Identification Of 2nd Order Digital IIR Filter From 4th Order Classical Type Using Bacterial Foraging Algorithm (BFA)

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Abstract—**This paper describes the identification of higher-order IIR filters using metaheuristic optimization. The transfer function coefficient and the gain response for second-order individual digital filters were identified from higher-order filters designed using the classical method. Minimization of an objective function through the mean square error between desired filters and identified filter was performed with the gradient-based iterative search algorithms. Fourth-order digital IIR filters designed using the Bilinear z transform method were specifically reduced to second-order digital IIR filters using Bacterial Foraging Optimization in identifying the optimal parameters a_0, b_1 and b_2 in the required second-order digital filters. The digital filters were individually tested with an input signal, designed to have multiple frequencies within and around the passband frequency of interest, in the case of the classical and optimal designed digital filters. The identified second-order filters discriminated unwanted frequencies at very close magnitude levels with the fourth-order filters. These are shown by the plots of the FFT responses for both sets of filters which cutoff frequencies were greatly attenuated with respect to the 3-dB points within the spectrum.**

Keywords—**Butterworth Analogue and Digital filter, Mean Square Error (MSE) Infinite Impulse Response (IIR) filter, System Identification, Bacterial Foraging Optimization (BFO), Bilinear Transformation (BLT).**

I. INTRODUCTION

The act of signal processing with respect to filter is the cutting down of undesirable parts of the signal, such as random noise in the extraction process of useful parts. Filters can be categorized into two main forms namely: analog and digital. Those filter circuits that use analog electronics such as resistors, capacitors and op amps to produce the required filtering effect are called analog filter while those that use a mathematical algorithm in hardware and/or software implementation and operation on an input signal to produce a digital output signal for the purpose of achieving a filtering objective is called a digital filter. The BLT maps the analog filter to the equivalent digital filters, using the function

$$s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) \quad (1)$$

It overcomes the effects of aliasing but is somewhat degraded by frequency warping. The classical method of designing IIR filters produces high order filter functions that have good characteristics but usually computationally intensive to implement. There must, therefore, be a tradeoff between response and cost of realization. Identification can be useful in finding a system function that can present a good tradeoff that satisfies minimum specifications and realizability. Metaheuristic optimization techniques give a good prospect of achieving such identification goals. This work considered the application of BFO for the identification of a second-order IIR filter. BFO algorithm is based on foraging strategies of E Coli bacterium cells that tend to eliminate poor foraging strategies. BFO formulate the foraging behavior by maximizing the energy intake per unit time the bacteria used. The algorithms consist of mainly four steps including chemotactic, swarming, reproduction and elimination/dispersal respectively.

II. METHODOLOGY

System identification is done by mathematically modeling an unknown system using its input-output data. The varying parameters of the model are set with a given input, in which the output matches that of the system under consideration. The system whose behavior is not known; the adaptive behavior of the modeled system which keeps adjusting the parameters continuously using an adaptive algorithm. The required parameter can be obtained adaptively using the BFO algorithms, the output of the plant and the model are the same for the same set of inputs, which is the goal of system identification. The identified model depicts the characteristics of the given system. The most problems of filter system identification are formulated using adaptive IIR filtering.

Considering an IIR filter with an input-output relationship given by

$$y(k) + \sum_{i=1}^{M} b_i y(k-i) = \sum_{i=0}^{L} a_i x(k-i) \quad (2)$$

where $x(k)$ and $y(k)$ are the filter's input and output, respectively, M ($\geq L$) is the filter order. The transfer function of this IIR filter can be written as:

$$H(z) = \frac{A(z)}{B(z)} = \frac{\sum_{i=0}^{L} a_i z^{-1}}{1 + \sum_{i=1}^{M} b_i z^{-1}} \quad (3)$$

The parameters $a_0, a_1, a_2, \cdots, a_L, b_1, b_2, \cdots, b_M$ appearing in Equation (2) and (3) are the filter coefficients, and they determine the characteristics of the filter. The design of this filter can be stated as the optimization problem of objective function $J(w)$.

$$\min J(\omega) \quad (4)$$

where $\omega = \{a_0, a_1, a_2, \cdots, a_L, b_1, b_2, \cdots, b_M\}$ is the filter coefficient vector. The aim is to minimize the cost function $J(w)$ by adjusting ω.

$$J(\omega) = \frac{1}{N} \sum_{k=1}^{N} (d(k) - y(k))^2 \quad (5)$$

where $d(k)$ and $y(k)$ are the desired and actual responses of the filter, respectively and N is the number of samples used for the calculation of objective function.

From the identification model in Figure 1, the minimization of the objective function is typically defined as the mean squared error (MSE) between filter output and the desired response given in Equation 3:

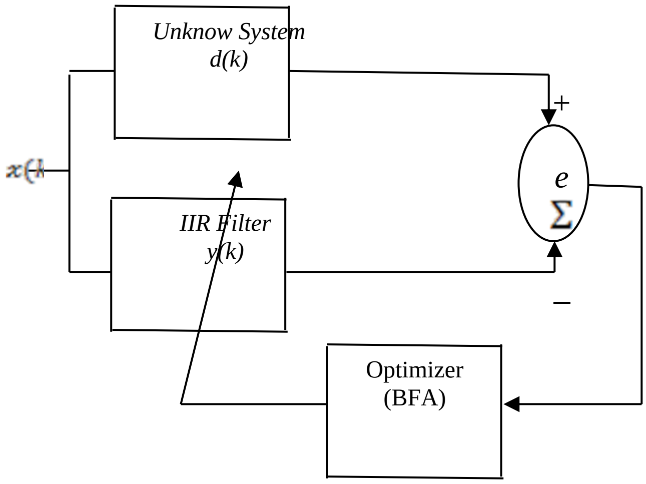


Figure 1: Schematic of IIR Filter for System Identification.

III. RESULTS AND DISCUSSION

➢ *Simulation Results*

The bilinear transformation is used to design the Lowpass, Highpass and Bandpass filters from their corresponding analog filter functions obtained in the proceeding section.

Table 1: Designed Analogue Filters Transfer Functions

Design Case	Transfer Function
Case1(LPF)	$H(s) = \frac{1}{s^4 + 2.61313s^3 + 3.41421s^2 + 2.61313s + 1}$
Case2 (BPF)	$H(s) = \frac{0.178366s^2}{s^4 + 0.597265s^3 + 2.000138s^2 + 0.54404s + 0.82971351}$
Case3 (HPF)	$H(s) = \frac{s^4}{s^4 + 1.64185e\,05s^3 + 1.34787e\,10s^2 + 6.48182e\,14s + 1.55854e\,10}$

The coefficients of filters obtained using the bilinear transform scheme and the corresponding transfer function are presented in Table 1. The coefficients identified using the BFO algorithm are presented in Tables 3. Filter type cases are as follows: Lowpass for case 1, Bandpass for cases 2 and Highpass filter for case 8.

Table 3 gives the transfer function with optimal coefficients calculated using the BFO algorithm. The numerator and denominator polynomials contain the optimal values of the filter's transfer function coefficients.

Table 2: Filters Transfer Functions Calculated using Bilinear Transform Technique

Design Case	Transfer Function
Case1(LPF)	$H(z) = \frac{8.8818e-16z^{-2} - 1.7764e-15z^{-3} + 3.3307e-16z^{-4}}{1 + -3.9999z^{-1} + 5.9998z^{-2} - 3.9998z^{-3} + 9.9993e-01z^{-4}}$
Case2(BPF)	$H(z) = \frac{3.73207e-10 - 1.33227e-15z^{-1} - 7.46417e-10z^{-2} + 3.55271e-15z^{-3} + 3.73205e-10z^{-4}}{1 + 3.99994z^{-1} + 5.99984z^{-2} + 3.99984z^{-3} + 0.99995z^{-4}}$
Case3(HPF)	$H(z) = \frac{1.2500e-05 - 2.4999e-05z^{-1} - 4.4409e-15z^{-2} + 2.4999e-05z^{-3} - 1.2500e-05z^{-4}}{1 - 3.9999z^{-1} + 5.9998z^{-2} - 3.9998z^{-3} + 9.9993e-01z^{-4}}$

Table 3: Filters Transfer Functions Calculated using BFO Algorithm

Design Case	Frequency Bands(Hz)	Transfer Function
Case1 (LPF)	60	$H(z) = \frac{-0.5216}{1 - 0.2935z^{-1} - 0.0506z^{-2}}$
Case 2 (BPF)	9989-13312	$H(z) = \frac{0.2567 - 0.2567z^{-1}}{1 + 0.1909z^{-1} - 0.0630z^{-2}}$
Case 3 (HPF)	19928	$H(z) = \frac{-0.2261 + 1.3208z^{-1} - 0.2261z^{-2}}{1 - 0.0418z^{-1} + 0.6251z^{-2}}$

As can be seen, Table 3 is second-order functions identified from the classically designed fourth-order filters. The filter performances were tested using generated test signals designed to have multiple frequencies within and outside the passband frequencies of each filter. Equations (6) to (8) gives the input signals with embedded frequencies in and around the passband of the corresponding filter.

For Case 1

$n = 0:1:200,$

$fs = 500,$

$p0 = 0, p1 = \pi/14, p2 = 2p1, p3 = 3p1, p4 = 4p1, p5 = 5p1, p6 = 6p1$

$$x = \sin\left(2\pi n \times \frac{30}{fs}\right) + \sin\left(2\pi n \times \frac{40}{fs+p1}\right) + \sin\left(2\pi n \times \frac{50}{fs+p2}\right) + \sin\left(2\pi n \times \frac{70}{fs+p3}\right) + \sin\left(2\pi n \times \frac{100}{fs+p4}\right) + \sin\left(2\pi n \times \frac{150}{fs+p5}\right) + \sin\left(2\pi n \times \frac{60}{fs+p6}\right) \quad (6)$$

For Case 2

$n = 0:1:200,$

$fs = 32000,$

$p0 = 0, p1 = \pi/14, p2 = 2p1, p3 = 3p1, p4 = 4p1, p5 = 5p1, p6 = 6p1$

$$x = \sin\left(2\pi n \times \frac{8000}{fs}\right) + \sin\left(2\pi n \times \frac{9000}{fs+p1}\right) + \sin\left(2\pi n \times \frac{10000}{fs+p2}\right) + \sin\left(2\pi n \times \frac{11000}{fs+p3}\right) + \sin\left(2\pi n \times \frac{12000}{fs+p4}\right) + \sin\left(2\pi n \times \frac{14000}{fs+p5}\right) + \sin\left(2\pi n \times \frac{15000}{fs+p6}\right) \quad (7)$$

For Case 3

$n = 0:1:200,$

$fs = 45000,$

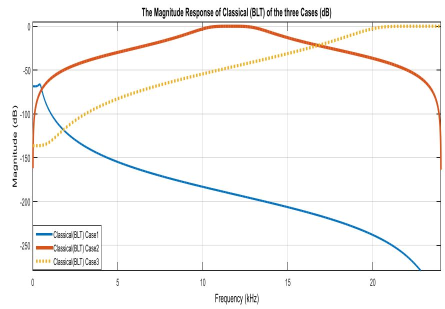
$p0 = 0, p1 = \pi/14, p2 = 2p1, p3 = 3p1, p4 = 4p1, p5 = 5p1, p6 = 6p1$

$$x = \sin\left(2\pi n \times \frac{15000}{fs}\right) + \sin\left(2\pi n \times \frac{16000}{fs+p1}\right) + \sin\left(2\pi n \times \frac{17000}{fs+p2}\right) + \sin\left(2\pi n \times \frac{18000}{fs+p3}\right) + \sin\left(2\pi n \times \frac{19000}{fs+p4}\right) + \sin\left(2\pi n \times \frac{20000}{fs+p5}\right) + \sin\left(2\pi n \times \frac{21000}{fs+p6}\right) \quad (8)$$

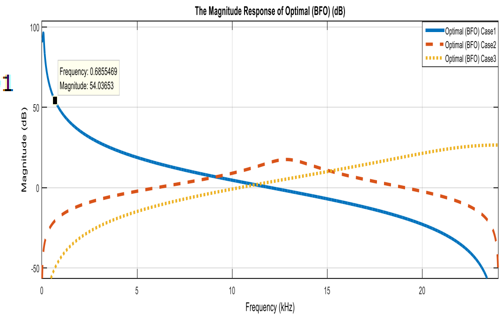
Figures 2 and 3 illustrate both classical and optimal design of all the filters magnitude and phase responses. Figure 4 illustrates the group delay responses for both classical and optimal design for all the filters. Figure 5 shows the pole-zero plots for classical and optimal design. The gain response, phase response, group delay and poles/zeros plot for each filter respectively.

➢ *FFT Test Response*

For each filter, the input/output response was tested by carrying out an FFT test on the signals. Figure 6 shows the Input and output response for filter 1 (LPF1), Figure 7 shows the Input and output response for filter 5 (BPF5), and Figure 8 shows the Input and output response for filter 8 (HPF8).



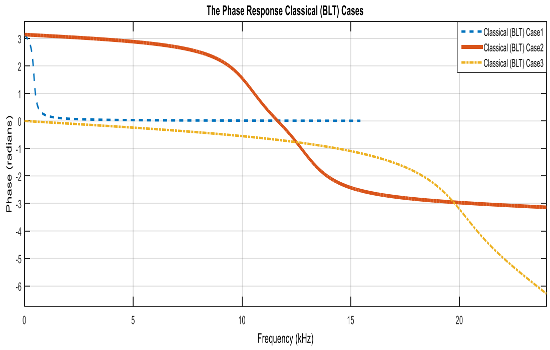
a) Classical



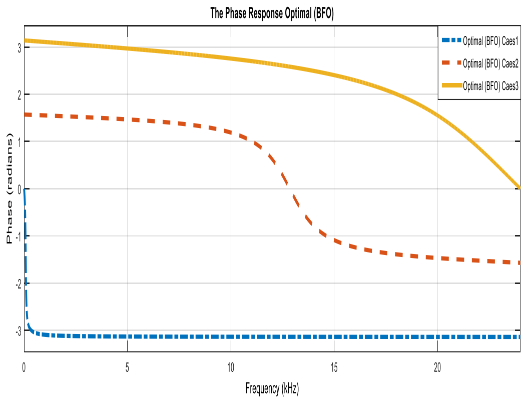
B

b) Optimal

Figure 2: Magnitude Responses of Classical and Optimal Designed Filters

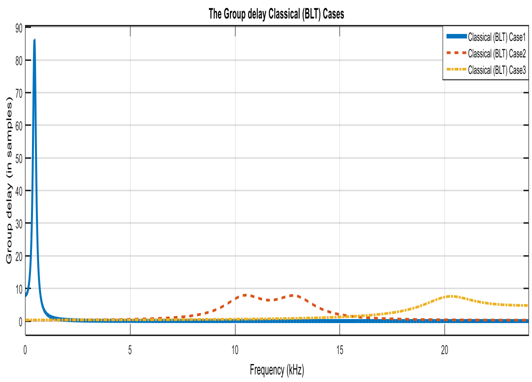


a) Classical



b) Optimal

Figure 3: Phase Responses of Classical and Optimal Designed Filters



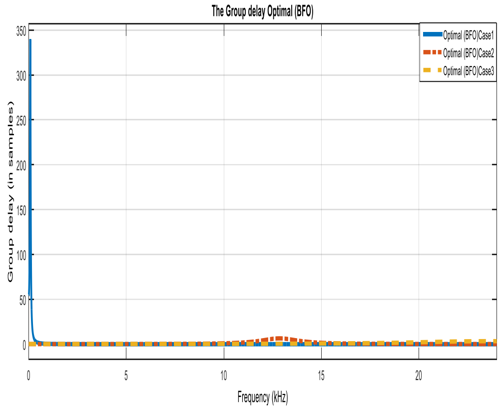
a) Classical

- *Discussion*

Each term of the transfer functions in Table 3 has lower order values when compared with the BLT design values in Table 2. The optimal design is responsible for the better amplitude response of filters designed using BFO than those designed using BLT.

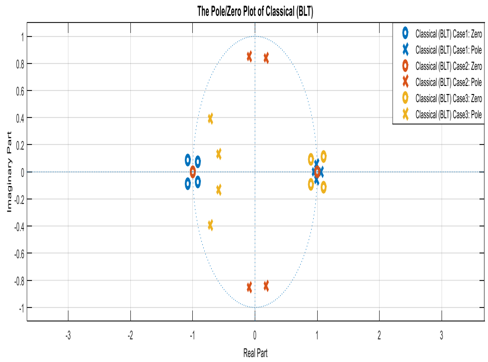
The obtained magnitude and phase response plots with respect to frequency presented in Figure 2 and 3 reveals that the designed IIR digital filters employing BFO possess flat passband and stopband characteristics. The designed filters are maximally flat at the passband with 3dB point as follows in Table 4 below as against the original specification in Table 1 with almost the same bandwidth in the case of a Bandpass filter.

The design of the IIR digital filter is feasible only if the designed filter is stable in nature. The pole-zero plots shown in Figure 5, verify that the designed IIR digital filters strictly follow the stability constraints because all the poles of designed LPF1, BPF5, and HPF8 IIR digital filter lie inside the unit circle which makes them lie with the condition for stability. The zeros of HPF8 and LPF1 digital IIR filter lie within the unit circle but the stability of filter is not affected by the position of zeros

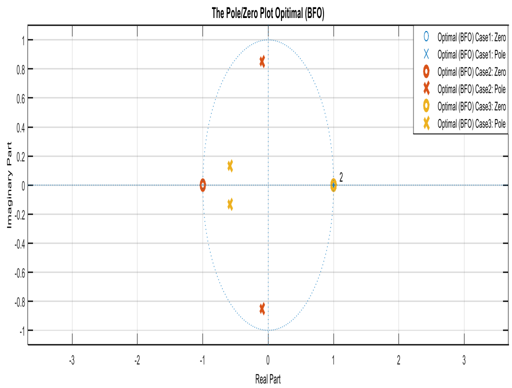


b) Optimal

Figure 4: Group Delay Responses of Classical and Optimal Designed Filters



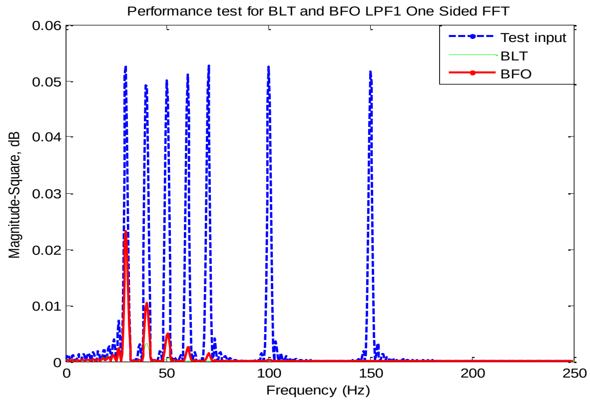
a) Classical



b) Optimal

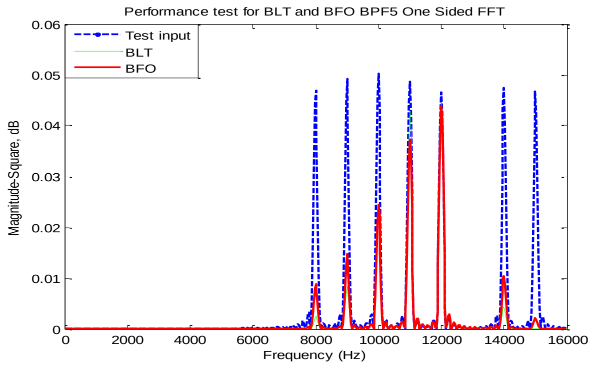
Figure 5: Poles-Zeros Plots for Classical and Optimal Designed Filters

The magnitude and angle of zeros for both the BLT and BFA design are all zero. The maximum magnitude of the pole in both the case of BLT and BFA are zeros for the LPF1 digital IIR filter, 0.8271, 0.2340 for BPF5 digital IIR filter, 0.8248, 0.5006 and 0.1324, 0.7904 for HPF8 digital IIR filter respectively. Further, the maximum magnitude pole value of all types of IIR digital filter testify that the designed IIR filter gives less quantization noise as the position of all poles is not within the region of the unit circle.



a) FFT for Input, BLT and BFO Output Response

Figure 6: Input and Output Response for Filter 1(LPF1*)*



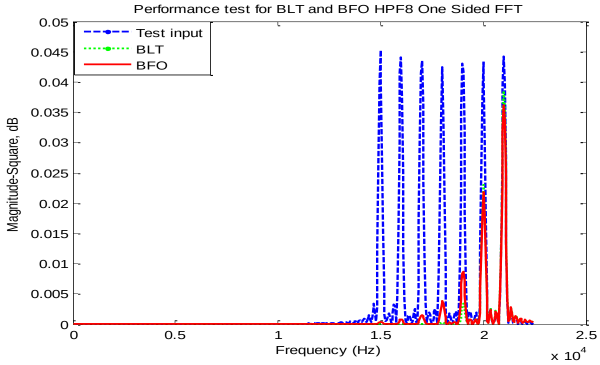
b) FFT for Input, BLT and BFO Output Response

Figure 7: Input and Output Response for Filter 5(BPF5)

The validity of the results obtained with the proposed BFA method has been established by comparing it with the BLT method. The compiled results in respect of lowest filter order, pass-band magnitude performance, stop-band magnitude performance and phase response error of all the above-mentioned methods are shown in Table 5.

The BFA designed filter has slightly wider bandwidth than its BLT counterpart, as shown in Table 5 above. Figure 8 shows both the BLT and BFA designs have minimum-phase responses, implying that there are stable and causal inverses.

The behavior of the BLT and BFA filters in discriminating input signals are here discussed in Table 6 with respect to FFT responses shown in Figures 6 to 8 for all the filters. In Table 6 above, it can be observed that all the multiple input frequencies obtained were discriminated properly in the passband region with different amplitudes, leaving those outside the bandwidth at the stopband region.



b) FFT for Input, BLT and BFO Output Response

Figure 8: Input and Output Response for Filter 8(HPF8)

Table 4: Table of 3dB Points of the Optimal Designed Filters

S/N	Band Number	3dB Point (dB)	Bandwidth (Hz)
1	LPF1	6.710	325
2	BPF5	12.3831	3554
3	HPF8	18.75133	18893

Table 5: The Behaviour of BLT and BFO Filters in Discriminating Input Signals

Band Name	Multiple Inputs frequencies obtained (Hz)	Discriminated frequencies (Hz)		Magnitude-Square (dB)
		passband	Stopband	
Case 1	30, 40, 50, 60, 70, 100, 150	30, 40, 50, 60	70, 100, 150	0.024, 0.01, 0.05, 0.025
Case 2	8000, 9000, 10000, 11000, 12000, 14000, 15000	10000, 11000, 12000	8000, 9000, 14000, 15000	0.025, 0.036, 0.046
Case 3	15000, 16000, 17000, 18000, 19000, 20000, 21000	20000, 21000	15000, 16000, 17000, 18000, 19000	0.022, 0.0351

IV. Conclusion

The obtained results were analyzed to observe the performance. The obtained IIR digital filters meet the stability criterion as all the poles lie within the unit circle. The effectiveness of proposed BFA has been also established for the identification of all IIR digital filter bands. The proposed BFA possess fast convergence speed in term of the number of function evaluations to achieve the global solution. Results obtained for the BFA justify the potential of the proposed algorithm for the design of IIR digital filter.

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