# Physical Insights for Vortex Modeling of Darrieus Vertical Axis Wind Turbine as a Rankine Vortex

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Abstract— In our previous study, the turbulent flow field around a Darrieus vertical axis wind turbine is simulated using two dimensional numerical solution then averaged through a time dependent solution for more than ten complete revolutions [1]. In this study, the distributions of the mean tangential velocity components on the points of the mean perpendicular axes of the turbine, represented by the mean x and y velocity components are plotted against the distance from the turbine rotation center. The free stream velocity value is excluded from the x-velocity component distribution. The results show that the mean tangential velocity distribution is approximately fitting on a perfect Rankine vortex velocity structure which can consider the flow field around a Darrieus vertical axis wind turbine as a Rankine vortex combined with the free stream. The two dimensional Reynolds Averaged Navier-Stokes (RANS) equations are solved using Fluent 14.5 commercial solver.

*Keywords*— Vertical axis wind turbines; Vortex modeling; Ansys fluent; Computational fluid dynamics; Rankin vortex.

## I. INTRODUCTION

Various methods have been developed to predict the performance and aerodynamic loads of the Darrieus vertical axis wind turbine from which are: Momentum model, Vortex model, Cascade model, Computational Fluid dynamics model, and Experimental measurements. The vortex methods are the most accurate, the blade element is replaced with a lifting line or surfaces that depend on the azimuth position, the wake dynamics and all the effects induced by the wake are built in. Vortex models are to be formulated as inviscid flow models in which viscous effects are added as corrections either a posteriori directly on the by introducing additional vorticity emission from the point of separation. New 2D and 3D free wake vortex models have been developed to introduce better representation for wake dynamics and modeling of dynamic stall. The vortex model is based on potential flow model, which calculate the velocity in the flow field around and the influence of vortices in the wake of the blades, The turbine blades are represented as a bound vortex filament called substitution vortex filament or a lifting line whose strengths are specified from airfoil coefficients and calculated relative flow velocity and angle of attack [2]. Marten et al used nonlinear lifting line free vortex wake code integrated into the wind turbine simulation software QBlade [3]. The study compared the lifting line theory (LLT) results with U-RANS and Double

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Multiple Stream tube (DMS) simulation results showing reasonable results for steady state performance as shown in figure 16. However, the code was lacking a dynamic stall mode. McIntosh et al. examined the performance of a swept bladed VAWT utilizing a Lagrangian based two dimensional free vortex model [4]. The model showed good agreement compared to 2-bladed VAWT wind tunnel experimental data carried out at West Virginia University (WVU). Balduzzi et al performed a 3D numerical computational fluid dynamic (CFD) simulation on a single blade Darrieus VAWT and compared the results with those obtained by an open source code based on a Lifting Line free vortex wake the (LLFVW) Model [5]. The comparison illustrated high agreement between the results. Ponta et al. combined the free vortex model with a finite element analysis (FEVDTM) to calculate aerodynamic characteristics of the flow around the blades. The combined model does not use airfoil coefficient data in order to avoid quasi-steady problems. The model includes pitching circulation effect, and the apparent mass effects are included in the momentum equation. However, it does not include stall phenomena [6]. The results confirm the advantages of FEVDTM compared against the earlier models in predicting instantaneous blade forces as well as wake constitution. Strickland et al. calculated an aerodynamic prediction for two and three-dimensional Darrieus VAWT using a vortex lattice of analysis [7]. Analytical and experimental were in good agreement with regard to the normal force (Fn) is quite. The vortex models are capable with some degree of confidence in predicting the dominant aerodynamic force component related to the structural integrity of the rotor. Yang et al. calculated the effect of tip vortex on the downstream region of VAWT using (CFD) simulations. Wind velocity curves calculated by CFD simulations are consistent with Laser Doppler Velocity (LDV) measurements [8]. The momentum models, actuator disk models coupled with the blade element method, the vortex free or prescribed wake models. Recently, due to the progress in computational capabilities, aerodynamic solutions of VAWT using CFD methods have been introduced.

#### II. METHODOLOGY

In this study the turbulent flow field around a Darrieus vertical axis wind turbine with straight bladed is numerically simulated. The numerical model is validated by comparing the computed results with the available experimental data. The flow field is averaged through a time dependent solution for more than ten complete revolutions. The mean tangential velocity distribution around the turbine is analyzed at different tip speed ratios. The two dimensional Reynolds Averaged Navier-Stokes (RANS) equations are solved using Fluent 14.5 commercial solver.

### III. RESULTS AND DISCUSSION

The average power coefficient of the single Darrieus turbine is calculated at different tip speed ratios ( $\lambda$ ) using time dependent solution for the rotating turbine. Figure (1) shows that the fully turbulent k-omega SST model under predicts the values of the power coefficient and that the transition SST turbulence model has the closest results to the experimental data, similar findings are reported in reference [1]. The maximum average power coefficient for the single turbine is found to be 0.345 corresponding to  $\lambda$  equal to 2.6.



Fig. 1: Cp at Different Tip Speed Ratios, Numerical Results Vs Experimental data [1]

Figure (2) shows the mean velocity contours around the Darrieus turbine blades, two regions of high velocity magnitudes are induced on both sides downstream the turbine. The rotating region could not be averaged due to the rotation of the blades which occupies different space in the rotating domain at each time step.



Fig. 2: Velocity Contours around the Darrieus Turbine Blades

The instantaneous stream function contours around the Darrieus turbine in figure (3) shows the expansion of the streamlines as they pass through the turbine disk. Figure (4) shows the mean static pressure around the Darrieus turbine, the large low pressure spot indicates how the flow passing across the rotor plane experiences a pressure drop. The flow slows down downstream of the rotor and causes the pressure to recover back to atmospheric pressure.



Fig. 3: Stream Function at zero Azimuth Angle for the Darrieus Turbine



Fig. 4: Mean Static Pressure Contours around the Darrieus Turbine

Figure (5) shows the vorticity vector at different azimuth angles from 0 to  $120^{\circ}$ , each turbine blade produces, convicts, and interacts with a vortex system generated by itself and by other blades. The blades in the second half of the turbine rotation interact with the shedding of the turbine shaft. The vortex structure shows close results to the vortex system introduced in reference [9]. The same structure is repeated every  $120^{\circ}$  of the turbine rotation. The turbine blade produces, convicts, and interacts with a vortex system generated by itself and by other blades as shown in figure 3, this is why the blade aerodynamics involves highly unsteady flow fields. In order to figure out how the unsteadiness in the flow field around the Darrieus turbine blade affects its performance a two

dimensional quasi-steady study is done for a single blade and the numerical results are compared to the unsteady performance of a single blade.



Fig. 5: Vorticity Vector at Different Azimuth Angles

The mean x-velocity and y-velocity distribution on the principle X-axis and Y-axis are plotted as shown in figure (6) and figure (7), where the turbine center is placed at the origin of the reference co-ordinate system and the blade rotates counter-clockwise. The mean x-velocity introduced in equation (1) is a combination of the undisturbed free stream velocity, the velocity induced due to the expansion of the streamlines and an additional component due to the rotational flow.

$$Vx = V_{x-undisturbed} + V_{x-induced} + V_{x-vortex}$$



Fig. 6: Mean y-velocity Component on the X-axis



Fig. 7: Mean x-velocity Component on the Y-axis

A perfect ranking vortex has a velocity structure similar to the shown in figure (8), the tangential velocity generated by a Rankine vortex is calculated by the following equation:

$$u_{\theta}(r) = \begin{cases} \Gamma r / (2\pi R^2) & r \leq R, \\ \Gamma / (2\pi r) & r > R. \end{cases}$$



Fig. 8: Velocity Structure of a Rankine Vortex

The mean x-velocity distribution on both the positive and negative Y-axes are combining on one plot as shown in figure (9). The distribution is found to approximately fit on a perfect Rankine vortex velocity structure having a circulation ( $\Gamma$ ) equal to 4.3 for r>0.5, where (r) is the radius of the turbine under study. Consistent results have been found at different tip speed ratios. This provides a description of the Darrieus turbine as a Rankine vortex combined with the free stream.



Fig. 9: Combined Mean x-velocity Distribution on the Y-axis Vs a Perfect Rankine Vortex

#### CONCLUSION

The turbulent flow field around a Darrieus vertical axis wind turbine is simulated using two dimensional numerical solution. The numerical model is validated by comparing the computed results with the available experimental data. The flow field is averaged through a time dependent solution for more than ten complete revolutions. The distribution of the mean tangential velocity component on the mean axes, represented by the mean x and y velocity components is plotted against the distance from the turbine rotation center. The free stream velocity value is excluded from the xvelocity component distribution. The mean tangential velocity distribution is found to approximately fit on a perfect Rankine vortex velocity structure. Consistent results are found at different tip speed ratios. The flow field around a Darrieus vertical axis wind turbine can be described as a Rankine vortex combined with the free stream

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