Vibrations Of Micro-Gyroscopes, Fundamentals And Review

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***Abstract*—In this work, the vibration analysis, dynamic characteristics, and fundamentals of optimized and traditional Micro electro mechanical systems (MEMS) gyroscope is reviewed based on different types of modeling and theories including modified couple stress theory (MCST), nonlocal elasticity theory, modified coupled displacement theory, nonclassical theories with mutable and changing nonclassical parameter. Several conditions affecting the performance and operation of MEMS gyroscopes including temperature effects, effects of the rotations and angular velocity, constrained relations coupling the translational and rotational motions, and further considerations are reported and gathered. It is reported by several research articles that consideration and scrutiny of the mentioned parameters in the design procedure is an important and influential key step in order to design the most optimal MEMS devices such as MEMS gyroscopes. Optimization of MEMS gyroscopes is highly important to the industrial sector as lots of expanses and additional costs can be obviated. Moreover, optimization also yields MEMS devices with longer lifespan and better effectiveness.**

Keywords— Optimized MEMS gyroscope, vibrations, Modified coupled displacement field.

I. Introduction

Advent of micro and nano-technology helped the research and development sectors of industrial corporations to achieve a new sphere of electromechanical products with wide applications in automotive, printing, energy harvesting, bio-medical sciences etc. Such small-scaled products are manufactured by means of micro-fabrication techniques. Most common micro/nano-electro-mechanical systems (M/NEMS) serve as sensors, actuators, resonators, accelerometers, gyroscopes and in the form of so many other coupled electro-mechanical elements. Gyroscopes are usually used to determine orientation and angular velocities of any moving (rotating) body. More specifically, manufacturing vibrating structure gyroscope (Coriolis vibratory gyroscope) with MEMS technology has a simpler design theme and cheaper expenses than conventional rotating gyroscopes. As a result, modeling and dynamical analysis of MEMS gyroscope is worthy of pursuit. In advanced automotive industry, implementation of MEMS gyroscope with integrated circuits along with sensors assembled at the steering wheel leads to aversion of car roll-over which increases the safety of the car. In industrial robotics, MEMS gyroscopes are hired to control and suppress excess vibrations on the robots which finally end up in high precision of robot end effector with the desired speed and intense declarations. In photography, MEMS gyroscopes are employed for image stabilization and obviation of the blurring effects due to unavoidable motions of the camera. Consequently, optimized and proper design of MEMS gyroscopes is advantageous to car safety, quality of imaging machines, and robots performance and precision along with several other applications and benefits. To design efficient MEMS gyroscopes, modeling is the first step and one the most commonly-used geometries to represent a mathematical/mechanical model is small-sized beams. The proposed model is to be analyzed based on dynamic and vibration concepts of mechanical engineering. In order to peruse the dynamic and vibration analysis, proper theorems should be applied to gain the governing equations of motion. It is proved that for such small-scaled models, classical theorems do not yield precise results and a non-classical theorem along with a theorem compatible with the geometry should be utilized instead [1-3]. Amongst the popular nonclassical theories is the modified couple stress theory (MCST). In this theory, a material length scale parameter captures the relation between symmetric part of the rotational gradient and the couple stress tensor. The mentioned material length scale parameter enables us to consider the size-dependency of the model of gyroscope. Besides to equilibrium equations of forces and moments, an augmented equilibrium equation of moments of couples is taken into account in MCST. MCST is established based on single nonclassical parameter making the theorem simple [4].

In this section, a concise literature review about mechanical analysis of small-sized elements is presented. Vibration analysis of a thermally-stressed Timoshenko beam undergoing external load is studied by Ghadiri and Shafiei [5]. They have proposed the external as a consequent of the rotational motion. Results of this study show the effects of material variations, boundary conditions, angular velocity, and thermal stresses upon dynamic analysis of the system. Bending and vibration analysis of a micro-beam concerning position of the neutral axis is reported by Al-Basyouni et al. [6]. They suggested a variable (mutable) nonclassical parameter. Thai et al. [7] studied static, bending and vibration analysis of a Timoshenko-beam, highlighting increment of natural frequency and stiffness of small-sized systems in comparison to macro-systems. Dynamic analysis of small-sized shells based on the first order shear deformation shell model is carried out by Beni et al. [8]. They reported the difference between the results derived from classical and nonclassical theorems. Babaei et al. [9] reported vibration-dynamic analysis of Euler-Bernoulli beams using MCST. They suggested variations of thermo-mechanical properties through thickness direction of the geometry along with power law. Ke et al. [10]. Studied buckling and vibration analysis of a Timoshenko-beam using MCST. They have used material variations based on Mori-Tanaka rule of distribution. They have reported the interval of nonclassical when it directly affects the stability characteristics of the system. Dynamic responses of a MEMS element with non-homogenous material properties is reported by Babaei and Ahmadi [11]. They have utilized the MCST along with Timoshenko beam theory. Based on the first-order shear deformation beam theory, Nateghi and Salamat-talab [12] investigated buckling and vibration analysis of micro-beams undergoing thermal stress. Using differential quadrature method, they showed the importance of material length scale parameter, material distribution, poison's ratio and temperature rises upon static and dynamic behavior of the system. Transverse vibration analysis of nonlocal beams is reported by Babaei et. al [13]. They have considered the system to be stressed initially. This initial stress is imposed to the system by increasing the temperature at both bottom and top surfaces. Among the mentioned papers, variations and mutations of the length scale parameter is considered by Babaei and Rahmani [14]. In this paper for the first time; mutable material length scale parameter effects' upon dynamic-vibration characteristics of Timoshenko beam is reported using MCST. To address principal characteristics of a MEMS gyroscope, rotating systems are mostly proper and popular case studies to model desired system both mathematically and mechanically. Thus, several researchers have proposed various rotation-based concepts to determine dynamic responses of a gyroscope model. Ilkhani and Hosseini-Hashemi [15] reported vibration analysis and stability response of rotating beams based on MCST. Results of this paper reveal the effects of nonclassical material length scale parameter, tangential load value and direction of this very load, and the value of angular velocity are the most influential factors for predicting system response. Vibration analysis of a rotary micro-beam with non-uniform cross section is carried out by Shafiei et al. [16]. Based on the Euler-Bernoulli beam theory, the non-uniform tapered structure is supposed to bear axial load caused by rotation of the subject assembled on the substrate. Effects of angular velocity, material length scale parameter, length and width of the beam, and rate of cross-section variations upon the frequencies and mode shape functions are considered and reported as influential parameters. Investigation of vibration behavior of rotating micro-beam is conducted by Ebrahimi and Mokhtari [17]. Material properties of the beam vary based on the power of law through thickness direction. In this paper, importance of rotational speed as well as effect of material distribution profile on the analysis are shown. Using iteration method, Chen et al. [18] reported vibration responses of a tapered Timoshenko beam. Effects of the rotational speed and cross section change rate over dynamic analysis are reported as the key-factors. Babaei and Yang [19] carried out a research regarding vibration analysis of a rotating rod using nonlocal theory. For the first time, they introduced a novel modified coupled displacement filed which restricts the translational and rotational motions of the system. In this paper, significance of angular velocity and rotations of the system are expressed as role-playing factors. Babaei [20] extended his previous research to present a more efficient and geometrically-friendly MEMS gyroscope. He used the mentioned modified coupled displacement field along with axial functional variations of the model to peruse oscillatory response of the gyroscope. Forcing term effects over the vibratory response of MEMS devices is carried out and presented by Babaei [21] to highlight the effects of such forcing loads.

II. FUNDAMENTALS OF THEORIES WITH MEMS GYROSCOPES

Modeling of the mentioned MEMS devices are using the continuum mechanics theories such as:

Modified couple is stablished based on the idea that Cauchy stress tensor multiplied by Cauchy strain tensor; and couple stress tensor conjugated with the curvature tensor, form strain energy density function. Accordingly, strain energy for an isotropic linear elastic material is as follows (Babaei et al. [7]):

$$U_s = \frac{1}{2}\int_V (\ddot{\sigma} : \ddot{\varepsilon} + \ddot{m} : \ddot{\chi})\, dV \quad (1)$$

Where, $\ddot{\sigma}, \ddot{m}, \ddot{\varepsilon}, \ddot{\chi}$, are Cauchy stress, deviatoric part of couple stress, Cauchy strain and symmetric part of curvature tensors. These tensors are defined as follows [3]:

$$\ddot{\sigma} = 2G\ddot{\varepsilon} + \lambda \mathrm{tr}(\ddot{\varepsilon}) I_d \quad (2a)$$

$$\ddot{m} = 2Gl^2\ddot{\chi} \quad (2b)$$

$$\ddot{\varepsilon} = \frac{1}{2}(\nabla u + (\nabla u)^T) \quad (2c)$$

$$\ddot{\chi} = \frac{1}{2}(\nabla \theta + (\nabla \theta)^T) \quad (2d)$$

where, ∇ represents gradient ($\nabla = e_x \frac{\partial}{\partial x} + e_y \frac{\partial}{\partial y} + e_z \frac{\partial}{\partial z}$), u is displacement vector, l is material length scale (nonclassical) parameter, G is shear modulus (modulus of rigidity), λ represents Lame's constant, I_d is identity tensor, and θ means rotation vector:

$$\theta = \frac{1}{2}\mathrm{curl}(u) \quad (2e)$$

In order to incorporate thermal effects, following nonlinear equation is used:

Thermo-mechanical properties (Young's modulus, Poison's ratio, modulus of rigidity, thermal expansion coefficient) are supposed to vary by temperature based on the following nonlinear equation [9, 14]:

$$P = P_0(P_{-1}T^{-1} + 1 + P_1T + P_2T^2 + P_3T^3) \quad (3)$$

In order to represent an optimized model of rotating beams, Babaei and Yang [19] has proposed a linear mathematical relation which restricts rotational displacements to translational displacements. Based on this assumption, angular displacement ($\varphi(x, z, t)$) is expressed in terms of linear combination of axial (longitudinal) and lateral (transverse) displacements. It is noteworthy that since deformations due to oscillations are supposed to stay in elastic region with small amounts, such an assumption is valid (superposition principle). [19]

$$x\varphi(x, z, t) = p_1 u_0(x, t) + p_2 w(x, t) \quad (4)$$

Several results pertaining to the above-mentioned papers show and prove the significant effects of temperature, angular velocities, material profile distribution, and nonclassical theories in the following:

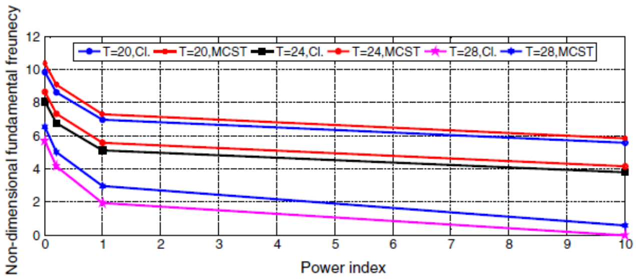

Figure 1 Effect of material profile distribution over vibratory response

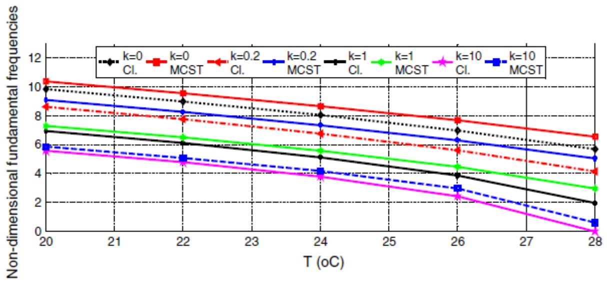

Figure 2 Effects of temperature over vibratory response

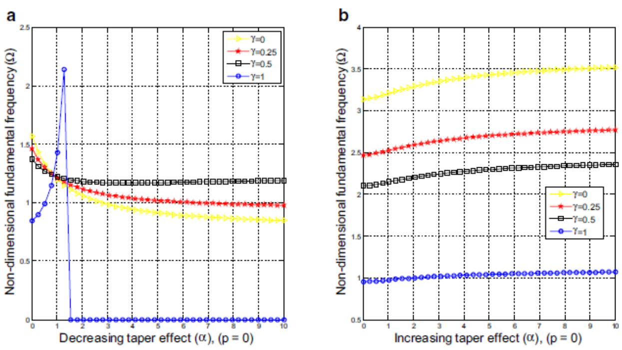

Figure 3 Effect of angular velocity over vibratory response.

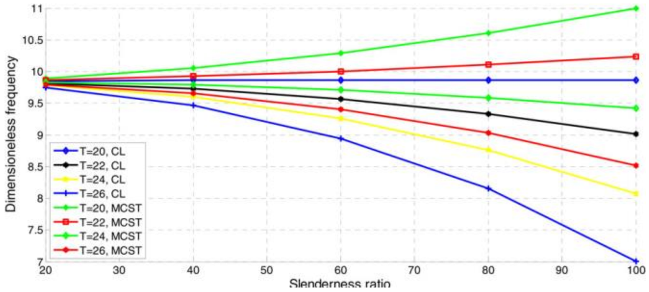

Figure 4 Effect of slenderness ration and nonclassical scale parameter over vibratory response

3. CONCLUSION

In this section, it is worth to mention and highlight the effect and impact of electro-mechanical parameters over vibratory response of MEMS devices and MEMS gyroscopes. It is proved that in order to design and benefit from high-quality MEMS devices in different industrial appliances, such as in automotive industry, bio-medical industry, and airplane industry, we can manipulate and shift behavior of MEMS devices by means of temperature effects, constrained rotating parameter and angular velocity, material profile distribution profile effect. Scrutiny of the mentioned parameters yield optimal and more effective MEMS devices and gyroscopes and guarantees save in extra costs along with longer durability. Further research efforts can be accomplished in this respect.

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