2.5)$$

Substituting for **H** from eqn. (2.4) and using eqns. (2.2) and (2.3), we have,

$$-(1/2m)\hbar^2\partial^2\psi_n(x)/\partial x^2+(1/2)m\omega_o^2x^2\psi_n(x)=E_n\psi_n(x) \qquad (2.6)$$

This is the wave equation for a Quantum Harmonic Oscillator [1,2]. Solution to the wave equation gives the eigenvalue-eigenfunction pairs (E_n, $\psi_n(x)$), $\forall n$, n=0,1,2, … for a Quantum Oscillator. Eigenfunction $\psi_n(x)$ is the state of the particle with the associated energy E_n. Since wavefunction has multiple solutions, Harmonic Oscillator can be at any state $\psi_n(x)$ with associated energy level E_n, $\forall n$, n=0, 1, 2, 3, ... What state a Quantum Oscillator is in is determined by the total energy of the particle.

When momentum is zero, p=0, the total energy is in the form of potential energy resulting the maximum displacement from which the particle cannot go beyond. Similarly, when the displacement is zero, x=0, the total energy is in the form of kinetic energy resulting the maximum momentum from which the momentum of the particle cannot exceed. As a result, the span Δx of the wavefunction $\psi_n(x)$ in position domain and the span Δp of the wavefunction $\psi_n(p)$ in the momentum domain are determined by the energy of the particle, nothing else, not by the Fourier Transform bandwidth limits. So, the span of the wavefunction, $\psi_n(x)$ in position domain and the span of the wave function $\psi_n(p)$ in momentum domain are strictly constrained by the energy of the Oscillator. As a result, unbound functions such as Gaussian functions, $\psi_n(x)=\exp((-1/2)y^2)$ cannot be a wavefunction of a Quantum Oscillator. The bounded nature of the solution to the wave equation of a Quantum Oscillator must have been incorporated into the wave equation as a constrain at the beginning.

However, it is important to note that Quantum Mechanics has disregarded the fact that the energy level E_n also limits the span of the eigenstate $\psi_n(x)$ in position domain and the span of the eigenstate $\psi_n(p)$ in the momentum domain. This is one of the major mistakes in Quantum Mechanics.

There are infinite number of solutions to a wave equation. Not all the solutions are a realistic representation of a Harmonic Oscillator. Unconstrained solutions do not represent real Quantum Oscillators. For solutions to the wave equation to represent a realistic oscillator, solutions must be obtained under the constraint that the maximum span of an eigenstate in position domain is limited by the maximum potential energy of the particle. Similarly, the maximum momentum span of a wavefunction in momentum domain is also limited by the maximum kinetic energy of the oscillator. Maximum potential energy is the total energy of the state of the Oscillator. The maximum kinetic energy is also the total energy of the state of the Oscillator. When an Oscillating particle is at maximum potential energy, its kinetic energy is zero and similarly, when an Oscillating particle is at maximum kinetic energy, its potential energy is zero.

For the time being, we concentrate on the unconstrained wavefunction for a Quantum Oscillator since it has held a predominant position in Quantum Mechanics for no apparent reason. Interestingly, it even has its own dedicated complex operator mechanics, even though the operators are real, that has no use anywhere else. You cannot find a Quantum Mechanics book that does not talk about the unconstrained solutions to the wave equation of a Quantum Oscillator without unnecessary complex operator maneuvers. In fact, there is a dedicated Complex Operator Mechanics for Quantum Oscillators themselves. We will introduce a simple one-line solution to the wave equation of a Quantum Oscillator.

III. UNCONSTRAINED SOLUTION TO THE WAVE EQUATION

As we did before, with the change of variables, the wave equation can be written as [2],

$$-\partial^2\psi_n(y)/\partial y^2+y^2\psi_n(y)=\varepsilon_n\psi_n(y) \tag{3.1}$$

where,

$$y=(m\omega_o/\hbar)^{1/2}x \tag{3.2}$$

$$\varepsilon_n=2E_n/\hbar\omega_o \tag{3.3}$$

x is the displacement of the particle from the equilibrium position and ω_o is the fundamental angular frequency of the particle, $\omega_o=(k/m)^{1/2}$, E_n is the energy of the Oscillator at state n.

Since ω_o is the fundamental frequency, it cannot oscillate at a fraction of ω_o and hence $\omega_o\neq(1/n)\omega_o$ where, n is an integer, n>1.

Angular frequency of an Oscillator can be at any integer multiple of fundamental frequency ω_o or at higher angular frequency $\omega=n\omega_o$, n is an integer. So, the fundamental angular frequency ω_o is the ground state of the Oscillator with energy ε_o. So, the task is to solve the unconstrained wave equation given in eqn. (3.1) to find the eigenvalue-eigenfunction pair $(\varepsilon_n,\psi_n(y))$ ∀n, n=0, 1, 2, 3, … . As we mention before, these unconstrained solutions do not represent Quantum Harmonic Oscillator states.

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None of the solutions to the wave equation represents a state of a Quantum Oscillator unless the span of the wavefunction is strictly limited by the energy of the Oscillator.

IV. NATURAL ONE-LINE DIRECT SOLUTION TO THE UNCONSTRAINED WAVE EQUATION

Theorem: All in One Line

If ∃ $\psi_n(y)$, a function of y, such that the second order derivative satisfies the condition,

$$\partial^2\psi_n(y)/\partial y^2=[y^2-(2n+1)]\psi_n(y),\ \forall n,\ n=0, 1, 2, \ldots \tag{4.1}$$

then, $\psi_n(y)$ is an unconstrained solution to the wave equation of a Quantum Oscillator,

$$-\partial^2\psi_n(y)/\partial y^2+y^2\psi_n(y)=\varepsilon_n\psi_n(y) \tag{4.2}$$

with eigenvalue ε_n given by,

$$\varepsilon_n=2n+1,\ \forall n,\ n=0, 1, 2, \ldots \tag{4.3}$$

Proof:

Assume we have $\psi_n(y)$ such that,

$$\partial^2\psi_n(y)/\partial y^2=[y^2-(2n+1)]\psi_n(y),\ \forall n,\ n=0, 1, 2, \ldots \tag{4.4}$$

Now, substituting eqn. (4.4) in the wave equation given in eqn. (4.2), we have,

$$-[y^2-(2n+1)]\psi_n(y)+y^2\psi_n(y)=\varepsilon_n\psi_n(y) \tag{4.5}$$

$$(2n+1)\psi_n(y)=\varepsilon_n\psi_n(y) \tag{4.6}$$

We, now have,

$$\varepsilon_n=(2n+1)],\ \forall n,\ n=0, 1, 2, \ldots \tag{4.7}$$

As a result, $\psi_n(y)$ is an eigenstate with eigenvalue ε_n, ∀n.

Corollary: Alternate Way

If the momentum operator of the Quantum Oscillator is **P**, then the eigenfunction $\psi_n(y)$ of $\mathbf{P}^2$ with eigenvalue $-[y^2-(2n+1)]$ is an unconstrained-solution to the Quantum Oscillator

$-\partial^2\psi_n(y)/\partial y^2+y^2\psi_n(y)=\varepsilon_n\psi_n(y)$, $\forall n$, n=0, 1, 2, …

where, $\mathbf{P}=j\partial/\partial y$, where $\mathbf{P}^2=-\partial^2/\partial y^2$.

What is left to do is to find out $\psi_n(y)$ that satisfies the condition,

$\partial^2\psi_n(y)/\partial y^2=[y^2-(2n+1)]\psi_n(y)$, $\forall n$, n=0, 1, 2, …

In other words, we have to find the eigenvectors of the square momentum operator $\mathbf{P}^2$ corresponding to eigenvalues $-[y^2-(2n+1)]$ $\forall n$, n=0, 1, 2, … First, we want to find the ground state $\psi_o(y)$. From eqn. (4.7), we already have the ground state energy, $\varepsilon_o=1$.

V. GROUND STATE SOLUTION TO THE UNCONSTRAINED WAVE EQUATION

We have the wave equation for Quantum Harmonic Oscillator,

$$-\partial^2\psi_n(y)/\partial y^2+y^2\psi_n(y)=\varepsilon_n\psi_n(y) \quad (5.1)$$

where,

$$\varepsilon_n=(2n+1), \forall n, n=0, 1, 2, \dots \quad (5.2)$$

At ground state, we have eigenstate $\psi_o(y)$ with eigen value $\varepsilon_o=1$. We already have the eigenvalues ε_n for all the states, n=0, 1, 2, …. We know, we can guess the ground state solution. However, we want to find the ground state without any guessing. How can we find the ground state from ground up?

Lemma: Inseparable Couple

If $\psi_o(y)$ is the ground state solution, then, its derivative $\psi_o'(y)$ is also a solution and $\psi_1(y)=\psi_o'(y)$ with eigen value $\varepsilon_1=2+\varepsilon_o$, where $\psi_o'(y)=\partial\psi_o(y)/\partial y$.

Proof:

Assume ∃ a function $\psi_o(y)$, which is a solution to the wave equation. Then, we have,

$$-\partial^2\psi_o(y)/\partial y^2+y^2\psi_o(y)=\varepsilon_o\psi_o(y) \quad (5.3)$$

We have no idea what $\psi_o(y)$ exactly is, but we know it satisfies the wave equation of a Quantum Harmonic Oscillator.

Now, let us take the derivative of eqn. (5.3) with respect to y,

$$-\partial^2\psi_o'(y)/\partial y^2+y^2\psi_o'(y)+2y\psi_o(y)=\varepsilon_o\psi_o'(y) \quad (5.4)$$

where, $\partial\psi_o(y)/\partial y=\psi_o'(y)$.

In order to maintain the structure of the equation, we represent only the extra derivative we are taking by $\psi_o'(y)$.

Now, let us assume $\psi_o(y)$ is a special function that satisfies the condition,

$$\psi_o'(y)=-y\psi_o(y) \quad (5.5)$$

Substituting for $2y\psi_o(y)$ in eqn. (5.4), we have,

$$-\partial^2\psi_o'(y)/\partial y^2+y^2\psi_o'(y)-2\psi_o'(y)=\varepsilon_o\psi_o'(y) \quad (5.6)$$

$$-\partial^2\psi_o'(y)/\partial y^2+y^2\psi_o'(y)=(\varepsilon_o+2)\psi_o'(y) \quad (5.7)$$

This says that if $\psi_o(y)$ is a solution, then, the first derivative $\psi_o'(y)$ is also an eigenstate with eigenvalue ε_o+2. We already know that the eigenvalue of any state is given by,

$$\varepsilon_n=(2n+1), \forall n, n=0, 1, 2, \quad (5.8)$$

where, $\varepsilon_o=1$.

So, we have for n=1,

$$\varepsilon_1=(2+\varepsilon_o) \text{ or } \varepsilon_1=(2+1). \quad (5.9)$$

Therefore, the first higher energy level eigenstate, $\psi_1(y)$ is the derivative of the ground state $\psi_o(y)$,

$$\psi_1(y)=\partial\psi_o(y)/\partial y \quad (5.10)$$

if $\psi_o(y)$ is such it satisfies the relationship,

$$\psi_o'(y)=-y\psi_o(y) \quad (5.11)$$

a) Derivation of Ground State $\psi_o(y)$

Although we started with the intention of finding the ground state $\psi_o(y)$, so far, we have not found $\psi_o(y)$. What we found was that if $\psi_o(y)$ is a solution, then, $\psi_o'(y)$ is also a solution, and it is the eigenstate $\psi_1(y)$ of the first higher energy level ε_1. This happens if $\psi_o(y)$ satisfies the condition,

$$\psi_o'(y)=-y\psi_o(y) \quad (5.1.1)$$

$$\partial\psi_o(y)/\partial y=-y\psi_o(y) \quad (5.1.2)$$

We make a further assumption that

$$\psi_o(y)>0, \forall y. \quad (5.1.3)$$

In other words,

$$\psi_o(y)=|\psi_o(y)| \quad (5.1.4)$$

Under this assumption, we have,

$$(1/\psi_o(y))\partial\psi_o(y)=-y\partial y \quad (5.1.5)$$

Integrating both sides,

$$\int(1/\psi_o(y))\partial\psi_o(y)=-\int y\partial y \quad (5.1.6)$$

$$\ln \psi_o(y)=-(1/2)y^2+C \quad (5.1.7)$$

$$\psi_o(y)=\exp(-(1/2)y^2+C) \quad (5.1.8)$$

where, ln is the natural logarithm.

Since $\psi_o(y)>0$, or $\psi_o(y)=|\psi_o(y)|$, $\psi_o(y)$ can also represent a probability density function provided it satisfies the condition,

$$\int\psi_o(y)dy=1 \quad (5.1.9)$$

Substituting for $\psi_o(y)$ from eqn. (5.1.8), we have,

$$\int\exp(-(1/2)y^2+C)dy=1 \quad (5.1.10)$$

$$e^C \int\exp(-(1/2)y^2)dy=1 \quad (5.1.11)$$

$$e^C=1/(2\pi)^{1/2} \quad (5.1.12)$$

Now, we have the ground state solution of the Quantum Harmonic Oscillator,

$$\psi_o(y)=(1/(2\pi)^{1/2})\exp(-(1/2)y^2) \quad (5.1.13)$$

For the time being, we can disregard the constant multiplication factor since it does not affect the solution to the wave equation and as a result we have,

$$\psi_o(y)=\exp(-(1/2)y^2) \quad (5.1.14)$$

b) Second Derivative of $\psi_o(y)$

The ground state $\psi_o(y)$ is given by,

$$\psi_o(y)=\exp(-(1/2)y^2) \quad (5.2.1)$$

$$\psi_o'(y)=-y\psi_o(y) \quad (5.2.2)$$

$$\psi_o''(y)=(y^2-1)\psi_o(y) \quad (5.2.3)$$

This must satisfy the general condition for natural one-line solution, $\psi_n''(y)$ for n=0. We have seen that for $\psi_n(y)$ to be a solution, $\psi_n''(y)$ must satisfy the condition,

$$\partial^2\psi_n(y)/\partial y^2=[y^2-(2n+1)]\psi_n(y), \forall n, n=0, 1, 2, \dots \quad (5.2.4)$$

Substituting n=0 in eqn. (5.2.4), we have,

$$\psi_o''(y)=(y^2-1)\psi_o(y) \quad (5.2.5)$$

So, $\psi_o''(y)$ satisfies the general condition for $\psi_n(y)$ to be a solution at n=0.

c) All About the Ground State

Now, we can summarize what we know about the ground state $\psi_o(y)$,

$\psi_o(y)=\exp(-(1/2)y^2)$ (5.3.1)
$\varepsilon_n=(2n+1)$, at n=0 (5.3.2)
$\varepsilon_o=1$ (5.3.3)
$\partial^2\psi_n(y)/\partial y^2=[y^2-(2n+1)]\psi_n(y)$ for n=0 (5.3.4)
$\psi_o(y)=g_o(y)\psi_o(y)$ (5.3.5)

where,

$g_o(y)=1$ (5.3.6)

Function $g_o(y)\psi_o(y)$ is the Hermite of order 0. $g_o(y)$ is the Hermite polynomial of order 0.

VI. FIRST HIGHER STATE, n=1

Using the fact that if $\psi_o(y)$ satisfies the wave equation with eigenvalue ε_o, then, the first derivative of $\psi_o(y)$ is also a solution, we have,

$\psi_1(y)=-y\psi_o(y)$ (6.1)
$\varepsilon_1=2+1$ (6.2)

where, $\varepsilon_o=1$ and

$\varepsilon_n=(2n+1)$ at n=1 (6.3)
$\psi_o(y)=\exp(-(1/2)y^2)$ (6.4)

a) **Second Derivative of** $\psi_1(y)$

In order to see if the condition for the general one-line solution is satisfied, we have to obtain the second derivative of $\psi_1(y)$,

$\psi_1(y)=-y\psi_o(y)$ (6.1.1)
$\psi_1'(y)=(y^2-1)\psi_o(y)$ (6.1.2)
$\psi_1''(y)=(-y(y^2-1)+2y)\ \psi_o(y)$ (6.1.3)
$\psi_1''(y)=(y^2-3)(-y)\psi_o(y)$ (6.1.4)
$\psi_1''(y)=(y^2-3)\psi_1(y)$ (6.1.5)

This must satisfy the condition for the general one-line solution for n=1. For any $\psi_n''(y)$ to be a solution to the unconstrained wave equation, the second derivative must satisfy,

$\partial^2\psi_n(y)/\partial y^2=[y^2-(2n+1)]\psi_n(y),\ \forall n$ (6.1.6)

At n=1, we have,

$\psi_1''(y)=(y^2-3)\psi_1(y)$ (6.1.7)

$\psi_n''(y)$ satisfies the condition at n=1

b) **All About State-1**, n=1

For the first higher state, we have,

$\psi_1(y)=-y\psi_o(y)$ (6.2.1)
$\psi_o(y)=\exp(-(1/2)y^2)$ (6.2.2)
$\varepsilon_1=2+1=3$ (6.2.3)
$\varepsilon_n=(2n+1)$, at n=1 (6.2.4)
$\varepsilon_o=1$ (6.2.5)
$\partial^2\psi_n(y)/\partial y^2=[y^2-(2n+1)]\psi_n(y)$ for n=1 (6.2.6)
$\Psi_1(y)=g_1(y)\psi_o(y)$ (6.2.7)

where,

$g_1(y)=-y$ (6.2.8)

Function $g_1(y)\psi_o(y)$ is the Hermite of order 1. $g_1(y)$ is the Hermite polynomial of order 1.

Any constant multiplication factor of an eigenstate does not have any effect on the solution to the wave equation and hence can be dropped safely.

VII. STEP-UP AND STEP-DOWN OPERATORS

The relationship between the ground state $\psi_o(y)$ and the first state $\psi_1(y)$ provides a mechanism for generating nearest lower state. If we have such a mechanism or an operator, we can operate on any state n and obtain the nearest lower state. Such an operator is a Step-Down operator **D**.

Similarly, we can also use the relationship between the ground state $\psi_o(y)$ and the first state $\psi_1(y)$ to find a mechanism or an operator that we can use to obtain the state that is one step higher than the current state. Such an operator is a Step-Up operator **U**. Step-Up operator **U** is also the same as the Inverse-**D** operator or $\mathbf{D}^{-1}$. **U** does the direct opposite of what **D** does. Similarly, the Step-Down operator **D** is also the Inverse-**U** operator or $\mathbf{U}^{-1}$. The product of Step-Up operator **U** and the Step-Down operator **D** is a constant,

$\mathbf{UD}=\beta$ (7.1)
$\mathbf{U}=\beta D^{-1}$ (7.2)
$\mathbf{D}=\beta U^{-1}$ (7.3)

where β is a constant.

Step-Up operator **U**, and Step-Down operator **D**, are inverse of each other except for a scalar factor.

a) **Step-Down Operator D**

We have ground state-0 and state-1,

$\psi_o(y)=\exp(-(1/2)y^2)$ (7.1.1)
$\psi_1(y)=-y\psi_o(y)$ (7.1.2)

The derivative of $\Psi_1(y)$ is given by,

$\partial\psi_1(y)/\partial y=[y^2-1)]\psi_o(y)$ (7.1.3)

We can write this as,

$\partial\psi_1(y)/\partial y=-y\psi_1(y)-\psi_o(y)$ (7.1.4)
$\psi_o(y)=-(\partial/\partial y+y)\psi_1(y)$ (7.1.5)
$\psi_o(y)=\mathbf{D}\psi_1(y)$ (7.1.6)

where,

$\mathbf{D}=-(\partial/\partial y+y)$ (7.1.7)

When operator **D** operates on state $\Psi_1(y)$, it gives one step down lower state $\psi_o(y)$. In other words, operator **D** is a Step-Down operator.

b) **Step-Up Operator U**

If the operator **D** steps down one state, its inverse operator **U** must exist. Operator **U** will reverse the operation **D**. It raises the one step down state back to its original state.

Consider the operation of operator **D** on state $\psi_n(y)$,

$\psi_{n-1}(y)=\mathbf{D}\psi_n(y)$ (7.2.1)

If we operate **U** on both sides, we have,

$\mathbf{U}\psi_{n-1}(y)=\mathbf{UD}\psi_n(y)$ (7.2.2)

Since **U**, and **D**, are inverse of each other, we have,

$\mathbf{UD}=\beta$ (7.2.3)

where, β is a constant.

Substituting in eqn. (7.2.2), we have,

$\mathbf{U}\psi_{n-1}(y)=\beta\psi_n(y)$ (7.2.4)

Lemma: Kind of Inverse

If $\mathbf{D}=-(\partial/\partial y+y)$, and **UD**=constant, where **U** is the inverse of **D**, then, **U** is given by,

$\mathbf{U}=\partial/\partial y-y$ (7.2.5)

Proof:

We have to show that the product **UD** is a constant. The product **UD** is given by,

$$\mathbf{U}\mathbf{D}\psi_n(y)=(\partial/\partial y-y)(-\partial/\partial y-y)\psi_n(y) \quad (7.2.6)$$

$$\mathbf{U}\mathbf{D}\psi_n(y)=(-\partial^2/\partial y^2+y^2)\psi_n(y) -\partial(y\psi_n(y))/\partial y+y\partial\psi_n(y)/\partial y \quad (7.2.7)$$

$$\mathbf{U}\mathbf{D}\psi_n(y)=(-\partial^2/\partial y^2+y^2)\psi_n(y)-\psi_n(y) \quad (7.2.8)$$

$$\mathbf{U}\mathbf{D}\psi_n(y)=\mathbf{H}\psi_n(y)-\psi_n(y) \quad (7.2.9)$$

where, **H** is the Hamiltonian,

$$\mathbf{H}=-\partial^2/\partial y^2+y^2 \quad (7.2.10)$$

$$\mathbf{H}\Psi_n(y)=\varepsilon_n\psi_n(y) \quad (7.2.11)$$

It is clear that the Eigenspace of Hamiltonian **H** is the same as the eigenspace of the product operator **UD**. Substituting in eqn. (7.2.9), we have,

$$\mathbf{U}\mathbf{D}\psi_n(y)=(\varepsilon_n-1)\psi_n(y) \quad (7.2.12)$$

$$\mathbf{U}\psi_{n-1}(y)=(\varepsilon_n-1)\psi_n(y) \quad (7.2.13)$$

In other words,

$$\mathbf{U}\mathbf{D}\psi_n(y)=\beta_n\psi_n(y) \quad (7.2.14)$$

$$\mathbf{U}\psi_{n-1}(y)=\beta_n\psi_n(y) \quad (7.2.15)$$

where, β_n is a constant,

$$\beta_n=(\varepsilon_n-1) \quad (7.2.16)$$

Since $\varepsilon_o=1$, we have, $\beta_o=0$.

This proves that the operator **U** is the inverse of the Step-Down operator **D**, or **UD**=constant. Inverse of the Step-Down operator **D**=-(∂/∂y+y), is the Step-Up operator **U**, **U**=∂/∂y-y.

Similarly, **DU** is given by,

$$\mathbf{D}\mathbf{U}\psi_n(y)=\mathbf{H}\psi_n(y)+\psi_n(y) \quad (7.2.17)$$

Subtracting eqn. (7.2.9) from eqn. (7.2.17), we have,

$$(\mathbf{D}\mathbf{U}-\mathbf{U}\mathbf{D})\psi_n(y)=2\psi_n(y) \quad (7.2.18)$$

$$[\mathbf{D},\mathbf{U}]\psi_n(y)=2\psi_n(y) \quad (7.2.19)$$

So, we have,

$$[\mathbf{D},\mathbf{U}]=2 \quad (7.2.20)$$

$$[\mathbf{U},\mathbf{D}]=-2 \quad (7.2.21)$$

This relationship [**D**, **U**]=2 becomes useful later.

c) **An Alternate Approach to Step-Up Operator U**

We have obtained Step-Up operator **U** as the inverse of **D**. There is an alternate way to look at the Step-Up operator **U**. We know that the state-1, $\psi_1(y)$ can be obtained as the first derivative of the ground state $\psi_o(y)$ as well as just the product of -y and $\psi_o(y)$. As a result, we can write $\psi_1(y)$ as,

$$2\psi_1(y)=(\partial/\partial y-y)\psi_o(y) \quad (7.3.1)$$

$$2\psi_1(y)=\mathbf{U}\psi_o(y) \quad (7.3.2)$$

where, the Step-Up operator **U** is given by,

$$\mathbf{U}=\partial/\partial y-y \quad (7.3.3)$$

Since $\mathbf{D}\psi_1(y)=\psi_o(y)$, eqn. (7.3.2) can be written as,

$$2\psi_1(y)=\mathbf{U}\mathbf{D}\psi_1(y) \quad (7.3.4)$$

First state, $\psi_1(y)$ is an eigenfunction of **UD** with eigen value of 2. From eq. (7.2.14),

$$\mathbf{U}\mathbf{D}\psi_n(y)=\beta_n\psi_n(y) \quad (7.3.5)$$

For n=1, we have,

$$\mathbf{U}\mathbf{D}\psi_1(y)=\beta_1\psi_1(y) \quad (7.3.6)$$

Comparing eqns. (7.3.4) and (7.3.6), we have,

$$\beta_1=2 \quad (7.3.7)$$

It is clear from eqn. (7.2.16) that for n=0, $\beta_o=0$ since $\varepsilon_o=1$. Further, the fact that $\beta_o=0$ is also clear since $\mathbf{U}\mathbf{D}\psi_o(y)=0$.

When operator **U** operates on $\psi_o(y)$, it gives the next higher state $\psi_1(y)$. The operator **U** represents the Step-Up operator. Since the operator **U** does not depends on any particular state, it is a general operator and applies to any state. Operator **U** takes any state $\psi_n(y)$ and gives the next higher state $\psi_{n+1}(y)$.

More importantly, every time Step-Up operator **U** operates on any state, it increases the eigenvalue of the state by 2. At state n, the eigenvalue of the state will be 2n,

$$\beta_n=2n \quad (7.3.8)$$

Substituting in eqn. (7.3.5),

$$\mathbf{U}\mathbf{D}\psi_n(y)=2n\psi_n(y) \quad (7.3.9)$$

The exact proof of this relationship is given later under the product operator **N**, where, **N**=**UD**. For now, we know that it holds true for n=0 and n=1.

The n^th^ state, $\psi_n(y)$ is the eigenfunction of the product operator **N**, **N**=**UD** with eigenvalue 2n. This is the reason for the factor 2n in the one-step solution to the Quantum Oscillator wave equation.

Eqns. (7.3.9) can also be written as,

$$\mathbf{U}\psi_{n-1}(y)=2n\psi_n(y) \quad (7.3.10)$$

d) **Step-Down Operator D on Ground State** $\psi_o(y)$

The operator **D** transforms a higher state to the next nearest lower state. If operator **D** operating on any state gives the state that is one step down, what happens when operator **D** operates on ground state $\psi_o(y)$. The ground state is given by,

$$\psi_o(y)=\exp(-(1/2)y^2) \quad (7.4.1)$$

Applying operator **D**, we have,

$$\mathbf{D}\psi_o(y)=-\partial(\exp(-(1/2)y^2))/\partial y-y\exp(-(1/2)y^2) \quad (7.4.2)$$

$$\mathbf{D}\psi_o(y)=0 \quad (7.4.3)$$

where,

$$\mathbf{D}=-(\partial/\partial y+y) \quad (7.4.4)$$

Step-Down operator **D** operating on the ground state $\psi_o(y)$ results in a null state. This is the case since there is no lower level below the ground state. This also proves that there are no fractional energy levels.

At the ground state, the angular frequency of the quantum oscillator, ω_o is a constant, $\omega_o=(k/m)^{1/2}$ that is determined by the parameters k and m which are constants for a particle in Harmonic Oscillation, not by the environment the particle is in. So, there is no fractional ω_o. Since there is no fractional energy quanta, the lowest energy possible at angular frequency ω_o is $\hbar\omega_o$. There can only be integer multiples of energy quantum, $n\hbar\omega_o$, where n is an integer. As a result, there will only be higher energy states above the ground state. Step-Down operator acting on ground state producing a null state confirms that. Of course, this is known for ages.

Operators **U** and **D** do not have subscript n attached to them and hence they are general operators that applies to any state n of Quantum Oscillators,

$$\mathbf{D}=-(\partial/\partial y+y) \quad (7.4.5)$$

$$\mathbf{U}=\partial/\partial y-y. \quad (7.4.6)$$

Operators **U** and **D** are absolutely real. Was this known for ages? I am not sure. If this is known, why did they stick j in Step-Up and Step-Down operators in Quantum Mechanics making them complex operators? In any case, Step-Up operator **U** and Step-Down operator **D** are real, that is what is important.

VIII. PRODUCT OPERATOR N=UD

Now, we have the Step-Up operator **U** and the Step-Down operator **D**. The product operator **N** is given by,

$$\mathbf{N}=\mathbf{UD} \tag{8.1}$$

Operating **N** on $\psi_n(y)$, we have,

$$\mathbf{N}\psi_n(y)=\mathbf{U}(\mathbf{D}\psi_n(y)) \tag{8.2}$$

$$\mathbf{D}\psi_n(y))=\psi_{n-1}(y) \tag{8.3}$$

Now, we have,

$$\mathbf{N}\psi_n(y)=\mathbf{U}\psi_{n-1}(y)) \tag{8.4}$$

Theorem:

The n^{th} eigenstate of **N** is given by,

$$\mathbf{N}\psi_n(y)=2n\psi_n(y),\ \forall n,\ n=0,1,2,\ldots \tag{8.5}$$

Proof:

We have already seen this holds true for n=0 and 1. Assume this holds true for n. Then, we have,

$$\mathbf{N}\psi_n(y)=2n\psi_n(y) \tag{8.6}$$

Now, for the state n+1, we have,

$$\mathbf{N}\psi_{n+1}(y)=\mathbf{N}(\mathbf{U}\psi_n(y)) \tag{8.7}$$

Now, we want to find a way to turn **NU** into **UN**.
Consider the operator **NU-UN**,

$$\mathbf{NU}-\mathbf{UN}=\mathbf{UDU}-\mathbf{UUD} \tag{8.8}$$

$$\mathbf{NU}-\mathbf{UN}=\mathbf{U}[\mathbf{D},\mathbf{U}] \tag{8.9}$$

From eqn. (7.2.18), we have,

$$(\mathbf{DU}-\mathbf{UD})\psi_n(y)=2\psi_n(y) \tag{8.10}$$

$$[\mathbf{D},\mathbf{U}]=2 \tag{8.11}$$

Substituting in (8,9), we have,

$$\mathbf{NU}-\mathbf{UN}=2\mathbf{U} \tag{8.12}$$

$$\mathbf{NU}=\mathbf{UN}+2\mathbf{U} \tag{8.13}$$

Substituting for **NU** in eqn. (8.7), we have,

$$\mathbf{N}\psi_{n+1}(y)=(\mathbf{UN}+2\mathbf{U})\psi_n(y)) \tag{8.14}$$

Since we assumed it to be true for n, we have,

$$\mathbf{N}\psi_n(y)=2n\psi_n(y) \tag{8.15}$$

Substituting for **N** in eqn. (8.14), we have,

$$\mathbf{N}\psi_{n+1}(y)=(2n+2)\mathbf{U}\psi_n(y)) \tag{8.16}$$

$$\mathbf{N}\psi_{n+1}(y)=2(n+1)\psi_{n+1}(y)) \tag{8.17}$$

If $\psi_n(y)$ is an eigenstate of **N** with eigenvalue 2n, then, $\psi_{n+1}(y)$ is also an eigenstate of **N** with eigenvalue 2(n+1).
The relationship,

$$\mathbf{N}\psi_n(y)=2n\psi_n(y) \tag{8.18}$$

is true for n=0 and n=1. It is also true for n+1 if it is true for n. As a result, it is true for $\forall$n, n=0, 1, 2, …

Substituting for **N** in eqn, (8.6), we have,

$$\mathbf{UD}\psi_n(y)=2n\psi_n(y) \tag{8.19}$$

$$\mathbf{U}\psi_{n-1}(y)=2n\psi_n(y) \tag{8.20}$$

Theorem:

The n^{th} eigenstate $\psi_n(y)$ of a Quantum Oscillator is an eigenfunction of the product operator **N** with eigenvalue 2n.

When operator **N** operates on the ground state $\psi_o(y)$, we have,

$$\mathbf{N}\psi_o(y)=\mathbf{U}(\mathbf{D}\psi_o(y)) \tag{8.21}$$

$$\mathbf{N}\psi_o(y)=0 \tag{8.22}$$

The ground state $\psi_o(y)$ is the eigenfunction of the product operator **N** with zero eigen value. Operator **N** nullifies the ground state $\psi_o(y)$. If **N** operates consecutively on states, the eigenvalue advances by 2 units in each state. As a result, at the state n, the eigenvalue will be 2n, $\forall$n,

$$\mathbf{N}\psi_n(y)=2n\psi_n(y) \tag{8.23}$$

From eqn. (7.2.9), we already have,

$$\mathbf{UD}\psi_n(y)=\mathbf{H}\psi_n(y)-\psi_n(y) \tag{8.24}$$

$$\mathbf{N}\psi_n(y)=\mathbf{H}\psi_n(y)-\psi_n(y) \tag{8.25}$$

$$\mathbf{H}\psi_n(y)=(\mathbf{N}+1)\psi_n(y) \tag{8.26}$$

It is clear that the eigenstate of the product operator **N**, **N**=**UD**, is also the eigenstate of the Hamiltonian **H**.

Lemma: Sharing Generously

The eigenspace of the product operator **N** is the same as the eigenspace of the Hamiltonian **H**,

Eigenspace(**N**)≡Eigenspace(**H**),

where, **N**=**UD**.

We have already seen that the operation of **N** increases the eigenvalue by 2 units. From eqns. (8.5) and (8.26), we have,

$$\mathbf{H}\psi_n(y)=(2n+1)\psi_n(y) \tag{8.27}$$

$$\mathbf{H}\psi_n(y)=\varepsilon_n\psi_n(y) \tag{8.28}$$

where,

$$\varepsilon_n=(2n+1),\ \forall n,\ n=0,1,2,\ldots \tag{8.29}$$

This shows that the states we obtained using Step-Up operator **U**, and Step-Down operator, **D** are eigenstates of the Hamiltonian operator **H** of the Quantum Oscillator and solution to the wave equation,

$$-\partial^2\psi_n(y)/\partial y^2+y^2\psi_n(y)=\varepsilon_n\psi_n(y) \tag{8.30}$$

Corollary: Always Doubling

The n^{th} eigenvalue of the product operator **N**, **N**=**UD** is 2n.

We have already seen that $\psi_n(y)$ that satisfy the condition,

$$\partial^2\psi_n(y)/\partial y^2=[y^2-(2n+1)]\psi_n(y) \tag{8.31}$$

satisfies the wave equation for a Quantum Oscillator for n=0 and n=1. Let us see if it satisfies for n=2.

IX. STATE-2, n=2

We already have the eigenstate n=1, $\psi_1(y)$ with the eigenvalue ε_1. Applying Step-Up operator **U** on $\psi_1(y)$, we have the eigenstate n=2, $\psi_2(y)$,

$$\psi_2(y)=\mathbf{U}\psi_1(y) \tag{9.1}$$

where, **U**=$\partial/\partial y$-y.
From eqn. (6.2.1), the eigenstate $\psi_n(y)$ for n=1 is given by,

$$\psi_1(y)=-y\psi_o(y) \tag{9.2}$$

Now, we have,

$$\psi_2(y)=(\partial\psi_1(y)/\partial y)-y\psi_1(y) \tag{9.3}$$

$$\psi_2(y)=2y^2\psi_o(y)-\psi_o(y) \tag{9.4}$$

$$\psi_2(y)=(2y^2-1)\psi_o(y) \tag{9.5}$$

Differentiating $\Psi_2(y)$, we get,

$$\psi_2'(y)=-y(2y^2-1)\psi_o(y)+4y\psi_o(y) \tag{9.6}$$

$$\psi_2'(y)=(-2y^3+5y)\psi_o(y) \tag{9.7}$$

The second derivative of $\psi_2(y)$ is given by,

$$\psi_2''(y)=(-y(-2y^3+5y)-6y^2+5)\psi_o(y) \tag{9.8}$$

$$\Psi_2''(y)=(2y^4-5y^2-6y^2+5)\psi_o(y) \tag{9.9}$$

$$\psi_2''(y)=(2y^4-11y^2+5)\psi_o(y) \tag{9.10}$$

$$\psi_2''(y)=(y^2-5)(2y^2-1)\psi_o(y) \tag{9.11}$$

$$\psi_2''(y)=(y^2-5)\psi_2(y) \tag{9.12}$$

This is same as,

$$\partial^2\psi_n(y)/\partial y^2=[y^2-(2n+1)]\psi_n(y) \text{ for } n=2 \tag{9.13}$$

This is the condition required for $\psi_2(y)$ to be a solution of the wave equation of the Quantum Oscillator. It is clear that the condition,

$\partial^2\psi_n(y)/\partial y^2=[y^2-(2n+1)]\psi_n(y)$ holds true for n=2.

All About State-2, n=2

For the second higher state, we have,

$$\psi_2(y)=(2y^2-1)\psi_o(y) \tag{9.14}$$

$$\psi_o(y)=\exp(-(1/2)y^2) \tag{9.15}$$

$$\varepsilon_2=4+1=5 \tag{9.16}$$

$$\varepsilon_n=(2n+1), \text{ at } n=2 \tag{9.17}$$

$$\varepsilon_o=1 \tag{9.18}$$

$$\partial^2\psi_n(y)/\partial y^2=[y^2-(2n+1)]\psi_n(y) \text{ for } n=2 \tag{9.19}$$

$$\psi_2(y)=g_2(y)\psi_o(y) \tag{9.20}$$

where,

$$g_2(y)=2y^2-1 \tag{9.21}$$

Function $g_2(y)\psi_o(y)$ is the Hermite of order 2. $g_2(y)$ is the Hermite polynomial of order 2.

X, STATE-3, n=3

We want to find out if the condition for $\psi_n(y)$ to be a solution to a Quantum Oscillator is satisfied for n=3,

$$\partial^2\psi_n(y)/\partial y^2=[y^2-(2n+1)]\psi_n(y) \text{ for } n=3 \tag{10.1}$$

We already have $\psi_2(y)$,

$$\psi_2(y)=(2y^2-1)\ \psi_o(y) \tag{10.2}$$

Applying Step-Up operator **U**, we have,

$$\psi_3(y)=\mathbf{U}\psi_2(y) \tag{10.3}$$

$$\psi_3(y)=(\partial\psi_2(y)/\partial y)-y\psi_2(y) \tag{10.4}$$

$$\psi_3(y)=(-2y(2y^2-1)+4y)\psi_o(y) \tag{10.5}$$

$$\psi_3(y)=(-4y^3+6y)\psi_o(y) \tag{10.6}$$

Disregarding the scale factor,

$$\psi_3(y)=(2y^3-3y)\psi_o(y) \tag{10.7}$$

First derivative is given by,

$$\psi_3'(y)=(-y(2y^3-3y)+6y^2-3)\psi_o(y) \tag{10.8}$$

$$\psi_3'(y)=(-2y^4+9y^2-3)\psi_o(y) \tag{10.9}$$

The second derivative is given by,

$$\psi_3''(y)=(-y(-2y^4+9y^2-3)-8y^3+18y)\psi_o(y) \tag{10.10}$$

$$\psi_3''(y)=(2y^5-17y^3+21y)\psi_o(y) \tag{10.11}$$

$$\psi_3''(y)=(y^2-7)(2y^3-3y)\psi_o(y) \tag{10.12}$$

$$\psi_3''(y)=(y^2-7)\psi_3(y) \tag{10.13}$$

$$\varepsilon_3=2(3)+1=7 \tag{10.14}$$

This is same as,

$$\partial^2\psi_n(y)/\partial y^2=[y^2-(2n+1)]\psi_n(y) \text{ for } n=3 \tag{10.15}$$

This is the condition required for $\psi_3(y)$ to be a solution for the wave equation of the Quantum Oscillator. It is true for n=3. Square value of a very specific null of the second derivative of the eigenstate is also an eigen value of the Hamiltonian.

All About State-3, n=3

For the third higher state, n=3, we have,

$$\psi_3(y)=(2y^3-3y)\psi_o(y) \tag{10.16}$$

$$\psi_o(y)=\exp(-(1/2)y^2) \tag{10.17}$$

$$\varepsilon_3=2(3)+1=7 \tag{10.18}$$

$$\varepsilon_n=(2n+1), \text{ at } n=3 \tag{10.19}$$

$$\varepsilon_o=1 \tag{10.20}$$

$$\partial^2\psi_n(y)/\partial y^2=[y^2-(2n+1)]\psi_n(y) \text{ for } n=3 \tag{10.21}$$

$$\psi_3(y)=g_3(y)\psi_o(y) \tag{10.22}$$

where,

$$g_3(y)=2y^3-3y \tag{10.23}$$

Function $g_3(y)\psi_o(y)$ is the Hermite of order 3. $g_3(y)$ is the Hermite polynomial of order 3.

For an iteration to holds true, we only have to show the condition, $\partial^2\psi_n(y)/\partial y^2=[y^2-(2n+1)]\psi_n(y)$ holds true for n=0 if we can show that it holds true for n+1 provided that it is true for n. So far, we have not shown that. It does not matter for how many n values the condition holds true, the proof is not going to be complete until we show that it is true for n+1 provided it holds true for n. Let us try one more n value before we consider the general case.

XI. STATE-4, n=4

We have seen that the condition for $\psi_n(y)$ to be a solution for a Quantum Oscillator holds true for n=0, 1, 2, 3. Now, we want to see if it holds true for the eigenstate 4, n=4.

We already have the state $\psi_3(y)$. We can obtain the state $\psi_4(y)$ using the Step-Up operator **U** on $\psi_3(y)$,

$$\psi_4(y)=\mathbf{U}\psi_3(y) \tag{11.1}$$

$$\psi_3(y)=(2y^3-3y)\psi_o(y) \tag{11.2}$$

where, $\mathbf{U}=\partial/\partial y-y$.

$$\psi_4(y)=(\partial\psi_3(y)/\partial y)-y\psi_3(y) \tag{11.3}$$

$$\psi_4(y)=(-2y(2y^3-3y)+6y^2-3)\psi_o(y) \tag{11.4}$$

$$\psi_4(y)=(-4y^4+12y^2-3)\psi_o(y) \tag{11.5}$$

First derivative is given by,

$$\psi_4'(y)=(-y(-4y^4+12y^2-3)-16y^3+24y)\psi_o(y) \tag{11.6}$$

$$\psi_4'(y)=(4y^5-28y^3+27y)\psi_o(y) \tag{11.7}$$

The second derivative is given by,

$$\psi_4''(y)=(-y(4y^5-28y^3+27y)+20y^4-84y^2+27)\psi_o(y) \tag{11.8}$$

$$\psi_4''(y)=(-4y^6+48y^4-111y^2+27)\psi_o(y) \tag{11.9}$$

$$\psi_4''(y)=(y^2-9)(-4y^4+12y^2-3)\psi_o(y) \tag{11.10}$$

$$\psi_4''(y)=(y^2-9)\psi_4(y) \tag{11.11}$$

This is same as,

$$\partial^2\psi_n(y)/\partial y^2=[y^2-(2n+1)]\psi_n(y) \text{ for } n=4 \tag{11.12}$$

This is the condition required for $\psi_4(y)$ to be a solution for the wave equation of the Quantum Oscillator. It is true for n=4. Since $\varepsilon_n=(2n+1)$, at n=4, we have,

$$\varepsilon_4=2(4)+1) \tag{11.13}$$

Eigenfunction $\psi_4(y)$ is the fourth state of a Quantum Harmonic Oscillator with eigenvalue,

$$\varepsilon_4=9. \tag{11.14}$$

All About State-4, n=4

For the fourth higher state, n=4, we have,

$$\psi_4(y)=(-4y^4+12y^2-3)\psi_o(y) \tag{11.15}$$

$$\psi_o(y)=\exp(-(1/2)y^2) \tag{11.16}$$

$$\varepsilon_4=2(4)+1=9 \tag{11.17}$$

$$\varepsilon_n=(2n+1), \text{ at } n=4 \tag{11.18}$$

$$\varepsilon_o=1 \tag{11.19}$$

$$\partial^2\psi_n(y)/\partial y^2=[y^2-(2n+1)]\psi_n(y) \text{ for } n=4 \tag{11.20}$$

$$\psi_4(y)=g_4(y)\psi_o(y) \quad (11.21)$$

where,

$$g_4(y)=(-4y^4+12y^2-3) \quad (11.22)$$

Function $g_4(y)\psi_o(y)$ is the Hermite of order 4. $g_4(y)$ is the Hermite polynomial of order 4. Now, it is time to show that it holds true for any n. It just occurred to me how to show it for a general case.

XII. STATE n

For any state $\psi_n(y)$, $\forall n$, n=0, 1, 2, … to describe the dynamics of a Quantum Oscillator, $\psi_n(y)$ must satisfy the wave equation,

$$-\partial^2\psi_n(y)/\partial y^2+y^2\psi_n(y)=\varepsilon_n\psi_n(y) \quad (12.1)$$

However, it is important to note that unconstrained solutions to the wave equation have infinite span whereas the span of any Quantum Oscillator is limited by the energy of the Oscillator. Therefore, unconstrained solutions do not represent Quantum Oscillators. It is only the solutions where the span is constrained to satisfy the energy level of an Oscillator that can represent the states of a Quantum Oscillator. We will consider span constrained solutions later.

For a given Oscillator, the fundamental angular frequency ω_o is a constant. There are no fractional ω_o. Frequency of any state can be integer multiples of ω_o, $n\omega_o$, where n is an integer.

Our goal is to find (ε_n, $\psi_n(y)$ pairs that satisfy the wave equation of the Oscillator given in eqn. (12.1). We found that if $\psi_o(y)$ is a solution to the wave equation, then, $\psi_o'(y)$ is also a solution under the constrain that $\psi_o'(y)=-y\psi_o(y)$. This allowed us to obtain the ground state solution $\psi_o(y)$ and the next higher state $\psi_1(y)$.

Once states $\psi_o(y)$ and $\psi_1(y)$ are found, we could see how to Step-Up to state $\psi_1(y)$ from state $\psi_o(y)$. It also allows us to see how to Step-Down from state $\psi_1(y)$ to state $\psi_o(y)$. This information led to the Step-Up operator **U** and Step-Down operator **D**. Both Step-Up operator **U** and Step-Down **D** are absolutely real. Operators **U**, and **D**, are inverse of each other or their product **UD** is a constant. No complex operators are required to find solutions to the wave equation for the Quantum Harmonic Oscillator.

Property: We are Real

Both Step-Up operator **U** and Step-Down operator **D** are real, not complex. **U** and **D** are inverse of each other; their product is a constant.

The Step-Up operator **U** and the Step-Down operator **D** are given by,

$$\mathbf{U}=\partial/\partial y-y \quad (12.2)$$

$$\mathbf{D}=-\partial/\partial y-y \quad (12.3)$$

As we can see, the operators, **U** and **D**, are not complex operators. When we generate consecutive eigenvalue and eigenfunction pairs (ε_n, $\psi_n(y)$), $\forall n$, n=0, 1, 2, …, a pattern immerges that we can generalize it to the state n. This generalized pattern obtained iteratively is the same as the general one-line solution.

The wave functions $\psi_n(y)$ $\forall n$, n=0, 1, 2, … or eigen states satisfy the wave equation,

$$-\partial^2\psi_n(y)/\partial y^2+y^2\psi_n(y)=\varepsilon_n\psi_n(y) \quad (12.4)$$

At any state n, eigenstate $\psi_n(y)$ also satisfies the condition,

$$\partial^2\psi_n(y)/\partial y^2=[y^2-(2n+1)]\psi_n(y) \quad (12.5)$$

where,

$$\psi_o(y)=\exp(-(1/2)y^2) \quad (12.6)$$

Eigenstate $\psi_n(y)$ can also be written as,

$$\psi_n(y)=g_n(y)\psi_o(y) \quad (12.7)$$

where, $g_n(y)\psi_o(y)$ is the Hermite of order n and $g_n(y)$ is the Hermite polynomial of order n.

The eigenvalue at eigenstate $\psi_n(y)$ is given by,

$$\varepsilon_n=(2n+1) \quad (12.8)$$

Eigen values have the pattern, (n=0, ε_0=1), (n=1, ε_1=3), (n=2, ε_2=5), (n=3, ε_3=7), (n=4, ε_4=9) … .

As we move to higher and higher states, the energy level increases by 2 units between two consecutive eigenstates. The relationship to the actual position x and energy E_n is given by,

$$y=(m\omega_o/\hbar)^{1/2}x \quad (12.9)$$

$$\varepsilon_n=2E_n/\hbar\omega_o \quad (12.10)$$

where, E_n is the mechanical energy of the particle (both kinetic energy and potential energy),

$\hbar=h/2\pi$, h is the Plank constant

$\omega_o=(k/m)^{1/2}$, k is the restoration force constant (Hook's coefficient) of the Harmonic Oscillator, m is the mass of the particle, and x is the displacement from the equilibrium position.

As we can see, ω_o is a constant that is determined by k and m that are constants. There cannot be fractional ω_o. As it is evident from the eigenvalues, $\varepsilon_n=(2n+1)$, $\forall n$, n=0, 1, 2, 3, …, a particle can only have integer multiples of the fundamental angular frequency of the particle, ω_o.

Substituting for ε_n, the actual energy level E_n is given by,

$$E_n=(1/2)\varepsilon_n\hbar\omega_o \quad (12.11)$$

Since $\varepsilon_n=(2n+1)$, we have,

$$E_n=(1/2)\hbar\omega_o(2n+1) \quad (12.12)$$

$$E_n=\hbar\omega_o(n+1/2) \quad (12.13)$$

Although this is the energy we obtained in the solution, there is an inherent problem associated with it since there cannot be fractional $\hbar$ or fractional ω_o. There cannot be fractional $\hbar\omega_o$. The minimum energy of the particle is $\hbar\omega_o$ if we make the invalid assumption that the mechanical energy is quantized. There can neither be fractional energy no fractional spins. We need to address this problem. This is not a problem that is intrinsic to a Harmonic Oscillator. This is a problem stems from the foundation of Quantum Mechanics itself [3].

XIII. STATE n+1

We have shown that the condition that the second derivative of eigenstate $\psi_n(y)$ must satisfy in order for it to be a solution to the wave equation is true for n=0, 1, 2, 3, 4. They do not prove that it is true for all n. We need one more general step in order to complete the proof.

Required Condition:
$\partial^2\psi_n(y)/\partial y^2=[y^2-(2n+1)]\psi_n(y)$ $\forall n$, 0, 1, 2, 3, 4, …

Lemma: Iteration

If condition $\partial^2\psi_n(y)/\partial y^2=[y^2-(2n+1)]\psi_n(y)$ holds true for any state n, then it holds true for state n+1.

Proof:

We have seen that the eigenfunction $\psi_n(y)$ is an unconstrained solution to the wave equation if the second derivative of $\psi_n(y)$ satisfies the condition,

$$\partial^2\psi_n(y)/\partial y^2=[y^2-(2n+1)]\psi_n(y) \quad (13.1)$$

We want to see if the condition holds true for the state n+1 given that it holds true for state n.

We have the Step-Up operator **U**,

$$\mathbf{U}=\partial/\partial y-y \quad (13.2)$$

Applying Step-Up operator **U** on state $\psi_n(y)$, we have,

$$\psi_{n+1}(y)=\mathbf{U}\psi_n(y) \quad (13.3)$$

$$\psi_{n+1}(y)=\psi_n'(y)-y\psi_n(y) \quad (13.4)$$

Taking the first derivative on both sides, we have,

$$\psi_{n+1}'(y)=\psi_n''(y)-y\psi_n'(y)-\psi_n(y) \quad (13.5)$$

Since the condition holds true for state n, substituting for $\psi_n''(y)$ from eqn. (13.1), we have,

$$\psi_{n+1}'(y)=(y^2-(2n+1))\psi_n(y)-y\psi_n'(y)-\psi_n(y) \quad (13.6)$$

Taking the derivative again, we have,

$$\psi_{n+1}''(y)=(y^2-(2n+1))\psi_n'(y)+2y\psi_n(y)-y\psi_n''(y)-2\psi_n'(y) \quad (13.7)$$

Since the condition holds true for state n, substituting for $\psi_n''(y)$ from eqn. (13.1), we have,

$$\psi_{n+1}''(y)=(y^2-(2n+1))\psi_n'(y)+2y\psi_n(y)$$
$$-y(y^2-(2n+1))\psi_n(y)-2\psi_n'(y) \quad (13.8)$$

From eqn. (13.4), we have,

$$y\psi_n(y)=-\psi_{n+1}(y)+\psi_n'(y) \quad (13.9)$$

Substituting in eqn. (13.8), we have,

$$\psi_{n+1}''(y)=(y^2-(2n+1))\psi_n'(y)-2\psi_{n+1}(y)+2\psi_n'(y)$$
$$-y(y^2-(2n+1))\psi_n(y)-2\psi_n'(y) \quad (13.10)$$

$$\psi_{n+1}''(y)=(y^2-(2n+1))(\psi_n'(y)-y\psi_n(y))-2\psi_{n+1}(y) \quad (13.11)$$

Substituting from eqn. (13.4), we have,

$$\psi_{n+1}''(y)=(y^2-(2n+1))\psi_{n+1}(y)-2\psi_{n+1}(y) \quad (13.12)$$

$$\psi_{n+1}''(y)=(y^2-(2(n+1)+1))\psi_{n+1}(y) \quad (13.13)$$

If $\psi_n''(y)=[y^2-(2n+1)]\psi_n(y)$ for state n, then,
$\psi_{n+1}''(y)=(y^2-(2(n+1)+1))\psi_{n+1}'(y)$.

If the condition $\partial^2\psi_n(y)/\partial y^2=[y^2-(2n+1)]\psi_n(y)$ holds true for state n, then, it also holds true for state n+1.

As a result $\psi_n''(y)=[y^2-(2n+1)]\psi_n(y)$ holds true for all n, n=0, 1, 2, ….. .

Unconstrained Solution to the Quantum Oscillator:

Here is the summary:

When $\psi_n(y)$ satisfies the relationship,
$\partial^2\psi_n(y)/\partial y^2=[y^2-(2n+1)]\psi_n(y)$, $\forall n$, n=0, 1, 2, 3, …
then, $\psi_n(y)$ is a solution with eigenvalue $\varepsilon_n=(2n+1)$ to the unconstrained Quantum Dynamics given by the wave equation,
$-\partial^2\psi_n(y)/\partial y^2+y^2\psi_n(y)=\varepsilon_n\psi_n(y)$,
where, $\psi_o(y)=\exp((-1/2)y^2)$,
$y=(m\omega_o/\hbar)^{1/2}x$, $\varepsilon_n=2E_n/\hbar\omega_o$,
x is the displacement of the particle from the equilibrium position and ω_o is the fundamental angular frequency of the particle of mass m and the restoration force constant k, $\omega_o=(k/m)^{1/2}$, E_n is the energy of the particle.
Step-Up operator $\mathbf{U}=\partial/\partial y-y$
Step-Down operator $\mathbf{D}=-\partial/\partial y-y$
U, and **D**, are inverse of each other, $\mathbf{UD}=2n$,
Hamiltonian, $\mathbf{H}=-\partial^2/\partial y^2+y^2$
$\mathbf{H}\psi_n(y)=(2n+1)\psi_n(y)$
$\mathbf{UD}\psi_n(y)=2n\psi_n(y)$
$\mathbf{U}\psi_{n-1}(y)=2n\psi_n(y)$.
All the operations are real.
The product operator $\mathbf{N}=\mathbf{UD}$
Eigenspace of the product operator **N** is the same as the eigenspace of the Hamiltonian **H**.

XIV. NO FRACTIONAL ENERGY QUANTA

If energy comes in quanta, there cannot be fractional quanta. Energy quantum hf at any frequency f is the minimum energy wave burst that can exist at that frequency.

We have seen that a Quantum Harmonic Oscillator comes with a ground state energy $E_o=(1/2)\hbar\omega_o$. This is impossible since the minimum possible energy at angular frequency ω_o is $\hbar\omega_o$. There is something fundamentally wrong here. As we are going to see, it is a result of a wrong assumption with regards to the wavelength of so-called particle waves.

First of all, particles (masses) do not behave as waves, and waves are not particles. Quantum Mechanics was founded upon the invalid assumption that particles behave as waves of deBroglie wavelength λ_{db} given by,

$$\lambda_{db}=h/p \quad (14.1)$$

where, h is the Plank constant and p is the momentum of the particle.

If we multiply both sides of eqn. (14.1) by frequency f_o of the Oscillator, where $\omega_o=2\pi f_o$,

$$f_o\lambda_{db}=hf_o/p \quad (14.2)$$

If the speed of the particle is u, the speed of the particle wave is also u and hence,

$$u=f_o\lambda_{db} \quad (14.3)$$

Substituting in eqn. (14.2), we have,

$$pu=hf_o \quad (14.4)$$

Since p=mu, substituting for p in eqn. (14.4), we have,

$$mu^2=hf_o \quad (14.5)$$

The ground state energy E_o is hf_o and hence, from eqn. (14.5), we have,

$$E_o=mu^2 \quad (14.6)$$

If we assume that mechanical energy is quantized and a particle of momentum p behaves as a wave of deBroglie wavelength, $\lambda_{db}=h/p$, then the energy of the particle at ground state, E_o should be mu^2. This is what the kinetic energy of a particle should be when potential energy is zero for that particle to be at deBroglie wavelength, $\lambda_{db}=h/p$. No mass has the kinetic energy mu^2. Free moving particle or particle at potential zero or in other words, a free-moving particle at ground state only has half the required energy or $(1/2)mu^2$. So, no particle wave can be at deBroglie wavelength. Energy of a particle is insufficient for it to be at deBroglie wavelength. This is the deBroglie particle wave blunder.

Particles do not behave as waves. It is the moving charge particles that generate electromagnetic radiation waves when they are stopped, accelerated, or decelerated. Once generated, propagation of these radiation waves is completely independent of the motion of the particle. They do not describe the state of a particle. They do not describe the probability of particle being at certain location.

XV. WAVELENGTH THAT A PARTICLE IS CAPABLE OF SUPPORTING [3]

No mass (a particle) behaves as a wave. It is simply impossible except in human fantasy. It does not happen in reality. If you want to mystify or voodoofy particles by assuming particles behave as waves, the wavelength of the wave must be the wavelength that the energy of a particle can support.

For a particle of mass m moving at speed u, the ground state energy E_o of the particle is the kinetic energy of the particle and hence,

$$E_o=(1/2)mu^2 \tag{15.1}$$

Since the momentum p of a particle of mass m and speed u is given by p=mu, we have,

$$E_o=(1/2)pu \tag{15.2}$$

If a particle is moving at speed u is behaving as a wave of wavelength λ and frequency f_o, we have,

$$u=f_o\lambda \tag{15.3}$$

Substituting for u in eqn. (15.2), we have,

$$E_o=(1/2)pf_o\lambda \tag{15.4}$$

The energy of a mass m is mechanical energy. Mechanical energy has no existence without an associated mass and as a result, mechanical energy cannot come in quanta [3]. Any entity that has a belonging cannot come in quanta since there is no mechanism for a quantum to carry belonging information. It is only the electromagnetic energy that comes in quanta since electromagnetic energy has no belonging.

If you are going to incorrectly claim that the mechanical energy of a particle is quantized, you have to use the correct energy of a particle. If you are going to assume that a mass behaves as a wave, the energy of the particle must be capable of supporting that wavelength. Under the invalid assumption that the energy of a particle is quantized, we have,

$$E_o=hf_o \tag{15.5}$$

From eqns. (15.4) and (15.5), we have,

$$hf_o=(1/2)pf_o\lambda \tag{15.6}$$

$$\lambda=2h/p \tag{15.7}$$

$$\lambda=2(\text{deBroglie wavelength}) \tag{15.8}$$

If you incorrectly assume that a particle of mass m and momentum p behaves as a wave, and also incorrectly assume that the mechanical energy is quantized, then the correct wavelength that the energy of a particle can support is twice the deBroglie wavelength, $\lambda=2\lambda_{db}$, where deBroglie wavelength λ_{db}=h/p. It is this wavelength error that has made havoc in Quantum Mechanics.

XVI. CORRECT ENERGY SPECTRUM FOR QUANTUM HARMONIC OSCILLATOR

As we have seen in eqn. (12.13), under deBroglie wavelength λ_{db}=h/p used in Quantum Mechanics, the energy spectrum of a Quantum Oscillator E_n, n=0, 1, 2, … is given by,

$$E_n=\hbar\omega_o(n+1/2),\ \forall n,\ n=0, 1, 2, \ldots. \tag{16.1}$$

The actual wavelength that any mass can support is twice the deBroglie wavelength and given by,

$$\lambda=2h/p \tag{16.2}$$

So, in order to correct the error due to incorrect deBroglie wavelength, all we have to do is substitute 2h in place of h in the energy spectrum E_n of the Quantum Oscillator obtained for the deBroglie wavelength λ_{db}=h/p. However, the h in the time progression operator is unaffected by the deBroglie wavelength error [3].

Now, we have the correct energy spectrum E_n for a Quantum Oscillator,

$$E_n=(2\hbar)\omega_o(n+1/2),\ \forall n,\ n=0, 1, 2, \ldots. \tag{16.3}$$

$$E_n=\hbar\omega_o(2n+1),\ \forall n,\ n=0, 1, 2, \ldots. \tag{16.4}$$

The energy levels of a Quantum Oscillator are,
$\hbar\omega_o(1, 3, 5, 7, 9, \ldots.)$.
The actual energy level increases by 2 units when one moves from one energy level to the next higher level.

The categorization of particles (masses) such as electrons and protons as Quantum 1/2 particles is incorrect and meaningless. There is no Quantum 1/2 [3]. Fractional energy quanta cannot exist by the very definition of energy quantum. Quantum is the smallest energy unit that can exist. Quantum is no longer a Quantum if there is a fractional quantum. There are no one half energy quanta. The smallest energy unit is $\hbar\omega_o$. The energy spectrum of a Quantum Oscillator of mass m and restoration force coefficient k cannot have $(1/2)\hbar\omega_o$ energy levels. The minimum ground state at zero potential energy is $\hbar\omega_o$. Any higher energy level must be an integer multiple of $\hbar\omega_o$ or $n\hbar\omega_o$, where n is a positive integer.

Corollary: No Fractional Quantum

Quantum is no longer a Quantum if there exists a fraction of a quantum. Quantum-half defies the very definition of Quantum.

It is the case for electromagnetic waves in a cavity. Fundamental frequency f_o of a cavity is determined by the geometry of the cavity. It is also the case for Quantum Harmonic Oscillator if particles are assumed to behave as waves, and the mechanical energy is assumed to be quantized; both assumptions are invalid. Particles (masses) do not behave as waves and mechanical energy cannot come in quanta.

Any entity that has a belonging cannot be quantized. Quantum Mechanics is a voodoo-science by design from its inception, not a science. Quantum Mechanics has turned into a new religion with unquestioning followers, just like all the blind-faith followers of religions. The fact that all the religions were founded by the people who believed either earth was flat, or sun goes around the earth, demonstrates the mockery of it. How can a guy who did not know what goes around what in a planetary system be a messenger of a creator? Quantum Mechanics is not much different from states sponsored dark-age barbaric religious dogma.

Corollary: Non-Quantizability

If an entity has an owner, that entity cannot come in Quanta since there is no mechanism to carry ownership information.

Mechanical energy cannot exist without a mass. Mechanical energy belongs to a particular mass. As a result, mechanical energy cannot come in quanta or cannot be quantized. You cannot quantize vectors either. Quantum field is an oxymoron.

It is only the electromagnetic energy that comes in Quanta since electromagnetic waves have no owner once they are out of a source. Electromagnetic energy quanta are not particles, they are wave bursts. There are no photons or light particles in nature. Photons by definition are spatially random and hence cannot represent directional light. The derivation of the photons also incorrectly assumes that the electromagnetic frequency spectrum in a cavity is continuous, which is not. Electromagnetic spectrum in a cavity is discrete and hence the derivation of photons is inconsistent with reality. If the electromagnetic energy is quantized, electromagnetic spectrum cannot be continuous.

There are no massless particles. There are no wave particles. What is there is wave bursts. Wave bursts have no momentum. Propagation of waves is not due to momentum. It is only motion of a mass that is due to momentum. There is no momentum in massless. Waves bursts are massless. Mass of a particle does not depend on observers. Path of light does not depend on observers. Observer perceptions do not determine the nature. Contrary to what the preachers in the Relativity cult are chanting, observer perceptions do not determine physics of the nature. No mountain is moving relative to a runner. It is only the object that does the work that is moving. Propagation of light does not depend on the observers. Light is not relative [6].

The direction of light is solely determined by the density gradient of the medium or the lack of it. Any entity with a mass cannot travel at constant speed in the universe since there is no place in the universe that is free of gravitational force; the gravitational field of any object is of infinite span. Gravity has no influence on massless; that is the reason why light can travel at constant speed in a gravitational field. Gravity cannot bend light. Gravity can create a density gradient in the medium. It is the density gradient in the medium that bends light. Photons exists only in misguided voodoofied human fantasy, not in reality.

XVII. REAL-OPERATOR MECHANICS FOR QUANTUM OSCILLATORS

Quantum Mechanics has its own dedicated operator mechanics for Quantum Harmonic Oscillators based on the commutation of complex operators [1]. However, there is no reason to use complex operator mechanics for solving a wave equation for a Quantum Harmonic Oscillator. Both Step-Up and Step-Down operators are absolutely real. The Hamiltonian is real. Wave equation is real. Solution to the wave equation of a Quantum Harmonic Oscillator can be achieved directly by using Real Operators.

Although you may find complex operator mechanics for a Quantum Harmonic Oscillator in any Quantum Mechanics Textbook, a good introduction to Quantum Harmonic Oscillators using Complex Operator Mechanics can be found in reference [1]. In fact, it is a good reference not just for Quantum Oscillators, but also for Quantum Mechanics in general. While reading it, rather than learning Quantum Mechanics, an attentive reader who has no religious attachment to Quantum Mechanics may discover at the very beginning of the book, especially in the section related to Spin, that Quantum Mechanics is simply bogus.

Stern-Gerlach Device or any other permanent magnetic field cannot be used to measure or set spin of a particle [3]. Permanent setting of the Spin of a particle is not possible. The Spin of a Single Atom always orients towards an external magnetic field as long as it is in the magnetic field. Once the Atom is out of the magnetic field, spin is no longer towards the magnetic field. Spin of an Atom set by an external magnetic field is volatile. It is only when magnetically coupled beam of atoms is used in the Stern-Gerlach experiment that the beam splits into Spin-Up and Spin-Down beams as long as the Atoms are in the Stern-Gerlach magnetic field.

Spin-Up and Spin-Down are observer dependent and hence cannot be states of a particle. Observer dependent quantities cannot come in quanta. One observer's Spin-Up can be another observer's Spin-Down and vice versa. If you want to know why the mere idea of Spin Quantization and a good part of reference [1] are so ridiculously laughable, you may find it in reference [3].

An attentive reader of reference [1] will discover by himself or herself that Quantum Mechanics cannot be right and why it cannot be right. You may discover yourself what has gone awry in Quantum Mechanics. If it is not for the blind-faith religious attachment of the authors to Quantum Mechanics, they would have discovered themselves what in fact had gone wrong in Quantum Mechanics and why? Religious attachments make people blind to the facts and make them to go on claiming it is the others who are blind.

What is important to realize is that Stern-Gerlach or any other magnetic field cannot be used to obtain the direction of a Spin. Stern-Gerlach Device cannot be used to obtain the components of a Spin along axes or in any direction. Stern-Gerlach Device cannot be used for permanent setting of a Spin to a desired direction. What Stern-Gerlach magnetic field or any other magnetic field does is that it aligns a Spin of an Atom along the direction of its magnetic field irrespective of its original orientation of the Spin as long as the Atom is within the magnetic field. Stern-Gerlach Device or any other magnetic field is blind to the actual direction of Spin of an Atom [3].

Working Principle of the Stern-Gerlach Device:

You are either with us or against us – Bushism. If you are not against us, we will torque you Up. If you are against us, it is Down you go.

Any single Atom enters into a Stern-Gerlach Device is always Spin-Up. It is only when magnetically coupled Atoms enter the Stern-Gerlach in sequence that the beam will be split into two beams of Spin-Up and Spin-Down. Once two beams are out of the Stern-Gerlach Device, they will no longer be Spin-Up or Spin-Down. The magnetic coupling between adjacent Atoms take over and reorient themselves just like the Atoms in the original beam. In the absence of an external magnetic field, the orientation of Atoms in a beam is such, the orientation of the Spins of any two neighboring Atoms will be against each other. More information on how and why that happens can be found in Reference [3].

Noteworthy Fact:

First Atom entering a Stern-Gerlach Device will always be deflected as Spin-Up.

If a beam of Atoms enters the Stern-Gerlach Device, since the Spins of the Atoms in the beam are magnetically coupled so that the Spins of adjacent Atoms are of opposite directions, the beam will be split such that all the odd Atoms will be Spin-Up while all the even Atoms are deflected as Spin-Down. A 50/50 Split of a beam by a Stern-Gerlach Device has nothing to do with probability. It is completely a deterministic process.

Quantum Mechanics is a collection of mathematical and conceptual blunders wrapped in incorrect and invalid experimental interpretations. Double-slit experiment and Stern-Gerlach experiment are two such misinterpreted and voodoofied experiments [3, 4].

Coming back to Quantum Oscillators, here, we introduce the solution to Quantum Harmonic Oscillator using Real Operator Mechanics since no real operator mechanics can be found anywhere.

a) **Real Operators**

By Using the ground state $\psi_o(y)$ and the first state $\psi_1(y)$, we already found the Step-Up operator **U** and the Step-Down operator **D**,

$$\mathbf{U}=\partial/\partial y-y \qquad (17.1.1)$$

$$\mathbf{D}=-\partial/\partial y-y \qquad (17.1.2)$$

Hamiltonian **H** for a Quantum Oscillator is given by,

$$\mathbf{H}=-\partial^2/\partial y^2+y^2 \qquad (17.1.3)$$

The Product Operator **N** is given by,

$$\mathbf{N}=\mathbf{UD} \qquad (17.1.4)$$

At eigenstate $\psi_n(y)$, we have,

$$\mathbf{N}\psi_n(y)=\mathbf{UD}\psi_n(y) \qquad (17.1.5)$$

Substituting for **U** and **D**, we have,

$$\mathbf{N}\psi_n(y)=(\partial/\partial y-y)(-\partial/\partial y-y)\psi_n(y) \qquad (17.1.6)$$

$$\mathbf{N}\psi_n(y)=(-\partial^2/\partial y^2+y^2)\psi_n(y) -\partial(y\psi_n(y))/\partial y+y\partial\psi_n(y)/\partial y \qquad (17.1.7)$$

$$\mathbf{N}\psi_n(y)=(-\partial^2/\partial y^2+y^2)\psi_n(y)-\psi_n(y) \qquad (17.1.8)$$

$$\mathbf{N}\psi_n(y=\mathbf{H}\psi_n(y)-\psi_n(y) \qquad (17.1.9)$$

where, **H** is the Hamiltonian,

$$\mathbf{H}=-\partial^2/\partial y^2+y^2 \qquad (17.1.10)$$

$$\mathbf{H}\psi_n(y)=(\mathbf{N}+1)\psi_n(y) \qquad (17.1.11)$$

This indicates that the eigenstates of the product of Step-Up operator **U** and the Step-Down operator **D**, or in other words the Product Operator **N**, are also eigen states of the Hamiltonian **H**. Further, the eigenvalue of Hamiltonian **H** at any state is one unit higher than the eigenvalue of the Product Operator **N** at the same state. As a result, finding the eigenvalues and the eigenstates of the Hamiltonian **H** is equivalent to finding the eigenvalues and eigenstates of the Product Operator **N** or **UD**. The only significant difference between the Product Operator **UD** or **N** and the Hamiltonian of the particle **H** is that the ground state eigenvalue of **N** is zero while the ground state eigen value of **H** is 1 unit.

b) **Commutation** [**D**, **U**]

$$\mathbf{U}=\partial/\partial y-y \qquad (17.2.1)$$

$$\mathbf{D}=-\partial/\partial y-y \qquad (17.2.2)$$

$$\mathbf{DU}\psi_n(y)=(-\partial/\partial y-y)(\partial/\partial y-y)\psi_n(y) \qquad (17.2.3)$$

$$\mathbf{DU}\psi_n(y)=(-\partial^2/\partial y^2+y^2)\psi_n(y) +\partial(y\psi_n(y))/\partial y-y\partial\psi_n(y)/\partial y \qquad (17.2.4)$$

$$\mathbf{DU}\psi_n(y)=(-\partial^2/\partial y^2+y^2)\psi_n(y)+\psi_n(y) \qquad (17.2.5)$$

$$\mathbf{DU}\psi_n(y=\mathbf{H}\psi_n(y)+\psi_n(y) \qquad (17.2.6)$$

From eqn. (17.1.9), **UD** is give by,

$$\mathbf{UD}\psi_n(y=\mathbf{H}\psi_n(y)-\psi_n(y) \qquad (17.2.7)$$

Subtracting eqn. (17.2.7) from eqn. (17.2.6), we have,

$$(\mathbf{DU}-\mathbf{UD})\psi_n(y)=2\psi_n(y) \qquad (17.2.8)$$

The Commutation [**D**, **U**] is given by,

$$[\mathbf{D}, \mathbf{U}]\psi_n(y)=(\mathbf{DU}-\mathbf{UD})\ \psi_n(y) \qquad (17.2.9)$$

Now, we have,

$$[\mathbf{D}, \mathbf{U}]\psi_n(y)=2\psi_n(y) \qquad (17.2.10)$$

$$[\mathbf{D}, \mathbf{U}]=2 \qquad (17.2.11)$$

Similarly, we can also obtain commutation [**U**, **D**],

$$[\mathbf{U}, \mathbf{D}]=-2 \qquad (17.2.12)$$

c) **Commutation [D, N]**

$$\mathbf{N}=\mathbf{UD} \qquad (17.3.1)$$

$$[\mathbf{D}, \mathbf{N}]=\mathbf{DUD}-\mathbf{UDD} \qquad (17.3.2)$$

$$[\mathbf{D}, \mathbf{N}]=[\mathbf{DU}-\mathbf{UD}]\mathbf{D} \qquad (17.3.3)$$

$$[\mathbf{D}, \mathbf{N}]=[\mathbf{D}, \mathbf{U}]\mathbf{D} \qquad (17.3.4)$$

From eqn. (17.2.11), we have [**D**, **U**]=2 and hence,

$$[\mathbf{D}, \mathbf{N}]=2\mathbf{D} \qquad (17.3.5)$$

Similarly, [**N**, **D**] is given by,

$$[\mathbf{N}, \mathbf{D}]=-2\mathbf{D} \quad (17.3.6)$$

d) **Commutation [U, N]**

$$[\mathbf{U}, \mathbf{N}]=\mathbf{UN}-\mathbf{NU} \quad (17.4.1)$$

Substituting **N=UD**, we have,

$$[\mathbf{U}, \mathbf{N}]=\mathbf{UUD}-\mathbf{UDU} \quad (17.4.2)$$

$$[\mathbf{U}, \mathbf{N}]=\mathbf{U}[\mathbf{UD}-\mathbf{DU}] \quad (17.4.3)$$

From eqn. (17.2.12), we have [**U**, **D**]=-2, and hence

$$[\mathbf{U}, \mathbf{N}]=-2\mathbf{U} \quad (17.4.4)$$

Similarly, [**N**, **U**] is given by,

$$[\mathbf{N}, \mathbf{U}]=2\mathbf{U} \quad (17.4.5)$$

e) **Action of Step-Down Operator D**

Let β_n and $\psi_n(y)$ be an eigenvalue eigenfunction pair of Product Operator **N**. Then, we have,

$$\mathbf{N}\psi_n(y)=\beta_n\psi_n(y) \quad (17.5.1)$$

Given the n^{th} state solution, β_n and $\psi_n(y)$ of the Product Operator **N**, we want to find $(n-1)^{th}$ state of the Product Operator **N**,

$$\mathbf{N}\psi_{n-1}(y)=? \quad (17.5.2)$$

Since $\mathbf{D}\psi_n(y)=\psi_{n-1}(y)$, we have,

$$\mathbf{N}\psi_{n-1}(y)=\mathbf{N}(\mathbf{D}\psi_n(y)) \quad (17.5.3)$$

Now, we want to change **ND** to **DN**. The reason for doing that will be clear eventually. To do that, consider the commutation [**D**, **N**], where,

$$[\mathbf{D}, \mathbf{N}]=\mathbf{DN}-\mathbf{ND} \quad (17.5.4)$$

$$\mathbf{ND}=\mathbf{DN}-[\mathbf{D}, \mathbf{N}] \quad (17.5.5)$$

From eqn. (17.3.5), we already have,

$$[\mathbf{D}, \mathbf{N}]=2\mathbf{D} \quad (17.5.6)$$

Now, we have,

$$\mathbf{ND}=\mathbf{DN}-2\mathbf{D} \quad (17.5.7)$$

Substituting for **ND** in eqn. (17.5.3), we have,

$$\mathbf{N}\psi_{n-1}(y)=(\mathbf{DN}-2\mathbf{D})\psi_n(y) \quad (17.5.8)$$

We already know,

$$\mathbf{N}\psi_n(y)=\beta_n\psi_n(y) \quad (17.5.9)$$

$$\mathbf{D}\psi_n(y)=\psi_{n-1}(y) \quad (17.5.10)$$

Substituting in eqn. (17.5.8), we have,

$$\mathbf{N}\psi_{n-1}(y)=(\beta_n-2)\mathbf{D}\psi_n(y) \quad (17.5.11)$$

Since **D** is the Step-Down operator, we have,

$$\mathbf{N}\psi_{n-1}(y)=(\beta_n-2)\psi_{n-1}(y) \quad (17.5.12)$$

$$\mathbf{N}\psi_{n-1}(y)=\beta_{n-1}\psi_{n-1}(y) \quad (17.5.13)$$

where,

$$\beta_{n-1}= \beta_n-2 \quad (17.5.14)$$

$$\beta_o= 0 \quad (17.5.15)$$

The eigen values of the Product Operator **N** decreases by 2 units when n goes down by 1. When Step-Down operator **D** operates on any state it decreases the eigenvalue of the product operator **UD** by 2 units and provide the next lower state. Since the Step-Down operator **D** is the reverse of the Step-Up operator **U**, the Step-Up operator **U** should do the opposite; operator **U** should raise the eigenvalue of the product operator **UD** by 2 units.

The eigenstates of Hamiltonian **H** are the same as the eigenstates of the Product Operator **N**. The eigen values of the Hamiltonian **H** is the same as the eigen values of Product Operator **N** except an added constant, which is the ground state eigenvalue, $\varepsilon_o=1$. The eigenvalue of the Product Operator **N** at n=0 is zero, $\beta_o=0$ while the eigenvalue of the Hamiltonian **H** at n=0 is $\varepsilon_o=1$. The action of the Step-Down operator **D** brings down an eigenstate of the Hamiltonian **H** by 2 units. As we are going to see, next, the action of the Step-Up operator **U** does the opposite.

f) **Action of Step-Up Operator U**

Let us consider the n^{th} state eigenvalue eigenfunction pair (β_n, $\psi_n(y)$ of the Product Operator **N**,

$$\mathbf{N}\psi_n(y)=\beta_n\psi_n(y) \quad (17.6.1)$$

Given the n^{th} state solution, β_n and $\psi_n(y)$ of the Product Operator **N**, we want to find $(n+1)^{th}$ state of the Product Operator **N**,

$$\mathbf{N}\psi_{n+1}(y)=? \quad (17.6.2)$$

Since $\mathbf{U}\psi_n(y)=\psi_{n+1}(y)$, we have,

$$\mathbf{N}\psi_{n+1}(y)=\mathbf{N}(\mathbf{U}\psi_n(y)) \quad (17.6.3)$$

Now, we want to represent **NU** using **UN**. To do that, consider the commutation [**U**, **N**], where,

$$[\mathbf{U}, \mathbf{N}]=\mathbf{UN}-\mathbf{NU} \quad (17.6.4)$$

$$\mathbf{NU}=\mathbf{UN}-[\mathbf{U}, \mathbf{N}] \quad (17.6.5)$$

From eqn. (17.4.4), we already have,

$$[\mathbf{U}, \mathbf{N}]=-2\mathbf{U} \quad (17.6.6)$$

Now, we have,

$$\mathbf{NU}=\mathbf{UN}+2\mathbf{U} \quad (17.6.7)$$

Substituting for **NU** in eqn. (17.6.3), we have,

$$\mathbf{N}\psi_{n+1}(y)=(\mathbf{UN}+2\mathbf{U})\psi_n(y) \quad (17.6.8)$$

We already know,

$$\mathbf{N}\psi_n(y)=\beta_n\psi_n(y) \quad (17.6.9)$$

$$\mathbf{U}\psi_n(y)=\psi_{n+1}(y) \quad (17.6.10)$$

Now, we have,

$$\mathbf{N}\psi_{n+1}(y)=(\beta_n+2)\mathbf{U}\psi_n(y) \quad (17.6.11)$$

Since **U** is the Step-Up operator, we have,

$$\mathbf{N}\psi_{n+1}(y)=(\beta_n+2)\psi_{n+1}(y) \quad (17.6.12)$$

$$\mathbf{N}\psi_{n+1}(y)=\beta_{n+1}\psi_{n+1}(y) \quad (17.6.13)$$

where,

$$\beta_{n+1}= \beta_n+2 \quad (17.6.14)$$

$$\beta_o= 0 \quad (17.6.15)$$

The eigen values of the product operator **N** increases by 2 units when n goes up by 1. When Step-Up operator **U** operates on any state it increases the eigenvalue of the product operator **N** by 2 units and provide the next higher state, which is the reverse of the Step-Down operator **D**. As a result, at $n^{th,}$ state, the eigen value of **N** will be 2n.

Further, as we have seen before, the product operator **N** operated on the ground state will nullifies it,

$$\mathbf{N}\psi_o(y)=0 \quad (17.6.16)$$

The ground state eigenvalue of **N** is zero, $\beta_o=0$, for n=0, at ground state. Starting from eigenvalue zero at the ground state, every time n is increased by one step, the eigenvalue of N increases by 2 units. As a result, the n^{th} eigenvalue, β_n of **N** is given by,

$$\mathbf{N}\psi_n(y)=\beta_n\psi_n(y) \quad (17.6.17)$$

where, $\beta_n=2n$, $\forall n$, n=0, 1, 2, …

$$\mathbf{N}\psi_n(y)=2n\psi_n(y) \quad (17.6.18)$$

From eqn. (17.1.11), we already have the eigen relationship between the Hamiltonian **H** and the product operator **N**,

$$\mathbf{H}\psi_n(y)=(\mathbf{N}+1)\psi_n(y) \quad (17.6.19)$$

Substituting for $\mathbf{N}\psi_n(y)$ from eqn. (17.6.18), we have,

$$\mathbf{H}\psi_n(y)=(2n+1)\psi_n(y) \quad (17.6.20)$$

$$\mathbf{H}\psi_n(y)=\varepsilon_n\psi_n(y) \quad (17.6.21)$$

where,

$$\varepsilon_n=(2n+1) \quad (17.6.22)$$

From eqn. (12.11), the actual energy, E_n of the n^{th} state is given by,

$$E_n=(1/2)\varepsilon_n\hbar\omega_o \quad (17.6.23)$$

Since $\varepsilon_n=(2n+1)$, we have,

$$E_n=(1/2)\hbar\omega_o(2n+1) \quad (17.6.24)$$

$$E_n=\hbar\omega_o(n+1/2) \quad (17.6.25)$$

where, $\omega_o=(k/m)^{1/2}$.

The lowest energy of the oscillator must be $\hbar\omega_o$. There cannot be fractional quanta. The very idea of fractional quanta goes against the definition of quanta.

Corollary: Ultimate Quantum

Quantum can no longer be a quantum if there is a fractional quantum.

Quantum by definition is an indivisible entity. As a result, there is something wrong about the n^{th} state energy given by eqn. (17.6.25). We have already pointed out and addressed this problem earlier. This problem is due to deBroglie wavelength error. The Spin-Half appears in Quantum Mechanics is also a result of the same wavelength error [3]. DeBroglie wavelength that the Quantum Mechanics was founded upon is incorrect by a factor of one half. No particle (mass) has energy required to be at deBroglie wavelength. As a result, true wavelength is twice the deBroglie wavelength. We can easily correct this error by substituting $2\hbar$ in place of $\hbar$ in eqn. (|17.6.25), so we have,

$$E_n=(2\hbar)\omega_o(n+1/2) \quad (17.6.26)$$

$$E_n=\hbar\omega_o(2n+1),\ \forall n,\ n=0,\ 1,\ 2,\ \ldots \quad (17.6.27)$$

Ground state energy E_o is one energy quanta,

$$E_o=\hbar\omega_o. \quad (17.6.28)$$

When you go up to higher states step by step, at each step, the energy level increases by $2\hbar\omega_o$,

$$E_o=\hbar\omega_o(1,\ 3,\ 5,\ 7,\ 9,\ \ldots) \quad (17.6.29)$$

The difference between two neighboring levels will be 2 Quanta. Complex Operator Mechanics [1] is not required in obtaining solutions to the wave equation for a Quantum Harmonic Oscillator. All the operators are real. If you disregard the phase shift introduced with time, all the eigenfunctions are real.

Property: Real States

With the exception of multiplication factor, all eigenstates of a Quantum Harmonic Oscillator are real at any given time.

g) **Ground State** $\psi_o(y)$

We know that the Step-Down operator **D** operating on ground state result in a null state,

$$\mathbf{D}\psi_o(y)=0 \quad (17.7.1)$$

where,

$$\mathbf{D}=-\partial/\partial y-y \quad (17.7.2)$$

$$-\partial\psi_o(y)/\partial y-y\psi_o(y)=0 \quad (17.7.3)$$

If $\psi_o(y)=|\psi_o(y)|$ and $\psi_o(y)>0$, we have,

$$(1/\psi_o(y))\partial\psi_o(y)=-y\partial y \quad (17.7.4)$$

Integrating both sides over the span of y, we have,

$$\int(1/\psi_o(y))\partial\psi_o(y)=-\int y\partial y \quad (17.7.5)$$

$$\ln\psi_o(y)=-(1/2)y^2+C \quad (17.7.6)$$

$$\psi_o(y)=\exp(-(1/2)y^2+C) \quad (17.7.7)$$

$$\psi_o(y)=e^C\exp(-(1/2)y^2) \quad (17.7.8)$$

where ln is the natural logarithm.

Since we assumed $\psi_o(y)>0$, $\psi_o(y)$ itself can represent a probability distribution if the area under $\psi_o(y)$ is unity, i.e.,

$$\int\psi_o(y)dy=1 \quad (17.7.9)$$

$$\int\exp((-1/2)y^2+C))dy=1 \quad (17.7.10)$$

$$e^C\int\exp((-1/2)y^2)dy=1 \quad (17.7.11)$$

Since $\int\exp((-1/2)y^2)dy=(2\pi)^{1/2}$, we have,

$$e^C=1/(2\pi)^{1/2} \quad (17.7.12)$$

The ground state wave function or assumed ground state probability distribution is given by $\psi_o(y)$, where,

$$\psi_o(y)=(1/(2\pi)^{1/2})\exp(-(1/2)y^2) \quad (17.7.13)$$

The ground state $\psi_o(y)$ itself represents a probability density function since $\psi_o(y)=|\psi_o(y)|$, $\psi_o(y)>0$ and

$$\int\psi_o(y)dy=1. \quad (17.7.14)$$

Since any scale factor of a state has no effect on the solution to the Quantum Harmonic Oscillator, we can disregard the scale factor and hence, we have,

$$\psi_o(y)=\exp(-(1/2)y^2) \quad (17.7.15)$$

It is only at the ground state $\psi_o(y)$ that the state itself can be considered as a probability distribution since $\psi_o(y)=|\psi_o(y)|$ and $\psi_o(y)>0$. No squaring is necessary. This is not the case for $n\neq0$. Wavefunction $\psi_n(y)$, $n>0$, has one or more zeros. In fact, state n, $\psi_n(y)$ has exactly n number of zeros. As we are going to see, wavefunctions that have zeros or nulls cannot represent probability distributions of particle being at certain location since particle will be trapped between nulls in the presence of nulls. However, even the ground state that is free of nulls cannot represent a probability distribution for a different reason, which we consider later.

XVIII. AN EIGENSTATE AND PROBABILITY OF PARTICLE BEING AT A CERTAIN LOCATION

Now, we have wave functions or the states for a Quantum Oscillator, $\psi_n(y)$, $\forall n$, n=0, 1, 2, … It has been incorrectly claimed that the probability of a particle being at a certain location y at any state $\psi_n(y)$ is $|\psi_n(y)|^2$, $\forall n$, n=0, 1, 2, … A particle in nature does not take decision by rolling dies or using a probability distribution. Nature does not do probability. Nature does not make decisions by throwing dies. It is we who use probability to explain certain phenomena when the true nature of the phenomena is unknown to us for the sole purpose of extracting some information in order to make decisions. Probability is a human tool, not a tool of nature.

Probability says nothing about reality. The use of probability is an indication that we are not doing science and we have no idea what the underline mechanism of the system is. You cannot discover the fundamental working of nature using probability. Probability is not a science. Probability is a tool of human decision making, not a design feature of nature. Mathematical Probability was originated in gambling to decide how to split a bet when a match had to be ended without a conclusion due to bad weather. Whether to rain or not is not decided by nature using probability. Not knowing whether it is going to rain or not, the chance of raining is determined by using probability. Chance says nothing about reality. Do not tell a lottery winner his/her chance of winning the lottery was nil, because it is not; it is hundred percent for the winner, we just did not know it. Probability has no meaning after an event. Probability only feeds into human expectation before an event. A lottery ticket gives few days of hope of striking it big. Probability itself is useless for the event itself or for our understanding of the mechanism of the event. In fact, it is a predicament or a hindrance to our understanding of the fundamentals of nature.

State of a charge particle in a population of a charge particles cannot be random since they are electrostatically bound. In addition, orbiting systems such as Atoms spin. Spinning atoms or charge particles generate a Spin Magnetic Moment [3]. Charge particles in a population are magnetically bound due to the spin magnetic moment. State of a mass in a population of masses cannot be random since they are gravitationally bound.

Corollary: Disadvantageous Probability

Probability is a predicament or a hindrance to our understanding of the fundamentals of nature.

Momentum or movement of a mass cannot exist without change of position. The change of position cannot take place without change of time. Time is absolute, not relative; propagation of light is not relative [6]. Mass is absolute, not relative [9]. Time and mass do not depend on observers; they are observer independent. Change of position cannot take place without a momentum and change of time. Nature of a particle does not depend on observers. It is true that the momentum does not depend on the position; momentum only depends on the rate of change of position. However, change of momentum cannot take place without change of position and change of time. The position of a particle depends on the momentum. Momentum has no existence if time is paused. Quantum Mechanics requires time to be paused. Time cannot be paused. Although nothing happens in reality if the time is paused, everything in Quantum Mechanics, Quantum behavior of observables, come alive only when time is paused.

Position cannot be fixed in the presence of a momentum. Position is determined by the momentum. There is no existence of a momentum without change of position; this prevents the position and momentum pair from being a Fourier Transform pair. A position of a particle cannot change without change of time. If position is constant, momentum has no existence. If the momentum is a constant, then, position of a particle cannot be random. Irrespective of the size of a particle, if the momentum is a constant, particle has a deterministic path, a path that is either linear or circular.

A particle (mass) cannot appear in one location and then disappear and reappear in another location randomly. If a particle at point A has to move to a point B, particle can take any path that connect point A to point B, but the particle has to cross all the in between points on the path to get to B. Motion of a particle is causal, not random. A particle (mass) cannot be in multiple locations at the same time. State of a particle must be certain, not probabilistic. For a particle at one location to appear in another location, particle must travel on a continuous path in between two locations. In 3-dimensional space, particle at position $\mathbf{r}_{initial}$ cannot reach position $\mathbf{r}_{final}$ without crossing all the spheres of radius r, where $r_{initial} \leq r \leq r_{final}$.

For the time being, let us leave the reality behind and enter the human created psycho-world of Quantum Mechanics. Quantum Mechanics is simply a prolific paper mill for university professors and graduate students, nothing more. University professors and graduate students have one and only one goal in mind, “how can I cook up another publication?” For whatever fortune or misfortune, if you happen to come across one of those guys, the first question you will be asked is “how many publications do you have?” For the sane, what matters is what you have done, yet for these insane and strange characters what matters the most is a number, “how many publications?” Interestingly, they have whole series of publishing junk-holes run by them for them to fill their needs, not for the advancement of anything else.

Under the incorrect assumption that the probability of a particle being at certain location y in a given state $\psi_n(y)$ is given by the square of that state, we have,

$$\text{Prob}(y)=|\psi_n(y)|^2,\ \forall n,\ n=0,\ 1,\ 2,\ \ldots \qquad (18.1)$$

where, Prob(y) is the probability of particle being at position y in state $\psi_n(y)$.

Further,

$$\int|\psi_n(y)|^2 dy=1. \qquad (18.2)$$

We also have the following properties of the states $\psi_n(y)$, $\forall n$, n=0, 1, 2, … for a Quantum Oscillator,

1. $\psi_n(y)$ is symmetric for even values of n, n=0, 2, 4, 6, …
2. $\psi_n(y)$ is asymmetric for odd values of n, n=1, 3, 5, 7, …
3. $\psi_o(y)=|\psi_o(y)|$ and $\psi_o(y)>0\ \forall y$.

However, for a free-moving particle, $\psi_n(y)$ is not square integrable and hence $|\psi_n(y)|^2$ does not represent a probability distribution.

a) **Probability of Particle Being at Location y when the Particle is at Ground State.**

As we mentioned earlier, if we want to represent the ground state wavefunction $\psi_o(y)$ as a probability distribution, we do not have to square it. The ground state at any time t has the property that,

$$\psi_o(y)=|\psi_o(y, t)|, \forall t \quad (18.1.1)$$

This is because that the change of time only affects the phase, not the magnitude. The magnitude of the ground state is time invariant and remains positive for the entire range of y at any time t. As a result, the magnitude $|\psi_o(y)|$ is differentiable at any time t.

The probability Prob(y, n=0) of particle being at location y when the particle is at the ground state $\psi_o(y)$ at any time t is given by,

$$\text{Prob}(y, n=0)=\psi_o(y) \quad (18.1.2)$$

where, $\int\psi_o(y)dy=1$.

b) **Probability of Particle Being at Location y When Particle is at Higher States** $\psi_n(y)$, n>0.

The wavefunction of a particle at a higher state has different characteristics than the ground state wavefunction. Higher state wavefunctions can take both positive and negative values since higher state wavefunctions for n>0 contain at least one or more nulls. In fact, the number of nulls contain in wavefunction $\psi_n(y)$ is equal to n. When n is odd number, one null is always at y=0 and the rest of the nulls are symmetric. For even n, all the nulls are symmetric. Since probability distribution must always be positive, higher state wavefunctions, $\psi_n(y)$ for n>0 themselves cannot represent probability distributions. So, the probability density function for higher states is defined as the square of the wavefunction,

$$\text{Prob}(y, n>0)=|\psi_n(y)|^2, n>0 \quad (18.2.1)$$

where,

$$\int|\psi_n(y)|^2dy=1 \quad (18.2.2)$$

However, this probability representation is not possible since wavefunctions at higher states are guaranteed to contain nulls. As we are going to see, no wavefunction containing zeros or nulls can represent probability of particle being at certain location since nulls entrap the particle in between nulls.

c) **Higher States Cannot Represent Probability Distribution of a Particle being at certain Location**

Higher eigenstates or wavefunctions of the higher states of a Quantum Oscillator originated from the forever differentiable Gaussian function. The n^{th} eigenstate $\psi_n(y)$ is a result of the Step-Up operator **U**, **U**=∂/∂y-y, operating on the $(n-1)^{th}$ state, $\psi_{n-1}(y)$,

$$\psi_n(y) =\mathbf{U}\psi_{n-1}(y) \quad (18.3.1)$$

$$\psi_n(y) =\partial\psi_{n-1}(y)/\partial y-y\psi_{n-1}(y) \quad (18.3.2)$$

$$\psi_o(y)=\exp(-(1/2)y^2) \quad (18.3.3)$$

As a result, each wavefunction is either symmetric or anti-symmetric and contains n nulls for n>0,

$$\psi_n(y)=0, \text{ for n values of y for } n>0 \quad (18.3.4)$$

$$\psi_n(y)\neq 0, \text{ for } n=0 \quad (18.3.5)$$

All the wavefunctions are guaranteed to contain nulls except the ground state wavefunction $\psi_o(y)$. Wavefunction $\psi_n(y)=0$ for n positions of y since $\psi_n(y)$, for n>0, is positive for certain ranges of y and negative for certain ranges of y. In the case of odd numbers of n, all the nulls are symmetric with extra null at y=0. For even numbers of n, there is no null at y=0 and all the nulls are symmetric on y. Probability distribution cannot be negative. So, $\psi_n(y)$ for n>0 itself cannot represent a probability distribution. We may think that we can overcome this difficulty simply by using the square of the wavefunction, $|\psi_n(y)|^2$ for n>0 as the probability distribution. However, $|\psi_n(y)|^2$ for n>0 still contains the same number of nulls. It is these nulls in $\psi_n(y)$ for n>0 that prevent the representation of higher state wavefunctions as probability of particle being at location y on state $\psi_n(y)$ for n>0.

If the square wavefunction $|\psi_n(y)|^2$ for n>0 is represented as a probability distribution, a null in the wavefunction $\psi_n(y)$ for n>0 indicates that a particle cannot be in that position where a null is. A particle on one side of the null cannot cross over onto the other side of the null. If particle cannot be at a null, no particle can cross a null. If particle is on one side of the null, particle has no way to cross to the other side. The other side become a prohibited zone for a particle since probability of particle being at a null is nil.

In addition, when the state n is an odd number, $\psi_n(y)=0$ when y=0. As a result, zero-mean particle cannot be at on average value <y>=0 with certainty when n is odd. If particle cannot be at the mean value <y>=0, then, Quantum Mechanics fails right there. The mean value must be possible for Quantum Mechanics to work. If the probability of a particle being at the mean value is zero, it is certain that the particle cannot be at the mean value.

In addition, when a Quantum Oscillator is in state $\psi_n(y)$, where n is an odd number, particle can never be at y=0. If particle cannot be at y=0, then, it is no longer a Quantum Harmonic Oscillator when it is in states where n is an odd number. In addition, the General Uncertainty Principle does not hold true either when a Quantum Oscillator is at a state $\psi_n(y)$ where n is odd since the General Uncertainty Principle is constructed based on the assumption that the observables are zero-mean. We will consider why that is the case in more detail later.

Corollary: Improbable States

If the position of a particle is assumed to be a zero-mean random variable, the probability of particle being at y=o cannot be zero. A particle cannot be at zero-mean average position <y>=0 when n is odd since $\psi_n(y)=0$ at y=0. The probability of a particle being at y=0 is nil when n is odd. As a result, there is no existence of a Quantum Oscillator or a General Uncertainty Principle when n is odd.

Corollary: Essential Requirement

No Oscillator has any existence if the Oscillator cannot be at the equilibrium position y=0.

So, if the wavefunction contains only one zero as in the case of the first state n=1, particle on one side of the null will be trapped on that side of the null. If wavefunction has more than one null, $\psi_n(y)$, n>1, then, particle that is located in between two nulls remains in between those two nulls. Particle will be trapped in between two nulls forever. Once particle is trapped on one side of a single null or in between two nulls, particle has no way to get out of that entrapped zone. A particle trapped between nulls of an eigenstate $\psi_n(y)$, n>0 cannot be a Harmonic Oscillator. Square wavefunction $|\psi_n(y)|^2$ for n>0 can no longer represent a probability distribution of particle being at location y in state $\psi_n(y)$ for n>0 since a particle that is trapped in between nulls cannot be at any location y in the entire range of $\psi_n(y)$.

In order for the wavefunction $|\psi_n(y)|^2$ for n>0 to represent a probability distribution, particle must have the ability to be at any location within the entire span of y on the wavefunction $\psi_n(y)$ without restriction. There is no way for a particle to cross a null of a wavefunction $\psi_n(y)$ for n>0 at any state if the probability of particle being at position y on any state is represented by a wavefunction $\psi_n(y)$ for n>0 that is certain to consist of nulls. If the square wavefunction $|\psi_n(y)|^2$ represents probability of particle being at any position y at a state described by wavefunction $\psi_n(y)$, then, the higher eigenstates $\psi_n(y)$, n>0 cannot represent probability distributions even as $|\psi_n(y)|^2$.

Only the ground state $\psi_o(y)$ can represent probability distribution. The probability distribution of ground state is $\psi_o(y)$ itself, no squaring is necessary. Probability distribution $\psi_o(y)$ and $|\psi_o(y)|^2$ are two completely different distributions. When $\psi_o(y)$ itself can represent a probability distribution, there is no reason to square it for a probability representation.

It is not possible to avoid the problems presented with the presence of nulls in the higher eigenstates simply by claiming a particle is whizzing through a null.
It does not matter how particle choses to cross a null, by whizzing through, crawling through, or sailing through a null, if a particle crosses a null, then, the probability of particle being at a null is no longer nil. Do not try to voodoofy particles. Particle on one side of a null cannot get to the other side without crossing a null, period. As a result, $|\psi_n(y)|^2$, n>0 cannot represent a probability distribution for particle being at any position y through the entire span of state $\psi_n(y)$, since eigenstate $\psi_n(y)$, n>o has nulls.

You cannot claim impossible possible just for the sake of justifying false theory of Quantum Mechanics, it is not science, it is crookery. It is the Quantum Mechanics itself that prevents the crossing of a null by a particle. If a particle cannot cross a null, the square wave function containing nulls cannot represent a probability distribution of particle being at a certain location within the entire span of the wavefunction. It is the Quantum Mechanics that is self-destructing itself. It is the Quantum Mechanics that is self-falsifying itself.

You have to become a voodoo practitioner to justify Quantum Mechanics, not a scientist. That is what so-called Modern Physicists have become, voodoo practitioners. How else can you justify a particle being at multiple states at the same instant? How else can you justify position of a particle from being independent of momentum? How else can you justify a particle having a momentum when the time being paused? How else can you justify position and momentum are being mutually independent? How else can you justify position and momentum pair being a Fourier Transform pair? How else can you justify particle being waves and waves being particles? How else can you justify a momentum without a mass? How else can you separate momentum and speed as two independent entities? How else can you justify multiple worlds? How else can you justify multiple universes? How else can you justify spatially random light particles or photons? How else can you justify massless particles? It is Simply not possible. They pull-out universes from nowhere just like magicians pulls out rabbits from a hat. Modern Physics requires a reawakening. Salvaging of Quantum Mechanics from this probabilistic blunder that turned physics into voodoo-physics not possible.

Corollary: Probability Unfriendly Null
The n^{th} state has n nulls, all of which are symmetric except the null at y=0 for odd n. No eigenstate consisting of nulls can represent the probability distribution of a particle being at a certain location y as the square of the eigenstate.

Corollary: Probability Friendly Ground
It is only the ground state, which is free of nulls, that is at least has the properties required for representing itself as a probability distribution of a particle being at a certain location y.

d) **General Uncertainty Principle Fails when Quantum Oscillator is at State** $\psi_n(y)$ **When** n **is Odd**
General Uncertainty Principle requires the observables to be zero-mean. The probability of particle being at zero-mean position should not be nil for the General Uncertainty Principle to hold. This requirement does not satisfy when a Quantum Oscillator is at state $\psi_n(y)$ when n is odd.

As we have seen, $\psi_n(y)$ always has a null $\psi_n(y)=0$ at y=0 when n is odd. As a result, if $|\psi_n(y)|^2$ represents the probability of particle being at position y, then the probability of the particle being at y=0 is always zero when n is odd. In the General Uncertainty Principle, it is assumed that the position is a zero-mean random variable and hence the probability of particle being at average position $\langle y \rangle=0$ should not be zero for the General Uncertainty Principle to hold. Since probability of particle being at y=0 is always zero for odd n, General Uncertainty Principle does not hold for a Quantum Oscillator at any state $\psi_n(y)$ when n is odd.

It is not just the General Uncertainty Principle, Quantum Oscillator itself has no existence when oscillator cannot be at the average position $\langle y \rangle=0$ for odd n. In Quantum Mechanics, what varies with time is the average observable. If probability of a particle being at average observable is zero, the application of Quantum Mechanics to a Quantum Oscillator fails when n is odd.

Lemma: Generalized Uncertainty Conundrum

Probability of a particle being at the average position <y>=0 is nil when the particle is at states where n is odd since the eigenstate $\psi_n(y)=0$ at y=0 for odd n. As a result, the General Uncertainty Principle no longer applies when a Quantum Oscillator is at a state $\psi_n(y)$ where n is odd since General Uncertainty Principle requires observables to be zero mean.

e) **Probability and Observers**

Probability of a particle being at any position y at any state $\psi_n(y)$ has nothing to do with an observer. Particles are not thieves that are conscious of possible observers. State of a particle does not depend on observers. It is the misinterpretation of the results from two Stern-Gerlach Devices placed in-phase and in series that lead to the false idea that the state of a particle is observer dependent [3]. If you send Spin-Up beam of Atoms from one Stern-Gerlach Device to another in-phase and in series second Stern-Gerlach Device, the beam of Atom will pass through without any splitting since the placement of second in-phase in series Stern-Gerlach Device is simply equivalent to an extension of the Stern-Gerlach magnetic field. It is not an effect of an observer that made the Spin-Up beam to pass through without splitting. State of a particle is not observer dependent.

Probability only says that the possibility of particle being at a certain position y at any state $\psi_n(y)$ irrespective of any observers. If the probability distribution is represented as a function of an eigenstate, that probability distribution is going to contain nulls independent of any observer. A probability distribution consisting of nulls says that particle cannot be present at position y where $\psi_n(y)=0$, $\forall n$.

If probability distribution of a particle contains nulls, it is not a probability distribution of a particle. This prevents Quantum Mechanics being a real representation of a nature of a particle. This is one of the fallacies of Quantum Mechanics. Quantum Mechanics has turned physics into voodoo-physics. The fact of the matter is that the wavefunction or the eigenstates of higher states cannot represent probability distributions. It is only the ground state $\psi_o(y)$ that has the properties required to represent as a probability distribution. As we will see later, even the ground state $\psi_o(y)$ cannot represent a probability distribution of particle being at any position due to a different reason. In fact, no function of infinite span can be a state of a Quantum Oscillator of finite energy. There is a hidden error in the solution to the wave equation of Quantum Oscillators.

f) **Limitations of Quantum Oscillator Solution**

The wavefunctions $\psi_n(y)$, $\forall n$, n=0, 1, 2, 3, 4 … obtained as solutions to the wave equation of a Quantum Oscillator is only applicable for particles, where particles are in a linear motion against a restoration force directly proportional to the displacement. For the application of these energy levels and wavefunctions $\psi_n(y)$, $\forall n$, n=0, 1, 2, …, derived using Step-Up and Step-Down operators, the particle must have the Hamiltonian of the form $\mathbf{H}=\mathbf{P}^2+y^2$. This does not apply to any moving particle under gravitational and electrostatic potentials since in both cases the potential energy is proportional to the reciprocal of the distance. An electron in an atom is under electrostatic potential that is proportional to the reciprocal distance and hence Quantum Oscillator solution does not apply to electrons in an Atom.

Simple Harmonic Oscillator applies for particles that have similar characteristics as Hook’s law. No electrically charge particle can be in a simple harmonic motion when it is electrostatically bound to another electrically charged particle unless the change of electrostatic force due to the displacement is negligible compared to the change of restoration force. Similarly, no mass can be in a Simple Harmonic Motion if that mass is gravitationally bound unless the change of gravitational force due to the displacement is negligible compared to the change of restoration force as it is in the case of a particle connected to a spring.

No such dominant restoration force exists in the case of electrons in an Atom. Quantum Oscillator energy levels do not apply to electrons in an Atom. Energy levels obtained as unconstrained solutions to a Quantum Oscillator cannot be used to derive energy spectrum of an Atom. Electrons in an Atom are not in a Simple Harmonic Motion. An Atom is not in a Simple Harmonic motion. Orbiting particle does not represent a Simple Harmonic Motion. Only the projection of orbiting motion on an axis represents a Simple Harmonic Motion. Quantum Oscillator cannot be represented as unconstrained solution to a wave equation of a Quantum Oscillator.

QM Contradiction: Assumption Oversight

Harmonic Oscillator assumes that the displacement from the equilibrium position is small. The infinite span Quantum states obtained as solutions to the wave equation of a Quantum Oscillator is a contradiction to this assumption.

Property: Finite Span

Position span of a Harmonic Oscillator of finite energy cannot be infinite. The energy of a Quantum Oscillator at state $\psi_n(y)$ of infinite span cannot be finite.

Property: Limited Bandwidths

Unconstrained solutions of a wave equation cannot represent states of a Quantum Oscillator. Span of wave functions in both position and momentum domains must be finite and strictly limited by the energy of the Quantum State.

Theorem: Impossibility

Two functions with strictly limited spans cannot be a Fourier Transform pair.

XIX. INABILITY OF BOTH POSITION AND MOMENTUM TO BE ZERO SIMULTANEOUSLY HAS NOTHING TO DO WITH HEISENBERG UNCERTAINITY PRINCIPLE

In the case of a Harmonic Oscillator, the force F is a restoration force. For small displacement x, the force F is given by,

$$F=-kx \tag{19.1}$$

where, k is the restoration parameter.

The Hamiltonian of an Oscillating Particle of mass m and restoration parameter k at displacement x is given by,

$$H=(1/2m)p^2(x)+(1/2)kx^2 \tag{19.2}$$

where p(x) is the momentum of the particle at x, momentum p is dependent on the displacement x from the equilibrium position.

H is a constant that is determined by the maximum displacement x_{max} and the restoration force constant k.

In the case of a Harmonic Oscillator, momentum is zero when the displacement is maximum, $p(x)=0$ $|x|=x_{max}$ and momentum is maximum when the displacement is zero, $p(x)=p_{max}$ at x=0. The sum of the weighted square position and weighted square momentum is always a constant, or time invariant. What keeps Quantum Oscillator oscillating is the back and forth conversion of kinetic energy to potential energy and potential energy to kinetic energy. It is this inherent characteristic of Harmonic Oscillator itself that prevents both position x and momentum p from being simultaneously zero.

Since the sum of the kinetic energy and the potential energy is a constant, when one is higher, the other is lower and vice versa. If both the position and the momentum are zero at the same time, then, the total energy is zero and hence the particle is not in a Harmonic Motion. In fact, the particle is at stand still if both the position and the momentum are zero simultaneously.

In any Harmonic Oscillator, when the displacement is at its maximum, the momentum is zero, and when the momentum is at its maximum, the displacement is zero. At any in between position, the total energy or the sum of the kinetic and potential energy must be a non-zero constant determined by the potential energy at the maximum displacement. It is this property of back and forth energy conversion from kinetic energy to potential energy, and potential energy to kinetic energy that prevents position and momentum of a Quantum Harmonic Oscillator from being zero simultaneously. There is no uncertainty principle here. Uncertainty Principle has nothing to do with it. Use of Quantum Harmonic Motion to justify Heisenberg Uncertainty Principle everywhere in physics [1] is simply ridiculous. You cannot find physics book in Quantum Mechanics anywhere that does not make this preposterous claim.

Even if position and momentum are assumed to be a Fourier Transform pair, its bandwidth limits apply only to the maximum span Δx of the wavefunction in position domain $\psi_n(x)$, and the maximum span Δp of the wavefunction in momentum domain $\psi_n(p)$. Uncertainty Principle cannot prevent the position x taking any value withing the range $-x_{max} \leq x \leq x_{max}$ and momentum p taking any value in the range $-p_{max} \leq p \leq p_{max}$. Uncertainty principle only limits Δx, $\Delta x=2x_{max}$ and Δp, $\Delta p=2p_{max}$.

In spite of the bandwidth limits imposed by the assumption that the position and the momentum are a Fourier Transform pair, position x can take any value within the range $-x_{max} \leq x \leq x_{max}$. On average position is certain irrespective of the size of the width Δx. The width Δx does not have to be zero for on average certainty of position x. Similarly, momentum p can take any value within the range $-p_{max} \leq p \leq p_{max}$. On average momentum is certain irrespective of the size of the width Δp. Spread has no effect on the on average simultaneous certainty of both position and momentum.

There is no theoretical barrier preventing the measurement of both position x and momentum p simultaneously to any precision on average since any Fourier Transform pair must have a common eigenspace. In addition, even though position and momentum operators do not commute, position and momentum operators have a shared eigenspace. Since the position and momentum have shared eigenspace, they are simultaneously measurable. Measurement of one does not affect the other since they both have shared eigenspace.

We can only measure run-time on average position and momentum experimentally. Quantum Mechanics is a paused time theory. Paused time Quantum behaviors of observables are not accessible for observers since time cannot be paused. On average run-time Position and momentum are mutually dependent and hence measurement of both is not necessary. Measurement of one must contain all the information for obtaining the other and hence on average simultaneity is guaranteed in run-rime experiments.

Since position and momentum are mutually dependent, they cannot be a Fourier Transform pair. If the position and momentum are assumed to be a Fourier Transform pair, that very assumption makes them to be simultaneously measurable. No two simultaneously non-measurable observables can be a Fourier Transform pair by the very nature of the Fourier Transform.

So called uncertainty principle or the lower bound of the product of the width of the wavefunction in position domain (Δx) and the width of the wavefunction in the momentum domain (Δp) is a result of the invalid assumption that the wavefunction in position domain and the wavefunction in momentum domain are a Fourier Transform pair. Heisenberg Uncertainty Principle is a bandwidth limit. True position is within the span Δx irrespective of the size of Δx. True momentum is within the span Δp irrespective of the size of Δp. As a result, on average position and on average momentum are the actual position and momentum. Position and momentum are certain on average simultaneously. Since position and momentum operators have a shared eigenspace, on average precise position and on average precise momentum can be obtain simultaneously. Presumed Quantum behaviors of observables at paused time are not accessible to observers and hence can never be measured either separately or simultaneously. What we have access is only to the on average run-time observables that have no Quantum behaviors.

a) **Uncertainty Principle Cannot Prevent the Measurability of** x **and** p **Simultaneously**

Bandwidth limits do not prevent observers from measuring both the position and the momentum simultaneously. If both position and momentum operators have the same eigenspace or common eigenstates, both the position and the momentum can be measured simultaneously irrespective of the bandwidth limitation of the Heisenberg Uncertainty Principle. The lower bound of the product of the bandwidth of the wavefunction in the position domain and the bandwidth of the wavefunction in momentum domain cannot prevent the position and the momentum operators having common eigenspace.

In fact, if the position and the momentum operators do not have common eigenspace, the position and the momentum of a particle cannot be a Fourier Transform pair, which prevents the existence of the Uncertainty principle. If position and the momentum of a particle are not a Fourier Transform pair, there would be no Uncertainty Principle. Since the position and momentum have a shared eigenspace, they are simultaneously measurable.

In general, Commutation of operators is neither sufficient nor necessary for the simultaneous measurement of observables. It is only when the eigenvalues of two operators are mutually independent that the commutation of operators is sufficient for the simultaneous measurement of observables but not necessary.

Position and momentum operators do not have to commute for them to have a shared eigenspace. Even though position and momentum operators do not commute, they still have a shared eigen space. If the commutation of two operators is a constant, they have a shared eigenspace. When that constant is zero, it is a special case where operators commute.

The eigenspace of any observable shares its eigenspace with the independent observable, where the operator is the observable itself. The position operator is position itself and the commutation of the position and momentum operators is a constant, and as a result, position and momentum have a shared eigenspace. Since position and momentum have a shared eigenspace, they are simultaneously measurable.

Theorem: No Commuting Required

Commutation of operators is neither necessary nor sufficient for them to have a shared eigenspace. Non-commuting operators can have a shared eigenspace.

Lemma: Non-Commuting Simultaneity

If the commutation of two operators is a constant, they have a shared eigenspace, and hence they are simultaneously measurable. When that constant is zero, we have the case where operators commute.

The proof is straight forward and left as an exercise.

Corollary: Simultaneity of position and momentum

Position and momentum of a particle are simultaneously measurable since the commutation of position and momentum operators is a constant.

For a Quantum Oscillator to exist, the energy of the oscillator must not be zero. In other words, the position and the momentum must not be zero at the same time since the energy $H=(1/2m)p^2(x)+(1/2)kx^2$, where, momentum p(x) is a function of x for an Oscillator. The inability of both position and the momentum to be simultaneously zero is an inherent property of a Harmonic Oscillator itself, not a result of some arbitrary and preposterous Uncertainty Principle. If both p and x are zero at the same time, the energy of the oscillator will be zero and hence Oscillator seizes to exist, it is as simple as that.

Lemma: Oscillator Characteristic

The inability of both position and the momentum to be simultaneously zero is an inherent property of a Harmonic Oscillator itself, not a result of mathematically and logically invalid some arbitrary Heisenberg Uncertainty Principle.

The effort to attribute the inability of both position and momentum of a Harmonic Oscillator to be zero simultaneously to Heisenberg Uncertainty Principle in Quantum Mechanics literature is simply preposterous and laughable; there is no real justification for that except in the broom riding voodoo world of Harry potter. Desire of the people to always associate anything and everything with Heisenberg Uncertainty Principle is understandable since it sounds brainy; there is no other reason. It is the same reason why people include the meaningless phrase “space-time continuum” everywhere even in the throne speech of the Governor General; yes, the use of that phrase really makes politicians brainy because nobody knows what it really means including the people who coin the phrase. What is there is the space. There is no space-time [6]. Another meaningless phrase that sounds brainy is the phrase "space-time fabric" or "fabric of space-time"; pseudo speech. How can the time make a fabric?

Time does not depend on space. Space does not depend on time. Space and time are mutually independent. There is a physical space. There is no time as such. Time is a definition, a human definition. What is there is the changes in the nature of objects in space. We use those changes in objects to define time.

Time is not relative. If time is relative time will not be unique. If time is relative, time will be directional. Time must be unique. Time must be non-directional. Therefore, time cannot be relative [6, 7, 9].

b) **No Fractional Quanta**

By definition, a Quantum of an entity is the smallest possible quantity of that entity that can exist. In the case of a Quantum Oscillator, an energy quantum is the minimum energy required for the existence of a Quantum Harmonic Oscillator and it is the ground state energy. If there is a fractional quantum, then the quantum is no longer a quantum by very definition of the quantum. If there is a quantum-half, then the quantum itself will be the quantum-half. There are no fractional quanta. If you are one of those who are still talking about Quantum-half, it could be result of Quantum-Memory-lapse syndrome. There is no other explanation for it.

The energy of a Quantum Oscillator is mechanical energy. Mechanical energy is not quantized. Mechanical energy does not come in quanta. Mechanical energy is continuous. Only the electromagnetic energy is quantized. It is the invalid assumption that the mechanical energy is quantized that lead to the Quantum Mechanics. If we assume incorrectly that the mechanical energy is quantized, the minimum required energy for its existence is one energy Quantum or the ground state energy in the case of a Quantum Oscillator. There are no fractional Quanta; fractional Quanta defy the very definition of quantum.

Corollary: No Fractions Allowed

Quantum-half and Spin-half are oxymorons.

Electromagnetic energy quantum is not a particle. Electromagnetic energy quantum is a wave burst of certain frequency and width, where, the energy of the burst is proportional to the electromagnetic frequency of the wave burst, e=hf. The relationship e=hf is meaningless unless it is a wave of finite time span. The relationship e=hf is meaningless if it is a particle. The relationship e=hf does not apply to mechanical energy, which has no existence without a mass (particle). Any entity that is belong to another entity cannot be quantized. Any entity with a belonging cannot come in quanta since nature has no built in mechanism to carry belonging information.

c) **Eigenstates** $\psi_n(x)$ **and** $\psi_n(p)$ **Cannot Represent Quantum Oscillators when** n **is odd,**

Wavefunction in position domain $\psi_n(x)$ has a null at x=0 for odd n. Wavefunction in momentum domain $\psi_n(p)$ also has a null at p=0 for odd n. If the square of the wave function $\psi_n(x)$ in position domain determines the probability of particle being at x, then the probability of particle being at x=0 is nil when n is odd. If particle cannot be at x=0 when n is odd, Oscillator can never reach the maximum kinetic energy when n is odd. If the Oscillator cannot reach the maximum kinetic energy, it is no longer a Harmonic Oscillator when n is odd.

If particle cannot be at x=0, it indicates that the particle cannot be at the equilibrium point of the Oscillator since x is the displacement from the equilibrium position. If particle cannot be at the equilibrium position, Quantum Oscillator has no existence when n is odd.

Similarly, if the square of the wavefunction $\psi_n(p)$ in momentum domain represents the probability of particle being at p, then the probability of particle being at p=0 is nil when n is odd since $\psi_n(p)=0$ at p=0 when n is odd. What that means is that no Harmonic Oscillator can ever reach the maximum kinetic energy when n is odd. If an Oscillator cannot reach the maximum kinetic energy, it cannot also reach the maximum potential energy. As a result, no oscillator can reach ($p=0$, $x=x_{max}$) when n is odd.

When n is odd, $\psi_n(x)$ has a null at x=0, and $\psi_n(p)$ also has a null at p=0. Oscillator can neither reach the maximum potential energy nor the maximum kinetic energy when n is odd. If Quantum Oscillator cannot reach ($x=0$, $p=p_{max}$) and ($p=0$, $x=x_{max}$), eigenstates $\psi_n(x)$ and $\psi_n(p)$ cannot represent a Quantum Oscillator at higher eigenstates for odd n.

Property: Oddity in odd

The eigenstates $\psi_n(x)$ and $\psi_n(p)$ cannot represent a Quantum Oscillator since both $\psi_n(x)$ and $\psi_n(p)$ contain nulls at y=0 and p=0 when n is odd, ∀n, n=1, 3, 5, 7 ...

XX. POSITION x AND MOMENTUM p CANNOT BE A FOURIER TRANSFORM PAIR

In a Harmonic Oscillator, the maximum width Δx of a wavefunction in position domain is determined by the maximum potential energy of the particle, which is equal to the total energy when kinetic energy is zero. The width Δx is a constant for a given energy level of an Oscillator. It is not observer dependent. The maximum width of a wavefunction in momentum domain Δp is determined by the maximum kinetic energy, which is the total energy when the potential is zero. The width Δp is a constant for a given energy level of an Oscillator and it is not left to be determined by the Fourier Transform pair. Δx and Δp at any energy state are determined by the energy of a single frequency.

Property: Limited Span

Δx and Δp are finite and they are pre-determined by the energy of the state.

Theorem: Direct opposition

The change of position x, δx and the change of momentum p, δp of a Harmonic Oscillator are linearly related with negative gradient.

Proof:

Consider the total energy E of a Harmonic Oscillator at any displacement x,

$$E=(1/2m)p^2+(1/2)m\omega_o^2x^2 \quad (20.1)$$

Differentiating with respect to x, we have,

$$\partial E/\partial x=(1/m)p\partial p/\partial x+m\omega_o^2x \quad (20.2)$$

Since ∂E/∂x=0,

$$(1/m)p\partial p/\partial x+m\omega_o^2x=0 \quad (20.3)$$

$$(1/m)p\partial p=-m\omega_o^2x\partial x \quad (20.4)$$

As a result, we have,

$$p\partial p=-(m\omega_o)^2x\ \partial x \quad (20.5)$$

$$\delta p\propto-(x/p)\delta x \quad (20.6)$$

The change of p is directly related to the change of x. The change in position and the change in momentum are linearly related and depend on the position and the momentum.

Lemma: Direct Related

The maximum momentum span Δp and the maximum position span Δx of a Harmonic Oscillator are directly related and hence position x and momentum p pair cannot be a Fourier Transform pair.

Proof:

Since the maximum displacement of position is Δx/2, the total energy E is given by,

$$E=(1/2)m\omega_o^2(\Delta x/2)^2 \quad (20.6)$$

Similarly, since the maximum momentum is Δp/2, the total energy E is also given by,

$$E=(1/2m)(\Delta p/2)^2 \quad (20.7)$$

Since the maximum kinetic energy is the same as the maximum potential energy, we have,

$$(1/2m)(\Delta p/2)^2=(1/2)m\omega_o^2(\Delta x/2)^2 \quad (20.8)$$

$$(\Delta p)^2=(m\omega_o\Delta x)^2 \quad (20.9)$$

$$\Delta p=m\omega_o\Delta x \quad (20.10)$$

What we now have is,

$$\Delta p\propto\Delta x \quad (20.11)$$

The width Δp is directly proportional to the width Δx, not inversely. For the position x and the momentum p pair to be a Fourier transform pair, Δx and Δp must be related inversely, and hence position x and momentum p cannot be a Fourier Transform pair.

Theorem: Impossible Co-Existence

If Δx and Δp are inversely related, they cannot represent a Harmonic Oscillator. If Δx and Δp are directly related, they cannot be a Fourier Transform pair. Quantum Harmonic Oscillator and a Fourier Transform pair cannot co-exist. Hence, position x and momentum p of a Harmonic Oscillator cannot be a Fourier Transform pair.

Lemma: Simultaneous Certainty

Span Δx does not have to be zero for on average certainty of the position x. Span Δp does not have to be zero for on average certainty of the momentum p. Both spans Δx and Δp do not have to be simultaneously zero for the simultaneous certainty of both x and p. Both Δx and Δp can be non-zero, yet x and p can be certain on average simultaneously.

XXI. POSITION SPAN ΔX AND MOMENTUM SPAN ΔP OF HARMONIC OSCILLATOR

In the case of a Quantum Harmonic Oscillator or any Harmonic Oscillator, the maximum displacement x_{max} is limited by the total energy of the oscillator. The maximum momentum p_{max} is also limited by the total energy of the oscillator. Neither the width $\Delta x=2x_{max}$ of a wavefunction in position domain nor the width $\Delta p=2p_{max}$ of a wavefunction in momentum domain can have an infinite span.

Eigenstates $\psi_n(x)$ and $\psi_n(p)$ of Quantum Oscillators cannot have infinite span. No finite energy harmonic oscillator can have infinite position span or momentum span. For a Quantum Harmonic Oscillator, span of position x is bound by,

$$\psi_n(x)>0,\ -x_{max}\leq x\leq x_{max}, \quad (21.1)$$

$$\psi_n(x)=0,\ \text{otherwise} \quad (21.2)$$

where, x_{max} is determined by the energy level of the Quantum Oscillator.

The constrain $\psi_n(x)>0$, $-x_{max}\leq x\leq x_{max}$ allows the representation of $\psi_n(x)$ itself as a probability distribution if you are so inclined to choose such an invalid representation as it is the case in Quantum Mechanics.; no squaring is necessary. The representation of square wavefunction $|\psi_n(x)|^2$ as a probability of particle being at position x is artificial since the nature does not carry out squaring and normalization. Even with the introduction of the constrain that the wavefunction is positive within the position span or $\psi_n(x)>0$, $-x_{max}\leq x\leq x_{max}$, probability representation is still artificial since the normalization has to be carried out, but it is little bit less unnatural since no squaring is required.

Although meaningless, now you can directly say that the wavefunction $\psi_n(x)$ of a particle represents the probability of particle being at location x. If you have to say that $|\psi_n(x)|^2$ represents a probability of particle being at position x, as it is done in Quantum Mechanics, it sounds like a human crafted prophesy just like those creators in religious non-sense.

Similarly, the span of momentum p is also bounded by,

$$\psi_n(p)>0,\ -p_{max}\leq p\leq p_{max}, \quad (21.3)$$

$$\psi_n(p)=0,\ \text{otherwise} \quad (21.4)$$

where, p_{max} is determined by the energy level of the Quantum Oscillator.

The wavefunctions that represent a Quantum

Oscillator must satisfy these bounds while they satisfy the wave equation of a Quantum Oscillator,

$$-(1/2m)\hbar^2\partial^2\psi_n(x)/\partial x^2+(1/2)m\omega_o^2x^2\psi_n(x)=E_n\psi_n(x) \quad (21.5)$$

As a result, the solution to the Quantum Oscillator is a constrained eigen decomposition problem that does not have a closed-form solution. Although unconstrained solutions to the wave equation of a Quantum Oscillator is easy to find, they do not represent a Quantum Oscillator since they are of infinite span.

a) **Width of a Wavefunction in Position Domain** Δx

Consider the n^{th} state of Quantum Oscillator $\psi_n(x)$, $\forall n$, n=0, 1, 2, … The energy E_n of the n^{th} state is given by,

$$E_n=\hbar\omega_o(2n+1),\ \forall n,\ n=0, 1, 2, \ldots \quad (21.1.1)$$

When the maximum displacement x_{max} is achieved, the kinetic energy is zero, and the momentum is zero, p=0. The total energy is equal to the potential energy. This is the case for any Oscillator whether it is a Quantum Oscillator or not. If the maximum span of an Oscillator does not satisfy the energy bounds, it is not an oscillator. The word Quantum does not allow us to escape this reality. It is ironic that people use the word Quantum to override the reality, turning physics into some kind of voodoo magic. Quantum Mechanics has turned into the magic wand of Harry-Potter where broom-riding is the reality.

Maximum displacement is achieved when,

$$(1/2)k(x_{max})^2=\hbar\omega_o(2n+1) \quad (21.1.2)$$

where, k is the restoration force constant.

$$x_{max}=(2\hbar\omega_o(2n+1)/k)^{1/2} \quad (21.1.3)$$

We also have,

$$\omega_o=(k/m)^{1/2} \quad (21.1.4)$$

Substituting for k in eqn. (21.1.3), we have,

$$x_{max}=(2\hbar(2n+1)/m\omega_o)^{1/2} \quad (21.1.5)$$

The width Δx of the n^{th} eigenstate or the wavefunction $\psi_n(x)$ is twice the x_{max},

$$\Delta x=2x_{max} \quad (21.1.6)$$

$$\Delta x=2(2\hbar(2n+1)/m\omega_o)^{1/2} \quad (21.1.7)$$

This is the maximum span of any Quantum Oscillator at state n. If the particle at state n is at position x, that position must be within the displacement bounds,

$$-\Delta x/2\leq x \leq \Delta x/2 \quad (21.1.8)$$

$$-(2\hbar(2n+1)/m\omega_o)^{1/2}\leq x \leq(2\hbar(2n+1)/m\omega_o)^{1/2} \quad (21.1.9)$$

Any eigenstate that extends outside these bounds cannot represent an eigenstate of a Quantum Oscillator. The state n does not have sufficient energy to be outside these bounds.

As we have seen before, irrespective of the size of the width Δx, on average position x is the precise position since precise position is always within the span Δx. The width Δx does not have to be zero for on average precision. On average precision of x does not depend on the width Δp. Irrespective of the Δp, on average position of a particle is certain. Similarly, irrespective of Δx, on average momentum is also certain.

b) **Width of the Wavefunction in Momentum Domain** Δ**p.**

The energy E_n of the state n is given by the eigenvalue of that state,

$$E_n=\hbar\omega_o(2n+1),\ \forall n,\ n=0, 1, 2, \ldots \quad (21.2.1)$$

When the maximum momentum $p=p_{max}$ is reached, displacement is zero, x=0, and hence all the energy of the particle is in the form of kinetic energy,

$$(1/2m)(p_{max})^2=\hbar\omega_o(2n+1) \quad (21.2.2)$$

$$p_{max}=(2m\hbar\omega_o(2n+1))^{1/2} \quad (21.2.3)$$

This is the maximum momentum of state n. Momentum of a Quantum Oscillator at state n is bound by $|p_{max}|$. The momentum associated with state n is bound by,

$$-p_{max} \leq p \leq p_{max} \quad (21.2.4)$$

$$-(2m\hbar\omega_o(2n+1))^{1/2}\leq p \leq(2m\hbar\omega_o(2n+1))^{1/2} \quad (21.2.5)$$

The width Δp of the n^{th} eigenstate or the wavefunction in the momentum domain $\psi_n(p)$ is twice the p_{max},

$$\Delta p=2p_{max} \quad (21.2.6)$$

$$\Delta p=2(2m\hbar\omega_o(2n+1))^{1/2}) \quad (21.2.7)$$

Any eigenstate that extends outside these bounds cannot represent an eigenstate of a Quantum Oscillator. The state n does not have sufficient energy to be outside these bounds.

As in the case of position x, irrespective of the size of the width Δp, on average momentum p is the precise momentum since precise momentum is always within the span Δp. The width Δp does not have to be zero for on average precision. Precision of p does not depend on the width Δx.

c) **Fourier Transform Bounds on** Δp

Although incorrect, in Quantum Mechanics, the position and the momentum pair (x, p) is assumed to be a Fourier Transform pair. For a free-moving particle of mass m, the wave function is given by,

$$\psi_n(x,t)=\exp(jpx/2\hbar)\exp(-jE_nt/\hbar) \quad (21.3.1)$$

If you are wondering why $2\hbar$ instead of $\hbar$ we usually come across in Quantum Mechanics, it is because we have used the wavelength as twice the deBroglie wavelength as it justifiably should be [3]. At any time, t, for a free-moving particle, the wave function of the n^{th} state, $\psi_n(x)$ is given by,

$$\psi_n(x,t)=\exp(jpx/2\hbar) \quad (21.3.2)$$

For the wavefunction $\psi_n(x)$ of the n^{th} state of energy E_n, let the width of the wavefunction in position domain be Δx, and the width of the wavefunction in momentum domain be Δp. If (x, p) pair is a Fourier Transform pair, we have [5],

$$\Delta p\Delta x/2h\geq 1 \quad (21.3.3)$$

The width Δx of the wavefunction in position domain is twice the maximum position x_{max}, and the width Δp of the wavefunction in momentum domain is twice the maximum momentum p_{max},

$$\Delta p=2p_{max}. \quad (21.3.4)$$

$$\Delta x=2x_{max}. \quad (21.3.5)$$

Since (x,p) pair is assumed to be a Fourier Transform pair, the width of the wavefunction in momentum domain is determined by the width of the wavefunction in position domain and vice versa by the relationship in eqn. (21.3.3). On the other hand, the maximum displacement x_{max} is determined by the energy of the state E_n. At state n, $E_n=\hbar\omega_o(2n+1)$. We already found the width of the wave function in position domain for the state n in eqn. (21.1.7),

$$\Delta x=2(2\hbar(2n+1)/m\omega_o)^{1/2} \qquad (21.3.6)$$

Substituting for Δx in eqn. (21.3.3), we have,

$$\Delta p[2(2\hbar(2n+1)/m\omega_o)^{1/2}]\geq 2h \qquad (21.3.7)$$

$$\Delta p\geq 2\pi(\hbar m\omega_o/2(2n+1))^{1/2} \qquad (21.3.8)$$

As, $n \rightarrow \infty$, we have $\Delta p \rightarrow 0$.

From eqn. (21.3.4),

$$\Delta p=2p_{max}. \qquad (21.3.9)$$

As a result,

$$p_{max} \geq \pi(\hbar m\omega_o/2(2n+1))^{1/2} \qquad (21.3.10)$$

When position x and momentum p are assumed to be a Fourier Transform pair, the span Δp of the wavefunction $\psi_n(p)$ in the momentum domain is lower bound by the relationship given in eqn. (21.3.8).

The maximum momentum p_{max} has a lower bound given in eqn. (21.3.10). This lower bound is for p_{max}, not for p, and hence it does not prevent the on average observed momentum p from being the precise momentum since the precise momentum is within the range of Δp. The momentum p can take any value within the range $-p_{max}\leq p\leq p_{max}$. The lower bound of Δp only says that p_{max} cannot go below the lower limit given in eqn. (21.3.10) if x and p are a Fourier Transform pair. Lower bound of p_{max} is not a restriction on p. The true value of momentum p still lies within the range of $-p_{max}\leq p\leq p_{max}$. As we have seen earlier, p_{max} also has an upper bound determined by the energy of the state.

Lemma: On average Precision

Heisenberg Uncertainty principle cannot prevent the on-average measured momentum from being the precise momentum irrespective of the size of the bandwidths Δp and Δx since the true momentum lies within Δp.

Property: On Average Certainty

The introductions of lower bounds of x_{max} and p_{max} by the assumption that x and p are a Fourier Transform pair cannot prevent x taking all the values in the range $-x_{max}\leq x\leq x_{max}$ and p taking all the values in the range $-p_{max}\leq p\leq p_{max}$.

The lower bound of p_{max} introduced by Fourier Transform bandwidth bounds does not prevent what is required for a Harmonic Oscillator to exist, which is the momentum must be zero, p=0, when the position is at maximum span, $x=x_{max}$ and the position must be zero, x=0, when the momentum is at maximum, $p=p_{max}$.

However, the p_{max} and x_{max} are constants for a given Oscillator since they are determined by the energy level. If position and momentum is a Fourier Transform pair this is not possible, and hence position and momentum of a Quantum Oscillator cannot be a Fourier Transform pair. In addition, the momentum of a Harmonic Oscillator is a function of displacement x, p=p(x), and hence position and momentum of a Quantum Oscillator cannot be a Fourier Transform pair. Any two mutually dependent pair of observables such as position and momentum of a particle cannot be a Fourier Transform pair.

Property: Mutually Dependent Non-Fourier Pair

Momentum of a Harmonic Oscillator is a function of the displacement x, p=p(x), and hence position, x and momentum, p of a Quantum Harmonic Oscillator cannot be a Fourier Transform pair.

d) **Fourier Transform Bounds on** Δx

For a given wavefunction $\psi_n(p)$ with width Δp in momentum domain at state n, the width Δx of the wave function $\psi_n(x)$ is also limited by the Fourier Transform constrain,

$$\Delta p\Delta x/2h\geq 1 \qquad (21.4.1)$$

From eqn. (21.2.3), we already have the limit of Δp introduced by the energy limit of the oscillator at eigenstate n,

$$p_{max}=(2m\hbar\omega_o(2n+1))^{1/2} \qquad (21.4.2)$$

Since $\Delta p=2p_{max}$, substituting for Δp, in eqn. (21.4.1), we have,

$$\Delta x(2m\hbar\omega_o(2n+1))^{1/2} \geq h \qquad (21.4.3)$$

$$\Delta x \geq h/(2m\hbar\omega_o(2n+1))^{1/2} \qquad (21.4.4)$$

$$\Delta x \geq 2\pi(\hbar/2m\omega_o(2n+1))^{1/2} \qquad (21.4.5)$$

Since $\Delta x=2x_{max}$, we have,

$$x_{max} \geq \pi(\hbar/2m\omega_o(2n+1))^{1/2} \qquad (21.4.6)$$

When position x and momentum p are assumed to be a Fourier Transform pair, for a given wavefunction in momentum domain, $\psi_n(p)$, it imposes a lower bound on the x_{max} of the wavefunction in position domain, $\psi_n(x)$ given by eqn. (21.4.6). It is a lower bound on x_{max}. It is not a lower bound on x. It does not prevent x from being zero. Position x can take any value within the range $-x_{max}\leq x\leq x_{max}$. Momentum p can take any value within the range $-p_{max}\leq p\leq p_{max}$. It only says that x_{max} cannot go below lower limit given in eqn. (21.4.6). As it is the case for on average momentum, on average position <x> is also certain irrespective of bandwidth limits. As we have seen earlier, x_{max} also has an upper bound determined by the energy of the state.

Lemma: Powerless Heisenberg Uncertainty

Heisenberg Uncertainty principle cannot prevent the on-average measured position <x> from being the precise position irrespective of the size of the bandwidths Δx and Δp since the true position lies within Δx.

Corollary: Powerless Fourier Bandwidths

Lower bound on x_{max} for a fixed p_{max} due to the assumption that the x and p are a Fourier Transform pair cannot prevent from on average x from being zero if x is zero mean. Similarly, a lower bound on p_{max} for a fixed x_{max} due to the assumption that the x and p are a Fourier Transform pair cannot prevent on average p from being zero if p is zero mean.

Lemma: Span-Limited Non-Fourier Pair

Position span x_{max} and the momentum span p_{max} are determined by the energy level of a Quantum Oscillator. This precludes the x and p from being a Fourier Transform pair.

As it was in the case for momentum, the lower limit on x_{max} does not prevent the displacement x of a Harmonic Oscillator from reaching zero, x=o which is required for the existence of a Quantum Oscillator; it is only the span of the position x_{max} that is bounded, not the x. As it is required for the existence of a Quantum Oscillator, the momentum can be zero, p=0, when the displacement is maximum, $x=x_{max}$, and the displacement x can be zero, x=0 when the momentum is maximum, $p=p_{max}$.

However, the maximum displacement x_{max} and the maximum span of the momentum p_{max} of a Harmonic Oscillator are constant, and they are determined by the energy level of the Quantum Oscillator. If position and momentum are a Fourier Transform pair, this is not possible, and hence position and momentum of a Quantum Oscillator cannot be a Fourier Transform pair. If the displacement or position, x, and the momentum, p represent a Fourier Transform pair, they cannot represent a Quantum Oscillator. On the other hand, if the position and momentum represent a Quantum Oscillator, they cannot represent a Fourier Transform pair; one precludes the other.

Lemma: Catch 22 of a Quantum Oscillator

Position x and momentum p pair of a Quantum Oscillator cannot be a Fourier Transform pair. Position and momentum Fourier Transform pair cannot be a Quantum Oscillator.

e) **Upper and Lower Bounds For** Δp

As we have seen, oscillating particle in a state with limited energy cannot have a wavefunction of unlimited span. For a particle oscillating at a finite energy level, the maximum momentum, p_{max}, of an Oscillating particle at state n is determined by the energy level of the n^{th} state, $E_n=\hbar\omega_o(2n+1)$, and it is given by eqn. (21.2.3),

$$p_{max}=(2m\hbar\omega_o(2n+1))^{1/2} \qquad (21.5.1)$$

The momentum p of a Quantum Oscillator must be less than p_{max},

$$p\leq p_{max}, \qquad (21.5.2)$$

In addition, we also have another bound that appears when we assume that the position x and the momentum p pair to be a Fourier Transform pair, which is a lower bound. Under this bound, maximum momentum p_{max} of a Quantum Oscillator at state n is limited by,

$$p_{max} \geq \pi(\hbar m\omega_o/2(2n+1))^{1/2} \qquad (21.5.3)$$

Combining these two limits, we have,

$$\pi(\hbar m\omega_o/2(2n+1))^{1/2}\leq p_{max}\leq(2m\hbar\omega_o(2n+1))^{1/2} \quad (21.5.4)$$

For the existence of Harmonic Oscillation, when the position or the displacement x reaches maximum, the momentum p must be able to reach zero. The upper and lower bounds on p_{max} cannot prevent p from being zero. However, x_{max} and p_{max} are constants determined by the energy of the Quantum Oscillator. When x_{max} and p_{max} are constants, position and the momentum cannot be a Fourier Transform pair. Without position and momentum being a Fourier Transform pair, Quantum Mechanics itself has no existence.

f) **Upper and Lower Bounds For** Δx

We have seen for a fixed p_{max} of a wavefunction in the momentum domain $\psi_n(p)$ for state n, the width Δp is fixed and the width Δx of the wave function has a lower bound given by the Fourier Transform bandwidth limits given in eqn. (21.4.5),

$$\Delta x \geq 2\pi(\hbar/2m\omega_o(2n+1))^{1/2} \qquad (21.6.1)$$

The maximum span of the wavefunction at state n is limited by the energy of the state and given by the eqn. (21.1.7),

$$\Delta x=2(2\hbar(2n+1)/m\omega_o)^{1/2} \qquad (21.6.2)$$

Since $\Delta x=2x_{max}$ and the position x_{max} must satisfy these two limits, we have,

$$\pi(\hbar/2m\omega_o(2n+1))^{1/2}\leq x_{max}\leq(2\hbar(2n+1)/m\omega_o)^{1/2} \quad (21.6.3)$$

As n increases, the energy of the Oscillator increases. With the increase of the energy level, both the maximum displacement x_{max} and the maximum momentum p_{max} will increase. It is the energy of the state that govern the displacement and the momentum spans. The width of the wavefunction $\psi_n(x)$ in the position domain and the width of the wavefunction $\psi_n(p)$ in momentum domain will increase as the energy level n increases.

However, the assumption that the position and the momentum are a Fourier Transform pair introduces lower bounds on x_{max} for a given p_{max} and vice versa. This lower limit decreases with the increase of the energy level n. Contrary to many claims in the literature, this lower limit does not introduce a measurement problem. It only limits the x_{max}. It cannot prevent x from being zero and hence the momentum of the particle from reaching the maximum momentum p_{max}. It cannot prevent precise measurement of the position x on-average since the precise position x is within the span Δx irrespective of the size of spans of Δx and Δp.

When the energy level of a Quantum Oscillator increases, both the span of position x_{max} and span of the momentum p_{max} increases. Span of the position and the span of the momentum of a Harmonic Oscillator do not have a reciprocal relationship that is expected to have in a Fourier Transform pair, and hence position and momentum cannot be a Fourier Transform pair.

Lemma: Non-Reciprocal Relationship

Harmonic Oscillator does not have a reciprocal relationship between Δx and Δp, which is required for them to be a Fourier Transform pair.

In Quantum Mechanics, even though both $\psi_n(x)$ and $\psi_n(p)$ have infinite spans, the span of the position x and the span of the momentum p of any oscillator are strictly limited by the energy of the state. As a result, infinite span $\psi_n(x)$ and $\psi_n(p)$ cannot represent the states of an Oscillator.

If x cannot reach zero, when p is a maximum, and p cannot reach zero when x is maximum, there would be no Quantum Oscillator. If the assumption that the square wavefunction or an eigenstate of an Oscillator represent the probability of particle being at a certain position, wavefunctions consisting of nulls prevent the free movement of the particle within the entire span of the wavefunction. The presence of nulls can prevent the position from being zero when the momentum is maximum, and the momentum from being zero when the displacement is maximum. As a result, the position, and the momentum wavefunctions consisting of nulls cannot be a Quantum Oscillator.

The good news is that the assumption that the position and momentum of a Quantum Oscillator are a Fourier Transform pair cannot prevent x reaching zero when $p=p_{max}$, and p reaching zero when $x=x_{max}$. However, if the position and the momentum are a Fourier Transform pair, it cannot be a Quantum Oscillator or an Oscillator of any kind since Δx and Δp of a Quantum Oscillator must be constants determined by the energy level of the Oscillator state.

Theorem: Non-Compatibility

In a Fourier Transform pair, the width in one domain determines the width in the other domain. In a Quantum Harmonic Oscillator width in both domains are determined by the energy level.

Property: One or the Other

Fourier Transform pair (x, p) cannot be a Quantum Oscillator, and conversely, a Quantum Oscillator (x, p) cannot be a Fourier Transform pair.

g) **There is no Measurement Problem Associated with position** x **and momentum** p

If $\psi_n(x)$ and $\psi_n(p)$ are a Fourier Transform pair, the measurement of both position x and momentum p are not required. When $\psi_n(x)$ and $\psi_n(p)$ are a Fourier Transform pair, all the information in $\psi_n(x)$ is contained in $\psi_n(p)$ and vice versa; one can be derived from the other.

Further, when x and p are a Fourier Transform pair, it indicates that x and p are also simultaneously measurable. Any two observables that cannot be simultaneously measurable cannot be a Fourier Transform pair. So, if the observable x and p are a Fourier Transform pair, it is guaranteed that both x, and p, are simultaneously measurable.

It is not necessary to know both $\psi_n(x)$ and $\psi_n(p)$ separately if they are a Fourier Transform pair since the momentum information is contained in $\psi_n(x)$. The knowledge of $\psi_n(x)$ is sufficient in determining the $\psi_n(p)$. No separate knowledge is required.

Uncertainty principle cannot prevent the measurability of both position and momentum simultaneously. In fact, there will be no uncertainty principle if the position and the momentum are not simultaneously measurable since position and momentum cannot be a Fourier Transform pair if they are not simultaneously measurable.

If $\psi_n(x)$ and $\psi_n(p)$ are a Fourier Transform pair, they must have a common eigenspace. If they do not have a common eigenspace, they cannot be a Fourier Transform pair. If position and momentum are a Fourier Transform pair, position and momentum operators must have a common eigenspace. If position and momentum operators have a common eigenspace, they must be simultaneously measurable.

Irrespective of the size of a particle, the momentum is proportional to the rate of change of the position. If position is measurable, that is all required for obtaining the momentum; no separate momentum measurement is required. One electromagnetic pulse is all that is required for the measurement of both the position and the momentum simultaneously. The frequency shift of the reflected pulse provides the momentum information while the time delay of the reflected pulse provides the position information. You can Quantum theorize a motion of a particle anyway you like, if observables of a particle behave randomly only when the time is paused, those Quantum characteristics are not observable since time cannot be paused.

In Quantum Mechanics, for position and momentum to be a Fourier Transform pair, time had to be paused. When time is paused, momentum has no existence. Momentum cannot exist in reality without change in time. You can stop a particle at certain position in reality, but you cannot stop momentum of a particle at certain momentum in reality; you can only do it in your notebook, in theory, not in reality. When time is paused, momentum seizes to exist in reality. For Quantum Mechanics to exist, time has to be paused. You cannot pause time in reality, and hence Quantum Mechanics itself is not a realistic theory.

It is only at a specific frozen time that the position and momentum can be assumed to be probabilistic in Quantum Mechanics. In order to test the predictions of Quantum Theory, you need to pause the time, which is not possible. In spite of many bogus claims of experimental verifications of Quantum Theory, the fact is that there are no ways to carry out a realistic Quantum Mechanics experiments since we have no ability to pause the time. We have no ability to measure Quantum behaviors of particles in Quantum Mechanics since all the measurements that are realistically possible are in run-time.

Lemma: Simultaneity of a Fourier Transform Pair

If position x and momentum p pair is a Fourier Transform pair, the position operator and the momentum operator must have a shared eigenspace. If position operator and momentum operator have a shared eigenspace, position and momentum are simultaneously measurable.

Corollary: A Must for Fourier Pair

If position x and momentum p cannot be measurable simultaneously, they cannot be a Fourier Transform pair. Any Fourier Transform pair must be simultaneously measurable.

Contrary Theory: Existence

There will be no Uncertainty Principle per se if the position and the momentum are not simultaneously measurable since position and momentum cannot be a Fourier Transform pair if they are not simultaneously measurable.

XXII. EIGENSTATES CANNOT REPRESENT PROBABILITY DISTRIBUTIONS

We have already seen that no wavefunction or eigenstate with nulls can represent a probability density function of a particle since a particle has no escape if the particle is in between nulls. Since all the eigen states except the ground state have nulls, none of the eigenstate except the ground state can represent a probability distribution. Since the ground state is the only state that is free of nulls, it appears as if the ground state can represent a probability distribution or probability density function. However, there is another reason why none of the eigenstates, including the ground state, cannot represent a probability distribution.

We have seen that the maximum displacement of a Harmonic Oscillator is limited by the energy of the eigenstate. For state n, the maximum displacement x_{max} is given in eqn. (21.1.5),

$$x_{max}=(2\hbar(2n+1)/m\omega_o)^{1/2} \tag{22.1}$$

For any given state n, the displacement x must satisfy,

$$|x| \le x_{max} \tag{22.2}$$

$$-(2\hbar(2n+1)/m\omega_o)^{1/2} \le x \le (2\hbar(2n+1)/m\omega_o)^{1/2} \tag{22.3}$$

As a result, the probability distribution of a particle being at any x will have a strictly limited range in x. A Quantum Oscillator cannot have an infinite span at any eigenstate since the energy of any state is finite. An eigenstate of infinite span cannot represent a state of a Quantum Oscillator of finite energy.

The eigenstate $\psi_n(x)$, $\forall n$, n=0, 1, 2, 3, … does not have a bound span. The eigenstate $\psi_n(x)$ for any n has an infinite span, $-\infty \le x \le \infty$. Infinite span eigenstate $\psi_n(x)$ is unnatural for an Oscillator since it requires infinite energy and does not concur with the small displacement assumption. As a result, no eigenstate including the ground state can represent a probability distribution. Neither a state of a particle $\psi_n(x)$ nor the $|\psi_n(x)|^2$ can be a probability distribution or probability density function of a Harmonic Oscillator. Only a function that is span limited between $-x_{max}$ and x_{max} can represent a probability distribution, where $|x_{max}|$ is the maximum displacement of the state n determined by the limited energy of that state,

$$-x_{max} \le x \le x_{max} \tag{22.4}$$

$$-(2\hbar(2n+1)/m\omega_o)^{1/2} \le x \le (2\hbar(2n+1)/m\omega_o)^{1/2} \tag{22.5}$$

Oscillating particle does not have energy to be beyond the maximum displacement allowed by the energy of the particle at any state.

No particle of finite energy can be of infinite span. This is also why the so-called Quantum Tunneling is not possible. There is no Quantum Tunneling. No Oscillating particle at any state has the energy required to be beyond the displacement limits allowed by the energy of the Quantum Oscillator at that state. State of a Quantum Oscillator cannot be an unconstrained solution to the wave equation.

Lemma: Reality Oversight

An eigenstate of infinite span cannot represent a state of a Quantum Oscillator of finite energy.

XXIII. THERE IS NO QUANTUM TUNNELING

The maximum displacement of a Quantum Oscillator at any state is limited by the energy of the state. Although the eigenstate $\psi_n(x)$ has infinite span of x, $-\infty \le x \le \infty$, Quantum oscillation cannot have a displacement x beyond the maximum displacement x_{max} allowed by the energy of the state. Maximum displacement allowed by the energy of the state is given by,

$$x_{max}=(2\hbar(2n+1)/m\omega_o)^{1/2} \tag{23.1}$$

As a result, the range of x is given by,

$$-x_{max} \le x \le x_{max} \tag{23.2}$$

$$-(2\hbar(2n+1)/m\omega_o)^{1/2} \le x \le (2\hbar(2n+1)/m\omega_o)^{1/2} \tag{23.3}$$

The span of the solution to the wave equation of a Quantum Oscillator $\psi_n(x)$ that is used in Quantum Mechanics is given by,

$$-\infty \le x \le \infty \tag{23.4}$$

The eigenstates, $\psi_n(x)$, $\forall n$, n=0, 1, 2, …, we obtained have an unbound span of x. Unbound span wavefunction cannot represent a state of a Quantum Oscillator of finite energy. It is simply preposterous even to think about calling unbound wavefunction as a state of a Quantum Oscillator of finite energy. This error is due to the mistake in the formulation of the Quantum Harmonic Oscillator as an unconstrained eigenvalue eigenvector decomposition problem.

When we solve the wave equation, we did not specify the position and the momentum bounds for the Oscillator of finite energy. We should have formulated the solution to the wave equation as a conditional solution to the wave equation. If we have done so, we would have obtained the eigenstates that has a limited span that agrees with the energy of the state.

Although the eigenstates have unbound span of x, Quantum Oscillator cannot have an unbounded span since the energy at any state is finite. As a result, so-called Quantum Tunneling is not possible. There is no such phenomenon called Quantum Tunneling. Quantum Mechanics has turned out to be a chain of blunders, one blunder supporting the next. That is exactly what we can expect when the foundation of Quantum Mechanics is based on a theoretical and conceptual farce supported by experimental interpretation blunders.

No eigenstate can represent a probability of particle being at a certain position. Probability cannot determine where a particle can be. Position and momentum of an Oscillating particle are deterministic, not probabilistic. The wavefunction of a Quantum particle is as meaningless as the invalid claim that particles behave as waves.

There is nothing waving in a moving neutral particle [3]. Particles (masses) do not behave as waves. Wavefunctions cannot represent a probability distribution of particle being at certain location. It is only when the moving charge particles are stopped, accelerated, or decelerated that they generate electromagnetic waves [4]. These generated electromagnetic waves travel at the speed of light. A particle (mass) of momentum p cannot follow the waves that travel at the speed of light. These electromagnetic waves do not describe the state of a particle since the wave is completely detached from the particle that generated the wave. A particle that is at a lower speed than the radiation waves cannot follow the radiation waves.

It is known that a neutrino travels at a quite high-speed close to the speed of light, yet its mass is quite negligible. If Special Relativity is true, as the speed of a particle reaches the speed of light, the mass of the particle must reach infinity. How can a neutrino have a negligible mass if it travels close to the speed of light? Some of the claims in so-called Modern Physics are self-contradictory; it shows the mockery of so-called Modern Physics. Mass of a particle does not depend on its speed. It is the volume of a particle that depends on its speed. Time and mass are independent of frame of reference [9]. If relativity holds, I should be able to lose weight just by sitting on the couch, because according to relativity I am always moving relative to somebody really moving even though I am still sitting. If relativity holds, I should be able to move a mountain by running away from it. It is who is doing the work that decides who is moving, not the relativity.

Electrically neutral moving particles (masses) do not generate waves. Neutral moving particles (masses) do not behave as waves. Momentum does not generate waves. No mass can be in multiple places simultaneously irrespective of the size of the particle. There is no wave particle duality. There are no light particles or so-called photons. No particle waves. Quantum tunneling is mythical; it only exists in the mind of the believer, not in reality. There is no Quantum tunneling.

Property: No Charge - No wave

Movement of electrically neutral particles do not generate waves. It is the movement of charge particles, chomentum, that generates waves [3]. Once generated, these electromagnetic waves are completely independent of the behavior of the particles and do not represent states of particles.

Impossibility Theorem:

Quantum Tunneling is not possible.

XXIV. LOGICAL FORMULATION OF QUANTUM HARMONIC OSCILLATION

Quantum oscillator of mass m and restoration force constant k have the Hamiltonian H,

$$H=(1/2m)p^2+(1/2)kx^2 \tag{24.1}$$

Since $\omega_o=(k/m)^{1/2}$, we have,

$$H=(1/2m)p^2+(1/2)m\omega_o^2x^2 \tag{24.2}$$

If the energy of Quantum Oscillator is E, then, at the maximum displacement x_{max}, the momentum p=0. As a result, at the maximum displacement, the total energy will be in the form of potential energy of the Quantum Oscillator,

$$(1/2)m\omega_o^2x_{max}^2 = E \tag{24.3}$$

$$x_{max} = (2E/m\omega_o^2)^{1/2} \tag{24.4}$$

The position x of Quantum particle at the state of energy E is bounded by,

$$-x_{max} \le x \le x_{max} \tag{24.5}$$

$$-(2E/m\omega_o^2)^{1/2} \le x \le (2E/m\omega_o^2)^{1/2} \tag{24.6}$$

If the width of the wavefunction or eigenstate is Δx,

$$\Delta x=2x_{max} \tag{24.7}$$

The width of the wavefunction at state with energy E is given by,

$$\Delta x =2(2E/m\omega_o^2)^{1/2} \tag{24.8}$$

Any state ψ(x) of a Quantum Oscillator of energy E must be within the displacement bound allowed by the energy E of the Quantum state. No particle has the energy to be outside the range of Δx.

The wave equation of Quantum Oscillator is given by,

$$-(1/2m)\hbar^2\partial^2\Psi_n(x)/\partial x^2+(1/2)m\omega_o^2x^2\psi_n(x)=E_n\psi_n(x) \tag{24.9}$$

Span unconstrained solutions to the wave equation do not represent a Quantum Oscillator since the solutions $\psi_n(x)$, ∀n are unbound in the span of x. Eigenstates $\psi_n(x)$, ∀n have an infinite span x even though the span of a Quantum Oscillator at any state must be limited by the energy of the state E_n.

Quantum Oscillator of finite energy cannot have an infinite displacement. Further, the wave equation applies only for small displacement x; it does not apply for large displacements, certainly not for infinite displacements. Therefore, span unconstrained solutions to the wave equation cannot represent the states of a Quantum Oscillator.

In order to represent the states of a Quantum Oscillator, the solutions to the wave equation must be found under the constrained that the spans of the eigenstates are bound by the energy of the particle such that $-x_{max}\le x\le x_{max}$, where, $x_{max} = (2E/m\omega_o^2)^{1/2}$.

Lemma: Perfect Non-Reversibility

Although the state of any Quantum Harmonic

Oscillator is an eigenstate of the Hamiltonian $\mathbf{H}=(1/2m)\mathbf{P}^2+(1/2)m\omega_o^2x^2$, the reverse is not true. Any eigenstate of the Hamiltonian **H** is not a state of a Quantum Harmonic Oscillator.

Corollary: Lot More to It

For any wavefunction $\psi_n(x)$ to be a state of a Quantum Harmonic Oscillator, although it is NECESSARY for it to be an eigenstate of the Hamiltonian $\mathbf{H}=(1/2m)\mathbf{P}^2+(1/2)m\omega_o^2x^2$, it is NOT SUFFICIENT. For any wavefunction $\psi_n(x)$ to be a state of an Oscillator, the position x of the wavefunction must be withing the bounds allowed by the energy of the state, $-(2E_n/m\omega_o^2)^{1/2}\leq x\leq(2E_n/m\omega_o^2)^{1/2}$.

Theorem: Law Abiding

Eigenstate $\psi_n(x)$, $\forall n$, n=0, 1, 2, … is a state of a Quantum Harmonic Oscillator if and only if $\psi_n(x)$ satisfies the wave equation,

$$-(1/2m)\hbar^2\partial^2\Psi_n(x)/\partial x^2+(1/2)m\omega_o^2x^2\psi_n(x)=E_n\psi_n(x)$$

under the constrain that the displacement x is bounded by the energy of the particle so that,

$\psi_n(x)>0\ \forall n$, for $-(2E_n/m\omega_o^2)^{1/2} \leq x \leq (2E_n/m\omega_o^2)^{1/2}$,

$\psi_n(x)=0\ \forall n$, otherwise.

If $\psi_n(x)$ is normalized so that $\int\psi_n(x)dx=1$ within the range of x given by $-(2E_n/m\omega_o^2)^{1/2}\leq x\leq(2E_n/m\omega_o^2)^{1/2}$, then, $\psi_n(x)$ itself can be used to represent a probability distribution, no squaring is necessary.

There is no known closed form solution to this constrained wave equation for Quantum Oscillators. Since Quantum Mechanics itself is a hypothetical theory that does not represent the reality, any effort to find solutions to this constrained wave equation will not be of any value, a waste of time. Quantum Mechanics is a collection of mathematical and experimental bloopers.

Lemma: Impossibility

The wavefunction $\psi_n(x)$ in the position domain and the wavefunction $\psi_n(p)$ in the momentum domain of a particle cannot be a Fourier Transform pair since the span of the position and the span of the momentum of a Harmonic Oscillator are strictly limited by the energy of the state,

$\psi_n(x)>0\ \forall n$, for $-(2E_n/m\omega_o^2)^{1/2}\leq x\leq(2E_n/ m\omega_o^2)^{1/2}$,

$\psi_n(x)=0\ \forall n$, otherwise.

$\psi_n(p)>0\ \forall n$, for $-(2mE_n)^{1/2}\leq p\leq(2mE_n)^{1/2}$,

$\psi_n(p)=0\ \forall n$, otherwise,

where, E_n is the energy of the state n and

$E_n=\hbar\omega_o(2n+1)$, $\forall n$, n=0, 1, 2, 3, ….

Theorem: Catch-22

Fourier Transform pair cannot have strict predefined bandwidth limits. Harmonic Oscillators have no existence without strict limits on the position span and the momentum span determined only by the energy of the particle. As a result, the position and momentum of a particle cannot be a Fourier Transform Pair.

XXV. THERE IS NO NEGATIVE ENERGY

The negative momentum, -p means the momentum is of opposite direction of +p. Positive or negative momentum exists relative to an observer. What is considered a positive momentum for one observer can be negative for another observer. The claim that the negative momentum is associated with negative energy [1] is simply meaningless, preposterous. Negative energy can only exist in human psychic, not in nature. The energy cannot be negative. It is observers who define the negative or positive direction, not the nature. Nature has no positive or negative directions. What is positive for one observer can be negative for another observer. The concept of negative energy comes from archaic cultural and religious baggage (yin and yang) some of us a still carrying unconsciously and has no place in science except in voodoo-science.

Fourier transform contains both positive and negative frequencies simply as a result of bipolar mathematical symmetry; there are no negative frequencies. Electromagnetic energy comes in frequency bursts and the energy of the burst is proportional to frequency, E=hf. However, mechanical energy is continuous, does not come in bursts and hence mechanical energy is not proportional to frequency, $E\neq hf$. The relationship E=hf has no meaning unless it is associated with a wave burst of specific time duration. E=hf does not apply for continuous waves.

In Quantum Mechanics, mechanical energy is assumed invalidly to be proportional to frequency, E=hf. As a result, since frequency can be both positive and negative due to mathematical bipolar symmetry, the energy also appears to be both positive as well as negative. Mathematical possibility is not always a realistic possibility. There are no negative frequencies or negative energies.

Writing mechanical energy of a particle E as E=pc [1], where c is the velocity of light does not make E negative when p is negative. The velocity of light c also has the same direction as p. If p is negative it is because the direction of c is negative. Momentum p and speed c cannot be mutually exclusive since speed is inherent in the momentum p. When E is written as E=pc, the direction of c is part of it since p must be mc; the momentum of a particle cannot be detached from the speed of the particle. Since a mass cannot travel at speed of light c, for any mass $p\neq mc$ and $E\neq pc$. You cannot separate momentum from speed. Speed is a part of momentum. As a result, pc can never be negative, pc>0. Although c can be a constant, it is still a velocity at the same direction as momentum p.

You cannot write energy of a particle E as pc since mass cannot travel at the speed of light. You cannot write electromagnetic energy E as pc since electromagnetic energy is not associated with a mass. There is no momentum without an associated mass. If there is a momentum p of a particle, you cannot get the energy of the particle by multiplying p with any constant or the speed of light c. Irrespective of whether the energy E is electromagnetic energy or mechanical energy, writing E as pc is simply incorrect, meaningless, nonsense, $E \neq pc$. If E is electromagnetic energy, then $E=hf$. If E is the kinetic energy of a particle (mass), then $E=(1/2)pv$, where $p=mv$ and v is the speed of the particle of mass m. You cannot mix those two energies. So-called Modern Physics is in fact voodoo-physics; nothing is impossible there, reality out the window, mathematical and logical correctness out the window; it is the place where anything goes – professional wrestling of physics.

Contrary to the claims in Quantum mechanics [1], negative momentum does not have negative energy. What is negative for one observer is positive for another observer; positive or negative direction is relative. There is no negative energy. Electromagnetic energy, E cannot be written as momentum p times c since electromagnetic energy has no association with a mass and hence has no momentum $E \neq pc$. Electromagnetic energy does not behave as golf balls. There is no momentum without a mass. Only the mechanical energy, the energy associated with a mass can have a momentum.

Electromagnetic energy has no association with a mass and hence no momentum. It is the mistaken belief that the light is relative, which lead to the view that light has a momentum. Light is not relative [6, 7]. When light is not relative, light has no momentum and hence $E \neq pc$ and $E \neq mc^2$. In Special Relativity, light is given a momentum by assumption, not as a fact. When you fire light burst vertically from a bottom of a train, the path light takes relative to the train is not vertical. It is the vertical representation of this path in Special Relativity by assumption without a proof that fell physics into the dark abyss of voodoo-science.

If you make the invalid assumption that mechanical energy is also quantized and proportional to the frequency f described by deBroglie waves, then for a particle of mass m, the energy relationship is given by $(1/2)pv=hf$, where v is the speed of the particle and p is the momentum given by $p=mv$. A wave can start at the same speed that it propagates since the speed is solely determined by the medium characteristic. However, speed of a mass is not solely determined by the medium characteristic and hence mass cannot start at a same constant speed. A mass has to start from stand still and gain kinetic energy to remain at constant speed. As a result, the energy of a particle cannot be written as momentum p times the speed v. For a particle (mass), $E \neq pc$.

You cannot portray momentum p and speed v as two separate entities since the speed of a particle v is a momentum per unit mass. If you multiply the momentum by a constant to get the energy, that constant must be the speed of the particle, it cannot be any arbitrary constant. Speed and momentum are not disconnected entities. Momentum has the same direction as the speed and speed has the same direction as the momentum. If the momentum is negative, the speed is also negative and hence negative momentum does not mean that energy is negative. Energy is always positive.

Writing the energy E as pc and claiming c as a directionless constant to introduce negative energy [1] is simply meaningless, nonsense. The energy of a particle cannot be the product of momentum p and speed of light c since no particle is able to reach the speed of light c. To obtain energy of a particle, what p should be multiplied is p/2m, not speed of light c.

Speed of light c has nothing to do with the energy of a moving particle. Speed of light c has nothing to do with the mass of a moving particle. Time has nothing to do with a motion of a particle. Time is independent of the motion of a particle. Mass is independent of the speed of a particle. Mass is independent of the speed of light c. Momentum is a property of a moving mass. Propagation of light has nothing to do with moving masses or momentum. Light does not propagate by the momentum; light has no momentum. Light is not an equivalent of a golf ball. Speed of a wave is solely determined by the medium or by the lack of it. Speed of a particle is not solely determined by the medium.

It is the volume of a particle that has anything to do with the speed of light c. Volume of a particle contracts with the motion of the particle. Volume of a particle reaches zero when the speed of the particle reaches the speed of light c [9]. Physics no longer applies when a particle reaches the speed of light. When a particle reaches the speed of light, its volume reaches zero or in other words its mass density reaches infinity, a black hole.

Particle and the particle wave must be at the same speed. Otherwise, particle and particle wave become incoherent, and particle wave cannot represent the probability of finding a particle at certain position. If particle wave travels at the speed of light, you can forget about representing a particle wave or wave function as a probability distribution.

At the speed of light, the volume of a particle approaches zero [9] while mass density approaching infinity and hence physics no longer applies when particle reaches the speed of light c. In other words, no particle can reach the speed of light. For a moving particle of mass m, the energy E must be $E=(1/2)pv$, where $p=mv$. The energy of a particle can never be negative. If the mechanical energy is assumed to be quantized, it is one-half of pv that is equal to energy, $(1/2)pv=hf$. The product of momentum p and speed v is not equal to energy, $pv \neq hf$. The product of pc is meaningless for a particle. Mechanical energy is not Quantized and hence Quantum Mechanics is invalid at its very foundation.

You cannot equate energy E to hf wherever you come across energy E. All the energies are not created equal. It is only the electromagnetic energy that comes as electromagnetic frequency bursts that can be written as $E=hf$. Mechanical energy E cannot be written as hf, or in other words, E≠hf for mechanical energy. Mechanical energy does not come as frequency bursts; mechanical energy of a particle exists as long as particle is moving. When mechanical energy E cannot be represented as hf, or E≠hf, Schrodinger wave equation has no existence.

Particles do not have wave representations. Motion of a mass, momentum does not generate waves. It is a motion of charge, chomentum [3] that generates electromagnetic waves when a moving charge is stopped, accelerated, or decelerated. It is the misrepresentation of the electromagnetic waves generated by a moving charge particle when the particle is abruptly stopped that lead to the ill-conceived notion of non-existent particle waves of deBroglie wavelength λ=h/p [4]. There are no particle waves or deBroglie waves. There is no direct connection between the mass and the Plank constant. Quantum Mechanics is founded and validated upon the misinterpretation of experiments, specifically the double-slit experiment and the Stern-Gerlach experiment [3, 4].

XXVI. QUANTUM MECHANICS IS NOT EXPERIMENTALLY TESTABLE

For the almighty hero of Quantum Mechanics, the wave function to come alive and takes a particle on a probabilistic journey, time has to be paused. For the Quantum observables in Quantum Mechanics to come alive and display Quantum behavior, time has to be paused. We can pause time only on paper, not in reality. In reality what we have is run-time. Quantum behaviors disappear in run-time. In run-time, what we have is on average observables, which are classical observables we can observe in experiments.

The theory of Quantum Mechanics is a paused-time theory. The quantum behavior of particles is a paused-time behavior. Any paused-time theory is hypothetical since time cannot be paused to observe its predictions. What we only have access is the run-time behaviors of particles. Run-time behaviors of particles are not Quantum behaviors. Run-time behaviors are on-average behaviors in Quantum Mechanics lingo or simply what we observe in reality.

Not surprisingly, all the actions in Quantum Mechanics take place at a fixed time. Position of a particle take random distribution at a fixed time. Momentum of the same particle take a random distribution at the same fixed time. If you want to carry out an experiment to test the average position, time has to be paused, which is impossible. If you want to test the average momentum, time as to be paused. Otherwise, average position and average momentum become a meaningless concept. Time is not in our control. We cannot stop time. We cannot pause the time to test the Quantum theory. On the other hand, momentum has no existence if the time is paused.

In reality, position cannot change without change of time. In fact, time is a definition based on the change of position. Time exists because of the change of a physical entity. What came first is the change of position, not the time. We use the change of position of an object to define an entity called time. Time is a human definition. If there exist a change of position, then, there must be a change of time. Unlike the chicken and egg, when it comes to change of the position of an object and time, we know exactly what came first, the change of the position of an object.

The claim that the “Quantum Mechanics has been proven experimentally” is simply bogus since no run-time experiment provide paused-time behavior of a particle. None of those experiment is carried out when time is paused or frozen. If time is paused, nothing will take place in a run-time experiment. On the other hand, you cannot have Quantum Mechanics experiment unless the time is paused. Notwithstanding many bogus claims, Quantum Mechanics cannot be experimentally proven. The very notion of testing the predictions of Quantum mechanics experimentally is simply preposterous, cannot be done.

Average position of a particle in Quantum Mechanics is not a run-time average. Average momentum of a particle in Quantum Mechanics is not a run-time average. By repeating an experiment over and over, you cannot generate a paused-time time average since repeating is done in run-time. Ensemble average is not a paused-time average.

Quantum Mechanics is a theory developed by pausing the time. If you want to test Quantum Mechanics, you have no option, but to carry out experiments by pausing time, which is not something that can be done. You cannot pause the time because time does not exist. Time is a definition. Any experiment carried out in run-time is not a Quantum Mechanics experiment. If you develop a theory by pausing the time and try to test it using an experiment in run-time, there is no sense to what you are doing; you have no grasp of what Quantum Mechanics is about. Presumed Quantum behavior of a particle in Quantum Mechanics is lost in run-time. You can run experiment only in run-time where presumed quantum behavior has no existence. Quantum behavior of a particle is hypothetical. Quantum Mechanics is hypothetical, not testable in real run-time.

Lemma: Non-Testability

Ensemble average is not a paused-time average. Paused time behavior of Quantum observables cannot be tested using run-time experiments.

XXV. CONCLUSIONS

Hamiltonian of a Quantum Oscillator has the form $\mathbf{H}=(1/2m)\mathbf{P}^2+(1/2)m\omega_o^2x^2$. Any state of a Quantum Oscillator is an eigenstate of the Hamiltonian **H**. However, the reverse is not true. Any eigenstate of Hamiltonian **H** of a Quantum Oscillator is not a state of the Quantum Oscillator. Only a very special eigenstates of the Hamiltonian **H** can be the states of a Quantum Oscillator.

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Similarly, any state or wavefunction of a Quantum Oscillator is a solution to the wave equation of the Quantum Oscillator,

$-(1/2m)\hbar^2\partial^2\Psi_n(x)/\partial x^2+(1/2)m\omega_o^2x^2\psi_n(x)=E_n\psi_n(x)$.

However, the reverse is not true. Any wavefunction that satisfy the wave equation is not a state of a Quantum Oscillator.

Irrespective of the size of a particle, any particle that is in Harmonic Oscillation has a limited position span that is determined by the energy of the Oscillator,

$-(2E_n/m\omega_o^2)^{1/2} \le x \le (2E_n/ m\omega_o^2)^{1/2}$

where, E_n is the energy of the state n and

$E_n=\hbar\omega_o(2n+1)$, $\forall n$, n=0, 1, 2, 3, ….

No particle in Harmonic Oscillation can go beyond the position span that is allowed by the energy E_n of the particle. The maximum span, x_{max} of the particle is achieved when the total energy is in the form of potential energy, which happens when the momentum is zero. So, a particle in a Harmonic Oscillation must remain within position span of the $-x_{max}$ and x_{max}, $-x_{max}\le x\le x_{max}$, or else it will not be a Harmonic Oscillator.

In addition, the position and the momentum of any Harmonic Oscillator is perfectly correlated negatively. Position and the momentum of a Harmonic Oscillator cannot be random. It is a perfectly choreograph motion and hence each instance of position cannot be on-average random positions; each instance of momentum cannot be on-average random momentums. On average observables at one instant of time must be in perfect coherence with the on-average observables in the adjacent instant of time. On-average position and on-average momentum of a Quantum Oscillator are mutually perfectly correlated at each instant of time. If the behaviors of Quantum observables at any instant of time are random, coherent on-average run-time Harmonic Oscillator relationships of the observables are not possible.

Wavefunction of a Quantum Harmonic Oscillator of finite energy cannot have infinite span wavefunction in the position domain or in the momentum domain. If you are getting infinite span wavefunctions as solutions to a wave equation of a finite energy Quantum Oscillator, those solutions cannot represent states of a real Quantum Oscillator; the formulation of the wave equation must be incorrect. No harmonic Oscillator of finite energy can have an infinite span of position or momentum. Properly formulated wave equation of any Quantum Harmonic Oscillator must always provide eigenstates that have position span bound by the energy of the Oscillator.

On the other hands, free-standing solutions to the wave equation of a Quantum Harmonic Oscillator have no finite position span, no bounds; they all have infinite position spans. Free-standing solutions to the wave equation of a Quantum Oscillator cannot represent states of a Quantum Oscillator since the position span and the momentum span of a Quantum Oscillator are strictly limited by the energy of the Quantum Oscillator. Although Gaussian function and its associated Hermite polynomials are solutions to the wave equation of a Quantum Harmonic Oscillator, they cannot represent states of a Quantum Oscillator of finite energy since they all have infinite spans,

$\Psi_n(y)=g_n(y)\exp(-(1/2)y^2)$,

where, $g_n(y)$ is the Hermite polynomial.

Quantum Oscillator with finite energy cannot have infinite position span. The position span of any state must comply with the energy of the state.

In addition, the wave equation for Quantum Harmonic Oscillator only applies for small displacement x from the equilibrium position. As a result, the state or wavefunction $\Psi_n(y)$ must also be limited only for small displacement y in order for $\Psi_n(y)$ represent a true realistic state of a Quantum Oscillator.

Similarly, the momentum span of a Quantum Oscillator must also be bound by the energy level of the state,

$-(2mE_n)^{1/2}\le p \le (2mE_n)^{1/2}$

where, E_n is the energy of the state n and

$E_n=\hbar\omega_o(2n+1)$, $\forall n$, n=0, 1, 2, 3, ….

No Quantum Oscillator be in a momentum state of infinite span. If the state of a Quantum Oscillator in position domain is a Gaussian function and the position and the momentum are assumed to be a Fourier Transform pair, then, the state in the momentum domain will also be a Gaussian function of infinite span. As a result, the position and the momentum cannot be a Fourier Transform pair when both position and momentum spans are limited. No Fourier Transform Pair can have predefined strict bandwidth limits in both domains.

A particle in Quantum Harmonic Oscillation at finite energy state cannot be described by an eigenstate that has an infinite span. For an Oscillating particle to be at a state with infinite span of position and infinite span of momentum requires infinite energy, yet the energy of any Quantum state is finite. Irrespective of the size of the particle, span of the state in position domain as well as the span of the state in momentum domain of an Oscillator must be strictly limited by the energy of the state. No such strict limits are possible if the position and the momentum are a Fourier Transform Pair. As a result, position and momentum of a Quantum Harmonic Oscillator consisting of strict predefined span limits cannot be a Fourier Transform pair. Conversely, position and momentum without strict span limits determined by the energy cannot be a Quantum Oscillator. To be a Quantum Harmonic Oscillator, both position span and the momentum span must be strictly bound by the energy of the particle.

In addition, linear relationship between the restoration force F and the displacement x, F=-kx that Harmonic Oscillators founded upon applies only for small displacements, and hence wavefunctions of infinite span do not represent a Quantum Harmonic Oscillators.

The restricted span of a particle by the energy of the particle must be incorporated into the solution of the wave equation. As a result, the solutions to the wave equation for a Quantum Oscillator will become finding position-constrained solutions to the wave equation. For realistic solutions, wave equation of a Quantum Oscillator,

$-(1/2m)\hbar^2\partial^2\psi_n(x)/\partial x^2+(1/2)m\omega_o^2x^2\psi_n(x)=E_n\psi_n(x)$,

must be solved under the strict constrain that the maximum displacement is limited by the energy of the state,

$\psi_n(x)>0$ for $-x_{max} \le x \le x_{max}$,
$\psi_n(x)=0$, otherwise,
where $x_{max} = (2E_n/m\omega_o^2)^{1/2}$.

In addition, the momentum of a particle in Harmonic Oscillation must also be strictly limited by,

$\psi_n(p)>0$, for $-p_{max} \le p \le p_{max}$,
$\psi_n(p)=0$, otherwise,
where $p_{max}= (2mE_n)^{1/2}$.

There is no known closed-form solution to this problem. If you disregard the span limits of the position and momentum, the solutions you get are the unrealistic solutions or solutions that are inconsistent with reality of the Quantum Oscillators giving rise to illogical concepts such as Quantum Tunneling. If the energy of a particle cannot support the displacement, it is not a realistic solution; that solution does not exist in reality.

Any realistic Harmonic Oscillator only work for small displacements. There are no infinite span Harmonic Oscillators. No realistic oscillator can be of infinite span of position and infinite span of momentum. Hamiltonian of a Quantum Oscillator, $\mathbf{H}=(1/2)\mathbf{P}^2+(1/2)kx^2$ only holds true for small displacements x.

The concept of Quantum Tunneling is false. There is no Quantum tunneling. There cannot be any Quantum Tunneling since the span of a particle in a Quantum Oscillator is limited by the energy of the state, which is finite. State of finite energy cannot have an infinite span and hence Quantum Tunneling is not possible. A particle cannot be at positions where the limited energy of the particle is insufficient to support. If all you can afford is the bus fare to get around in your home-city, you cannot be anywhere in the world even though you have the freedom to do so. You cannot build a small span model and expect it to work for an infinite span; that is non-sense, not common-sense.

There is no reason to use complex operator mechanics for solving the unconstrained wave equation for Quantum Oscillators. Solution to the unconstrained wave equation for Quantum Oscillators does not require complex operator mechanics. Step-Up and Step-Down operators of Quantum Oscillators are real, not complex. Step-Up operator is the inverse of the Step-Down operator and vice versa. As a result, the product of the Step-Up and Step-Down operators is a constant. However, this product constant is 2, not 1 as in the case of a perfect inverse.

Hamiltonian of a Quantum Harmonic Oscillator is real and can be decomposed into a product of two real operators of first order, which are also the Step-Up and Step-Down operators. Since the Hamiltonian of a given Oscillator is a Constant, the product of Step-Up and Step-Down operators is also a constant, and hence for Step-Up and Step-Down operators are inverse of each other as expected.

There exists a one-line solution to the unconstrained wave equation for Quantum Oscillators. Any unconstrained solution, $\psi_n(y)$ must satisfy the condition,

$\partial^2\psi_n(y)/\partial y^2=[y^2-(2n+1)]\psi_n(y)$.

Hermite satisfies this condition. As a result,
$\psi_n(y)=g_n(y)\exp(-(1/2)y^2)$
is a solution, where $g_n(y)$ is a Hermite polynomial of any order n, ∀, n=0, 1, 2, ...

If a ground state solution exists for an unconstrained wave equation of a Quantum Oscillator, its first derivative is also a solution under certain condition. This fact itself is sufficient in obtaining both Step-Up (**U**) and Step-Down (**D**) operators. Step-Up and Step-Down operators are absolutely real, not complex. One-line solution gives both eigenvalues and eigenfunctions simultaneously. The eigenvalues of the Hamiltonian are the energy levels of the states, while the corresponding eigenfunctions are the unconstrained states or wavefunctions. Unconstrained here means the solutions to the wave equation without span bound; it does not mean free moving, which is completely different thing.

The inability of the position and the momentum of Quantum oscillator to be zero simultaneously has nothing to do with the Heisenberg Uncertainty principle. It is exclusively an inherent property of a Harmonic Oscillator that prevents the position and momentum from being zero simultaneously. In a Harmonic Oscillator, when the displacement is maximum, the momentum is zero, and when the momentum is maximum displacement is zero. When the magnitude of the position increases, the magnitude of the momentum decreases, and when the magnitude of the momentum increases the magnitude of the position decreases. It is this property that prevents the position and the momentum from being zero simultaneously, not the Heisenberg Uncertainty Principle.

If both position and the momentum are zero simultaneously, the energy of the Harmonic Oscillator will be zero, and hence there will be no Oscillator. For a harmonic Oscillator to exist, the total energy must be a non-zero constant. When the kinetic energy is maximum the potential energy is at its minimum (zero), and when the kinetic energy is minimum (zero), the potential energy is maximum. As a result, position and momentum of an Oscillator cannot be random. Position and momentum are perfectly correlated negatively. The maximum displacement is achieved when the potential energy is maximum, that is when the kinetic energy is zero. So, the displacement of any state of a Quantum Oscillator is limited by the energy of the state of the Quantum Oscillator. This is the reason why the solution to the wave equation for Quantum Oscillators must be handled as a maximum displacement constrained solution to the wave equation. The use of the word Quantum does not allow to override the energy requirement. It is not possible to use the Quantum Harmonic Oscillator to justify some arbitrary non-existent Heisenberg Uncertainty Principle; there is no link between them.

Heisenberg Uncertainty Principle is a result of invalid forcing of the position and the momentum of a particle in Harmonic Oscillation to be a Fourier Transform pair. Heisenberg Uncertainty Principle is a bandwidth limit between two domains when two domains are a Fourier Transform pair. Heisenberg Uncertainty Principle cannot prevent the simultaneous measurability of observables or their simultaneous certainty.

If the operators of observables have a common eigenspace, then, those observables are simultaneously measurable irrespective of what Uncertainty Principle declares. For two observables to be a Fourier Transform pair, they must have a common eigenspace. As a result, if two observables are assumed to be a Fourier Transform pair, that very assumption make those observables to be simultaneously measurable, otherwise they will not be a Fourier Transform pair. For two observables to be a Fourier Transform pair, the operators of two observables must have a shared eigenspace.

The claim in Quantum Mechanics that the operators must commute for observables to be simultaneously measurable is false; it is a mathematical and theoretical oversight. For observables to be simultaneously measurable, all they have to have is a shared eigenspace. Commutation of operators is not necessary for operators to have a shared eigenspace. It is perfectly possible for non-commuting operators to share an eigenspace. If the commutation of two operators is a constant, they have a shared eigenspace. The misguided claim that operators must commute for them to have a shared eigenspace [1] is absolutely false; it is a result of a theoretical error. Non-commutation of operators cannot prevent them having shared eigenspace.

“For operators to have a shared eigenspace, the commutation of operators neither necessary nor sufficient.”

Eigenvalues and Eigenfunctions obtained by solving the wave equation for Harmonic Oscillator only applies to Harmonic Oscillators, where a motion of a particle under a potential proportional to the square of the displacement. It does not represent the motion of electron in an Atom. In an Atom, the motion of electrons is under an electrostatic potential proportional to the inverse distance. Harmonic Oscillator motion and the motion of electrons in an Atom are two completely different motions. Eigenstates and Eigenvalues of a Harmonic Oscillator cannot represent the energy levels of an Atom. Applications of Quantum Oscillator solutions are limited only to particles with small displacements under Hook’s law. Microscopic charge particles do not behave under Hook’s law.

If you want to represent a wavefunction as the probability of particle being at a given position, then the ground state of a Quantum Oscillator itself can represent a probability distribution; no squaring of the ground state is necessary. The ground state of a Quantum Oscillator is positive in the infinite span of the position and contains no nulls. So why square? The ground state is magnitude integrable at any instant of time. The squaring of the ground state alters the probability distribution unnecessarily giving unfair emphasis to smaller displacements.

All the higher eigenstates contain nulls since higher states eigenfunctions are negative for ranges of positions. As a result, higher eigenstates themselves cannot be represented as probability distributions since probability distributions cannot be negative. It appears that the wavefunctions of higher states can represent probability distributions as their squares, but this is not possible since all the higher state wavefunctions contain nulls.

If any eigenfunction with nulls is represented as a probability distribution, the probability of a particle being at a null will be nil. As a result, particle will be trapped in between nulls with no mean to liberate itself from that entrapment. The situation is just like the situation of a wrongfully convicted person who also happens to be poor; there is no escape from the situation. For this reason, no higher states can represent a probability distribution even as squares.

For eigenstates or their squares to represent a probability distribution, particle must be able to be at any position within its entire range of positions. In the case of unconstrained solutions to the wave equation, the range of positions is not finite. The presence of any null in the eigenstates prevents the ability of a particle to be at any place within its range by entrapping the particle in between the nulls. As a result, no higher eigenstates can represent a probability distribution of a particle being at a certain location withing the entire range of positions of the eigenstates. It is only the ground state itself that appears to be able to represent a probability distribution for the case of unconstrained solution to the wave equation. However, unconstrained solutions even when they are free of nulls do not represent a Quantum Harmonic Oscillator.

In addition, Hook’s law only applies to small displacements. Harmonic Oscillation applies only for smaller displacements; it does not apply for large displacements. It certainly does not apply for infinite displacements. Only the constrained solutions to the wave function can represent a Quantum Oscillator provided that the span of the Oscillator is within the bound allowed by the energy of the state.

Ground state energy of a Quantum Oscillator cannot be a one-half of an energy quantum since fractional quanta cannot exist. Fractional quanta defy the very definition of quanta. If quantum-half exists, then, the quantum-half itself should be the quantum, not the quantum itself since the quantum by definition is a non-divisible quantity any further. If quantum is divisible to smaller units, that quantum is not a quantum. If a fractional quantum exists, the smallest fractional quantum that is in existence will be the new Quantum.

"Quantum-Half is an oxymoron."

Fractional quanta in Quantum Mechanics is a result of deBroglie wavelength error [3]. No particle has the energy required to be at deBroglie wavelength. Particles only have one-half of the energy that is required for them to be at deBroglie wavelength. As a result, if a particle is assumed to behave as a wave, the wavelength of the particle must be twice the deBroglie wavelength. The fractional energy quanta and fractional Spins simply disappear when the wavelength that the energy of a particle can support is used. Quantum is back to its true fake glory.

States of a Quantum Oscillator are not obtainable as free-standing solutions to the wave equation since the wave equation does not take into account the finite position span limits and the finite momentum span limits into account. Although any state $\psi_n(y)$ of a Quantum Oscillator is an eigenstate of the Hamiltonian $\mathbf{H}=\mathbf{P}^2+y^2$, any eigenstate $\psi_n(y)$ of a Hamiltonian **H** is not a state of a Harmonic Oscillator. Hamiltonian **H** of a Harmonic Oscillator is unique to any given state of a Quantum Oscillator $\psi_n(y)$ that satisfy Hamiltonian **H**; however, any $\psi_n(y)$ that is an eigenfunction of the Hamiltonian **H**,

$$\mathbf{H}\psi_n(y).=E_n\psi_n(y)$$

$$\mathbf{P}^2\psi_n(y).+y^2\psi_n(y)=E_n\psi_n(y)$$

is not unique to the Quantum Oscillator. An eigenstate of the Hamiltonian **H** does not describe the dynamics of the Quantum Oscillator unless the span of $\psi_n(y)$ is constrained to match the energy E_n of the Oscillator in question.

"Although any state of a Harmonic Oscillator is an eigenfunction of the Hamiltonian, any eigenfunction of the Hamiltonian is not a state of a Harmonic Oscillator."

Eigenstates of the form, $\psi_n(y)=g_n(y)\exp(-(1/2)y^2))$, where $g_n(y)$ is a Hermite polynomial of any order, cannot represent a Harmonic Oscillator since they all have infinite span requiring infinite energies, while any actual Oscillator has a finite position span with finite energies. State of finite energy cannot be described by an eigenfunction with infinite energy requirement. Although the position of the eigenfunction $\psi_n(y)$ is free to have an infinite position span, the position of a Quantum Oscillator of finite energy is not free to have an infinite span. Unless the states of a Quantum Oscillator is obtained as a position-span constrained solution to the wave equation, what you get as free-standing solutions to the wave equation of a Quantum Oscillator is simply unrealistic and useless; any conclusion drawn from the results is going to be unrealistic, mystical and spooky; an utterly useless exercise.

Position and momentum of a particle cannot be assumed to be a Fourier Transform pair because they are not a Fourier Transform pair [5]. You cannot force somethings to be what they are not. For a position and momentum to be a Fourier Transform pair, a particle must be able to be at infinitely many positions simultaneously for any given momentum. Similarly, the same particle must also be able to be at infinitely many momenta simultaneously for a given position. No mass can fulfill this task. No mass can be at infinitely many positions and momenta simultaneously except in voodoo-physics that exists only in human psychic and in some university textbooks in physics, not in reality. If you want to demonstrate that a particle can be at infinitely many positions and momenta simultaneously experimentally, you have to pause the time, which is not possible. No run-time experiment can demonstrate paused time Quantum properties of observables. So called Modern Physics is out of touch with reality. Universities are out of touch with reality; that is understandable since the only goal of academician is to increase the number of publications, to get more of it. Professors get up in the morning with one thing in their mind, "how can I cook up some publications today". There is nothing more annoying than the question "how many publications do you have?" How does the number matter unless it is money or votes we are talking about?

Position and momentum of a particle must be unique at any time. Irrespective of size, the position and momentum of a particle cannot change without change of time and without a cause. Any event in the nature is causal. Nothing is random in the universe. It is we who impose the randomness on the universe due to the lack of our understanding of the real working of the universe. Heisenberg Uncertainty has no effect on the simultaneous measurability of observables. Heisenberg Uncertainty Principle does not hold since position and momentum cannot be a Fourier Transform pair. When position and momentum are not a Fourier Transform pair, Quantum Mechanics seizes to exist. Position and momentum of a particle are not a Fourier Transform pair.

Particles do not behave as waves. Waves are not particles. Motion of a mass does not generate waves. It is the irregular motion of a charge that generates electromagnetic radiation waves. Since charge has no existence without a mass, we get the wrong impression that mass generates waves. It is always the charge that generates waves. Mass is just a chauffeur for a charge. Particle cannot follow these generated electromagnetic radiation waves since the radiation waves travel at the speed of light and a particle can never reach the speed of light. Once generated, these generated electromagnetic radiation waves have no attachment whatsoever to the particle and they say nothing about the state of a particle. It is the misinterpretation of this electromagnetic waves resulted from the stopping of a moving charge that had led to the concept of particle waves.

The undeniable fact is that Electrically neutral moving particles do not generate waves of any kind when they are stopped, accelerated, or decelerated. It is only the moving charge particles that generate electromagnetic radiation when they are stopped, accelerated, or decelerated.

Energy of a particle is mechanical energy. Mechanical energy does not come in quanta. Mechanical energy is continuous. Mechanical energy has no associated frequency f and hence cannot be represented as hf. Mechanical energy does not exist without a mass. As a result, mechanical energy cannot come in quanta. Anything that has a belonging such as mechanical energy cannot come in quanta since there is no way to carry the belonging information in a quantum.

Schrodinger equation is nothing more than the derivative of the plane wave with respect to time under the assumption that particles behave as waves of deBroglie wavelength and the energy of a particle is quantized. Since particles cannot behave as waves, Schrodinger equation has no existence. Since mechanical energy of a particle cannot come in quanta, Schrodinger equation cannot exist [4, 5]. Particles do not behave as waves and mechanical energy cannot come in Quanta, $E \neq hf$ for mechanical energy.

Light or Electromagnetic waves are not relative and hence there are no light particles or photons [6, 7]. If light consists of spatially random photons, directional light is not possible. Photons have no existence without being spatially random. Light cannot be spatially random. Since the light is not relative, light has no momentum. When the light is not relative, there is no space-time function. Special relativity and General Relativity have no existence since light is not relative. If the light is relative, the speed of light cannot be a constant due to the presence Shear Electromagnetic waves of which speed depends on the frame of reference [6]. Quantum Mechanics and Relativity are half-baked human crafted prophesies based on mathematical mistakes and experimental misinterpretations, theoretical and experimental deceptions at inception.

There is nothing more ridiculous than the use of light deflection near the sun to justify some ad hoc theory of General Relativity. Deflection of light near the sun is due to the density gradient of the material medium near the sun [7]. It is always the density gradient of a material medium that deflects light, not some space-time contortion of a made-up theory. Gravity has no effect on light. How can a space-time hold an object? How can the time create a fabric? How can the time create an axis that hold something? Does the time exist if nothing changes? Can you tell time if you are in an empty box or in an empty underground bunker unless you count the heat-beats? Light always follow the density gradient of the medium. Gravity cannot bend light.

The recurrent claim in physics that every predictions of Quantum Mechanics have been experimentally proven is simply laughable, an insult to scientific method of discovery. An experiment is as good as human interpretation of the result. It is always possible to misinterpret experimental result to support any theoretical blunder. Quantum Mechanics is one such theoretical blunder that had been justified by misinterpreted experiments.

Quantum behavior is a hypothetical pause-time behavior, not a run-time behavior. Quantum Mechanics is a paused-time theory that can only exist on paper or in the muddled human psychic, not in reality. For all the probabilistic nature of observables to appear, time has to be paused. In reality, nothing take place when time is paused by the very definition of time. Yes, time is a definition, a human definition. It is we who defined the time based on the repetitive natural changes in the environment. You can pause the time on paper and in your mind, not in reality.

All our experiments are on run-time experiments. All our observations are on run-time observations. We have no access to paused-time behaviors of observables; access denied. If any changes to the observables are taking place at paused-time, it is no longer a paused-time. A place in paused time is a place where no actions or changes are taking place. We only have access to run-time. We do experiments in run-time. In run-time, what we have is on average observables that are causal. We have no access to the Quantum behavior of observables in run-time. We only have access to on average behavior of observables. A paused-time theory such as Quantum Mechanics cannot be tested using run-time experiments. A paused-time theory can only exist on paper not in reality.

Paused-time Quantum Mechanics cannot be tested using a run-time experiment.

Double-Slit experiment and Stern-Gerlach Experiments are two such simple experiments that have been misinterpreted to support mathematically invalid, voodoo-theory of Quantum Mechanics [3,4]. This is no different from what TV evangelists do to justify bogus miracles of religious faith by putting perfectly mobile healthy people on wheelchairs secretly for a payment and making them to walk out of the wheelchair in front of an audience in front of TV cameras after some never-ending chanting at the command of a phony preacher. Unsuspecting audience and TV viewers open their wallets in disbelief as wheelchair-bound people walk away at the command of TV evangelists. If they are capable of making such miracles, why do we still have wheelchair bound. Can't they heal them all by their command?

Undoubtedly, People, whose livelihood depends on Quantum Mechanics and Relativity, are always going to hold on to them in the same way as some people hold on to religions. No matter how obvious and convincing the fallacies of Quantum Mechanics and Relativity are, people who benefits from those fallacies are going to hold on to them just like religious institutions are trying to hold on to flat-earth and earth-centric era meaningless bogus non-sensical doctrines using every imaginable brutal force they can muster, including the incorporation of the religious ideology into penal code, to subdue the truth. Similarly, mainstream journals take every step to prevent publication of any paper that contradicts the current state of physics in safeguarding their vested financial interest.

Nobody wants to hear that Large Hadron Collider (LHC) is a Billion Dollar Blunder even though it is [5, 8]. You cannot generate mass by colliding particles. When charge particles are collided at high speed, they generate electromagnetic radiation, not mass. This radiation bursts are not a product of the disintegration of particles in the collision. It is the misinterpretation of these extraneous radiation bursts as particles that lead to the impression of generating mass [5, 8]. It is not possible to separate the extraneous radiation due to the stopping of the moving charge particles at a collision from the intrinsic radiation burst due to the disintegration of particles in a collision. As a result, the constituent elements of particles cannot be obtained by colliding charge particles in a Large Hadron Collider (LHC). Although it is possible to obtain the constituent elements of electrically neutral and stable particles by colliding them, electrically neutral particles cannot be accelerated in LHC. LHC is a Billion Dollar Blunder hidden in Swiss Alps.

If collision of particles generate mass, mass of the sun must be increasing continuously since there are trillions and trillions of high-speed particle collisions are taking place in the sun. Mass of the sun is decreasing, not increasing. The recurrent claim that you can generate mass by colliding particles is simply non-sense, not science. You cannot generate mass by colliding particles. It is the misinterpretation of the extraneous electromagnetic burst due to the stopping of charge particles at a collision as particles that gave the impression of a false mass generation. Electromagnetic radiation wave bursts are not particles and they do not constitute a mass. By accelerating a charge particle further and further what you can increase is mass density, not the mass; mass remains unchanged, it the volume that contracts with the speed [9].

Gravity cannot bend light; it is the density gradient of the medium that bends light [7], yet some people go on making that claim because it is their job. Time cannot be relative and Global Positioning System (GPS) has nothing to do with relativity [8], yet some go on preaching that GPS is not possible without Relativity because it is their job. If you are hired to teach relativity, you have no option but to teach it. If you are hired to teach particles can be at multiple places simultaneously, you have no option but to teach it. If you are hired to light some candles, carryout some ancient rituals and preach God created heaven and earth, you have no option but to preach it, because it is your livelihood. If you question the validity of what is in the flat-earth and earth-centric era non-sensical archaic religious text, you are out of a job. If you are hired to analyze LHC data and publish the result in junk journals run by editors and reviewers whose head swelled up not by knowledge but by assumed self-importance and tones of ego (they have their PHDs gone right into their head in big time), you are going to do it because it is your job; if you question the validity of LHC you are out of the job because nobody want to accept it as a blunder. If the number of foreign students in Graduate Schools is an indication, the real reason why people go for PHD is that they cannot find real jobs. A PHD is in fact a disqualification for a job rather than other way around as it justifiably should be.

LHC is like fortuneteller's 8th ball, you can prove anything, any crafted prophesy with that. All you have to do is keep colliding until you get a matching data set to prove what you want to prove. If LHC really can be used to find the elementary component of particles, one collision is all that is required. In fact, if you collide neutral particles at high speed, what you get after each collision would be the same. The reason for different results after each and every collision is that the extraneous electromagnetic radiation due to the stopping of the particles at each LHC collision differs while the outcome due to the disintegration of particles remain unchanged in each collision. Extraneous radiation due to the stopping of moving charges at the collision is a contaminant that must be removed from the crash site. This cannot be done since it is not possible to separate the extraneous radiation due to the stopping of moving charges from the intrinsic radiation due to the disintegration of the particles by the collision.

However fallacious the established mainstream ideologies in science, religions, or politics are, it is safer to sail through life in compliance with the establishment than pointing out the mockery of mainstream ideologies; it is specially the case with some outdated, barbaric, gender discriminatory archaic religious doctrine based governing systems. It is the same reason why people tolerate kings, queens, autocratic rulers, despotic military dictators, and countries run by outdated stone-age merciless religious doctrines. It is the same reason why majority keep quite while some carryout activities that are unhealthy for the wellbeing of the only known planet that can support life and the species that live on it.

Do not wait for the people who are preaching Relativity, Quantum Mechanics, outdated flat-earth and earth-centric era religious dogmatic non-sense, or who are working on Large Hadron Collider (LHC) to accept that they are wasting life on fallacious theories and doctrines because no theoretical or experimental justification is going to change their mind since their livelihood depends on propagation of fallacy. When it comes to survival and the truth, survival matters the most naturally.

If you are praying for a creator, ask yourself, why am I praying for an entity who created so much junk real estate than suitable planets that can support life. Just in our solar system alone, all the planets are junk real estate except a very small portion of a negligibly small planet that can support life. If creator created the universe and life, that creator has failed miserably in doing at least a minimally acceptable job. Not a praiseworthy job by any mean. If you are a messenger of a creator, should you not have known at least it is the earth that orbit the sun not the other way around. Yet, all the founders of religions who claimed themselves that they were messengers of a creator were the people who thought sun goes around the earth or earth was flat. Doesn't that show the mockery of claims by some ancient individuals that they were the messengers of a creator?

If you are preaching that particles (masses) can be at multiple places simultaneously, just look in a mirror, what do you see, a scientist or a fraud? Not a scientist, definitely. Quantum Mechanics is a mathematically and conceptually invalid human crafted prophesy dictated down by misinterpreted Double-Slit experiment and Stern-Gerlach experiment just like state-enforced gender discriminatory flat-earth and earth-centric era illogical religious doctrines that claim it is blasphemous or a heresy to express any opinion against them; they both only exist in misguided and hypnotized human fantasy, not in reality.

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