Journal of Multidisciplinary Engineering Science and Technology (JMEST)
ISSN: 2458-9403
Vol. 7 Issue 3, March - 2020

Modification of analytical description of the converter-fed induction machine

Grzegorz Skarpetowski
Ret. TU, Kraków
Untersiggenthal, Switzerland
skarpetowski@hispeed.ch

Andreas Krättli
STF Winterthur
Mastrils, Switzerland
a.kraettli@gmx.ch

***Abstract*— A modified equivalent circuit and suitable analytical description of converter-fed induction machines is presented. The modification simplifies the demonstration of the universal control properties of the widely used electro-mechanical energy converter. The modification allows describing not only the classical currently used mechanical characteristic of the induction machine, but the derivation of one today still not used mechanical characteristic similar to the characteristic of series-excited direct current machine. Such type of the mechanical characteristic realised in converter-fed induction machines drives will help to reduce problems connected with slip and slide dynamical condition in adhesion traction drives.**

***Keywords*— Converter–fed induction machines, traction drives, series-excitation mechanical characteristic, modified equivalent circuit of induction machine, modified analytical description of induction machine**

I. INTRODUCTION

Converter-fed induction machines are currently widely applied in the modern drive systems, but not all universal properties of the machine available in this type of supply are already utilised. In the modern converter-fed traction drives with induction machines a mechanical characteristic similar to the characteristic of series-excited DC machines can be realised. Such mechanical properties of the drive can help to avoid the sudden change from driving into braking, thus increasing the developed traction force of the drives and reducing the dynamic of slip and slide states. This results in reduction of the dynamic overload causing fast destruction and devastation of drive's mechanical parts.

II. STEPS IN THE MODIFICATION OF INDUCTION MACHINES EQUIVALENT CIRCUIT

The classical equivalent circuit of induction machines in form of network of impedances shown at Figure 1 was introduced by Kloss and specially adapted for almost uncontrolled operation of the machine in special supply conditions accessible in industrial network which delivers sinusoidal voltage with constant amplitude and frequency. The classic analytical description connected with the Kloss equivalent circuit doesn't specifically support the modern feeding of the induction machines with variable voltages and frequency prepared in conditioning apparatus of power electronics (named static converter).

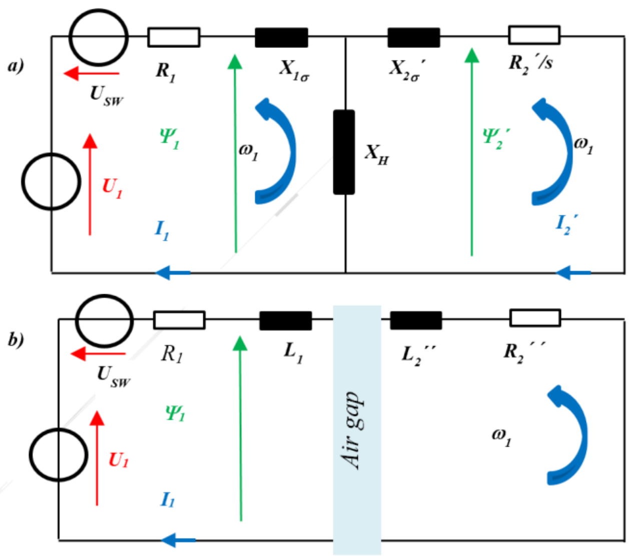

Figure 1. Single phase equivalent circuits of polyphase induction machine, a) classical, valid only for constant frequency with impedance organised in two meshes, b) first modification of the equivalent circuit valid for multifrequency supply with impedances switched in a single mesh with rotor equivalent parameter summarised in R_2'' and L_2''. So defined rotor parameters are depending of operating point coordinates ω_1 and ω_2. The schema b) is a graphical interpretation of the voltage to current immittance presented in analytical way in equation (1) which is the output of proper rearrangement of classical equation of induction machine. In the closed electrical circuit, in frequency domain, the mechanical switch is replaced by equivalent voltage source U_{SW}

First modification of the equivalent circuit needed for description of machine fed with variable stator frequency and variable voltage was introduced in [4]. The very useful analytical description carried out in the first modification of machine equivalent circuit was practically used in dimensioning of traction drives for years. The operating point of the converter-fed induction machine in this form of equivalent circuit depends on parameter R_1, L_1, T_1, T_2, σ and is controlled by the frequency of stator current $\omega_1=2\cdot\pi\cdot f_1$ and rotor current $\omega_2=2\cdot\pi\cdot f_2$.

$$Z_1(\omega_1,\omega_2) = R_1 + j\cdot\omega_1\cdot L_1 - j\cdot\omega_1\cdot L_1\cdot\frac{(1-\sigma)\cdot(\omega_2\cdot T_2)^2}{1+(\omega_2\cdot T_2)^2} + R_1\cdot\frac{(1-\sigma)\cdot\omega_1\cdot T_1\cdot\omega_2\cdot T_2}{1+(\omega_2\cdot T_2)^2} \quad (1)$$

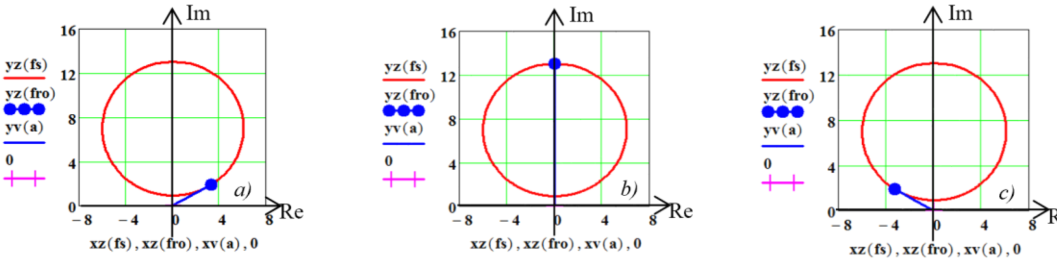

Figure 2. Complexor circle diagrams of voltage to current immittance of induction machine in three operating states. a) nominal motoring operation, b) freewheeling, c) nominal braking operation. Immittance U/I is called $Z_1(f_1,f_2)$. Following symbols are used: yz(fs) = $Im[Z_1(f_{1n},f_s)]$, xz(f_s) = $Re[Z_1(f_{1n},f_s)]$. Complexor of presented states begins in (0,0) and ends at the blue point yz(fro) = $Im[Z_1(f_{1n},f_{ro})]$, xz(f_s) = $Re[Z_1(f_{1n},f_{ro})]$. Values yv(a) and xv(a) draw the immittance complexor. f_s – scan frequency, f_{ro} – the rotor frequency in operating point. Red line presents the circle of the immittance changes with f_2

In compact form the voltage to current immittance can be written as:

$$Z_1(\omega_1,\omega_2)=\frac{U_1(\omega_1,\omega_2)}{I_1(\omega_1,\omega_2)}= \\ =R_1\cdot\frac{1-\sigma\cdot\omega_1\cdot T_1\cdot\omega_2\cdot T_2+j\cdot(\omega_1\cdot T_1+\omega_2\cdot T_2)}{1+j\cdot\omega_2\cdot T_2} \quad (2)$$

Both, the classical Kloss equivalent circuit and the equivalent circuit in the form of single mesh impedances, do not explain the way of energy flow through the induction machine.

The Kloss equivalent circuit consequently suggests that the input electrical power does not flow into the mechanical space but will be completely converted in the heat in the resistances of the equivalent circuit. The observation of the phase angle change of $Z_1(f_1,f_2)$, shows that inside of the induction machine an equivalent of power sink should be introduced. The phase shifts higher than *90°* in braking operation cannot be caused only by passive elements of equivalent circuit.

The second modification is directly connected with the searching for reasonable solution for problems occurring in slip and slide dynamic condition in adhesion traction drives. The modification made is a successful output of carried out investigation.

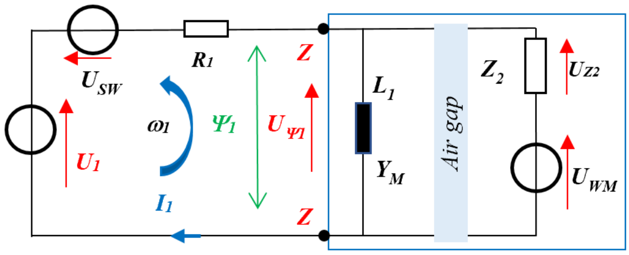

Figure 3. Modified equivalent circuit of converter - fed induction machine, valid in frequency domain for variable frequency and amplitude of fundamental harmonic in the converter output voltage. The voltage, current and flux values are presented in the form of the complex amplitude of fundamental harmonic.

- U_1 – Complex amplitude of stator voltage fundamental delivered by converter
- $U_{\Psi 1}=j\cdot\omega_1\cdot\Psi_1$ – Complex amplitude of voltage induced by stator field flux linkage
- $U_{WM}=j\cdot\omega_M\cdot\Psi_1$ - Complex amplitude of voltage at voltage sink representing the speed coordinate of mechanical power
- $U_{Z2}=j\cdot\omega_2\cdot\Psi_1$ - Complex amplitude of voltage drop on the equivalent impedance of rotor circuit in Fig.3.

The machine data of rotor mesh are transformed to the voltage level of stator winding. The symbols represent:

R_1, R_2', L_1, L_2'	Resistances and inductances of stator and rotor windings in [Ω] and in [H]
$X_H=j\omega_1 L_H$	Coupling reactance between stator and rotor windings in [Ω]
$X_{\sigma1}=j\omega_1 L_{\sigma1'}$ $X_{\sigma2}'=j\omega_1 L_{\sigma2'}$	Leakage reactance of stator and rotor winding in [Ω]
$\omega_1=2\cdot\pi\cdot f_1$, $\omega_2=2\cdot\pi\cdot f_2$	Angular frequency of stator and rotor current in [1/s]
$\omega_M=2\cdot\pi\cdot f_M$	Mechanical angular frequency of rotor in [1/s], $f_M=f_1-f_2$
$\sigma=1-L_H^2/L_1\cdot L_2$	Coefficient of total leakage of machine windings
U_1, I_1, I_2'	Complex amplitude of stator voltage in [V] and stator and rotor current in [A]
$U_{\Psi1}$, Ψ_1	Complex amplitude of stator field voltage in [V] and stator field flux linkage in [Vs]

III. THE USE OF THE MODIFIED EQUIVALENT CIRCUIT IN PRAXIS

The layout of the impedances describing the operating point of machine and used in the modified equivalent circuit allows to emphasise the main role of the flux linkage of the stator winding $\Psi_1(\omega_1,\omega_2)$ in the analytical description of induction machines. The value of stator flux linkage $\Psi_1(\omega_1,\omega_2)$ is a function of converter output voltage fundamental harmonic, pulsation in stator ω_1 and rotor ω_2 and of machine equivalent parameters.

$$U_1(\omega_1,\omega_2)=\left(\frac{1}{T_1}\cdot\frac{1+j\cdot\omega_2\cdot T_2}{(1+j\cdot\sigma\cdot\omega_2\cdot T_2)}+j\cdot\omega_1\right)\cdot\Psi_1(\omega_1,\omega_2) \quad (3)$$

The cross admittance in the equivalent circuit connected with the magnetic circuit of the machine is described by equation:

$$Y_M(\omega_1,\omega_2)=\frac{1}{j\cdot\omega_1\cdot L_1} \quad (4)$$

Impedance in the rotor equivalent circuit is described by equation:

$$Z_{Z2}(\omega_1,\omega_2) = R_1 \cdot \frac{T_1}{T_2} \cdot \frac{1 + j \cdot \sigma \cdot \omega_2 \cdot T_2}{(1-\sigma)} \tag{5}$$

The value of stator flux linkage $\Psi_1(\omega_1,\omega_2)$ describes in characteristic way the operating point of the machine. The voltage induced in energetic part of equivalent circuit (without R_1) is proportional to the fundamental frequency of static converter output terminals:

$$U_{\Psi 1}(\omega_1,\omega_2) = j \cdot \omega_1 \cdot \Psi_1(\omega_1,\omega_2) \tag{6}$$

The voltage induced in the voltage sink placed in rotor equivalent branch is proportional to the mechanical angular frequency $\omega_M = \omega_1 - \omega_2$ of the machine shaft:

$$U_{WM}(\omega_1,\omega_2) = j \cdot \omega_M \cdot \Psi_1(\omega_1,\omega_2) \tag{7}$$

Voltage drop on the rotor impedance depends on the slip pulsation ω_2:

$$U_{Z2}(\omega_1,\omega_2) = j \cdot \omega_2 \cdot \Psi_1(\omega_1,\omega_2) \tag{8}$$

Stator current as a velocity coordinate of electric input power of the machine is described by equation:

$$I_1(\omega_1,\omega_2) = \frac{U_1(\omega_1,\omega_2) - j \cdot \omega_1 \cdot \Psi_1(\omega_1,\omega_2)}{R_1} = \\ = \frac{\Psi_1(\omega_1,\omega_2)}{L_1} \cdot \frac{1 + j \cdot \omega_2 \cdot T_2}{1 + j \cdot \sigma \cdot \omega_2 \cdot T_2} \tag{9}$$

Current $I_2(\omega_1, \omega_2)$ through the equivalent voltage sink in rotor is converted in a torque coordinate of mechanical power developed in the rotating magnetic field of induction machine:

$$I_2(\omega_1,\omega_2) = j \frac{\Psi_1(\omega_1,\omega_2)}{L_1} \cdot \frac{(1-\sigma) \cdot \omega_2 \cdot T_2}{1 + j \cdot \sigma \cdot \omega_2 \cdot T_2} \tag{10}$$

The values obtained in frequency domain separately for all frequencies of the *3* phase system calculations are inverse transformed by Fourier Transformation into the time domain. For further calculation in the time domain for all *3* phase values the mathematical concept of rotating vector is used. The rotating vector definition of the stator current is repeated, as:

$$\overrightarrow{Vi_1(t)} = \frac{2}{3}\left(i_{11}(t) \cdot e^{j \cdot \frac{2\cdot\pi}{3} \cdot 0} + i_{12}(t) \cdot e^{j \cdot \frac{2\cdot\pi}{3} \cdot 1} + \\ i_{13}(t) \cdot e^{j \cdot \frac{2\cdot\pi}{3} \cdot 2}\right) \tag{11}$$

The time domain description of power consumed by the induction machine is represented by equation (12). The power average value derived from frequency domain and calculated only for fundamentals frequencies in stator and rotor is equal P_{10}:

$$p_1(t) = \frac{3}{2} \cdot Re\left\{\overrightarrow{Vu_1(t)} \cdot \overrightarrow{Vi_1{}^*(t)}\right\} \quad where \\ P_{10} = \frac{3}{2} \cdot \frac{|\Psi_1(\omega_1,\omega_2)|^2}{R_1 \cdot T_1{}^2} \cdot \frac{1 + (1-\sigma) \cdot \omega_1 \cdot T_1 \cdot \omega_2 \cdot T_2 + \omega_2^2 \cdot T_2^2}{1 + \sigma^2 \cdot \omega_2^2 \cdot T_2^2} \tag{12}$$

Mechanical power at the shaft of the machine is equal to the power consumed by the voltage sink existing in rotor branch. The mechanical power average value derived from frequency domain and calculated only for fundamental frequencies in stator and rotor is equal P_{WM0}:

$$p_{WM}(t) = \frac{3}{2} \cdot Re\left\{\overrightarrow{Vu_{WM}(t)} \cdot \overrightarrow{Vi_1{}^*(t)}\right\} \quad where \\ P_{WM0} = \frac{3}{2} \cdot \frac{\left|\Psi_{1(\omega_1,\omega_2)}\right|^2}{L_1} \cdot \frac{(1-\sigma) \cdot \omega_M \cdot \omega_2 \cdot T_2}{1 + \sigma^2 \cdot \omega_2^2 \cdot T_2^2} \tag{13}$$

I. INDUCTION MACHINE AS ENERGY CONVERTER

The converter-fed induction machine has become the most universal electro – mechanical energy converter and is widely used in diverse drive systems and especially in high power traction drives. Converter deliver to the stator windings of the induction machine phase shifted voltage blocks which are created as 3 series of width modulated pulses. Each series of voltage blocks can vary amplitude and frequency of the fundamental harmonic. The generation of *3* phase voltages means conditioning of electrical “force” coordinate of electric input power. The conditioning of both electric coordinates is described with equation system based at the theory of rotating vector of converter control distribution [5] $Vtsd_{3PH}(t)$:

$$\begin{cases} \overrightarrow{Vu_1(t)} = u_{DC}(t) \cdot \overrightarrow{Vtsd_{3PH}(t)} \\ i_{DC}(t) = \frac{3}{2} \cdot Re\left[\overrightarrow{Vi_1(t)} \cdot \overrightarrow{Vtsd_{3PH}{}^*(t)}\right] \end{cases} \tag{14}$$

Block diagram of the energy conversion in simple traction drive has been presented on Figure 4. Analytical description of the power conversion presented has been introduced in [5] and is described with equation system (15). There are two equations describing the energy conversion. Voltage and current as input coordinates of electrical power are converted in mechanical power coordinates: torque and angular frequency of the shaft. The conversion is described as a cross connection between “force coordinates” (voltage, torque) and “speed coordinates” (current, angular frequency) of the two different power (or energy) types. Input values of the converter black box are the “speed coordinates”: rotating vector of current $Vi_1(t)$ and angular stator frequency ω_1. Output values are the “force coordinates” rotating vector of voltage $Vu_{\Psi 1}(t)$ and torque $m_E(t)$. The input values of the converter are multiplied by the box contents and become the output values of the black box. Further necessary conditioning of the mechanic power parameter from $m_E(t)$ to values $m_M(t)$ and $\Omega_M(t)$ to $\omega_M(f)$ is executed in the “p” box of parametrizes. The box represents the change of the mechanical power coordinate to the mechanical power coordinates done by the pole pair of the magnetic circuit of the machine. The number of the pole pairs increases the value of the developed torque and reduces the angular speed of the motor shaft.

$$\begin{cases} m_M(t) = \frac{3}{2} \cdot p \cdot Re\left[j \cdot \overrightarrow{V\psi_1(t)} \cdot \overrightarrow{Vi_1{}^*(t)}\right] \\ \overrightarrow{Vu_{\Psi 1}(t)} = (\omega_2 + p \cdot \Omega_M) \cdot j \cdot \overrightarrow{V\psi_1(t)} \end{cases} \tag{15}$$

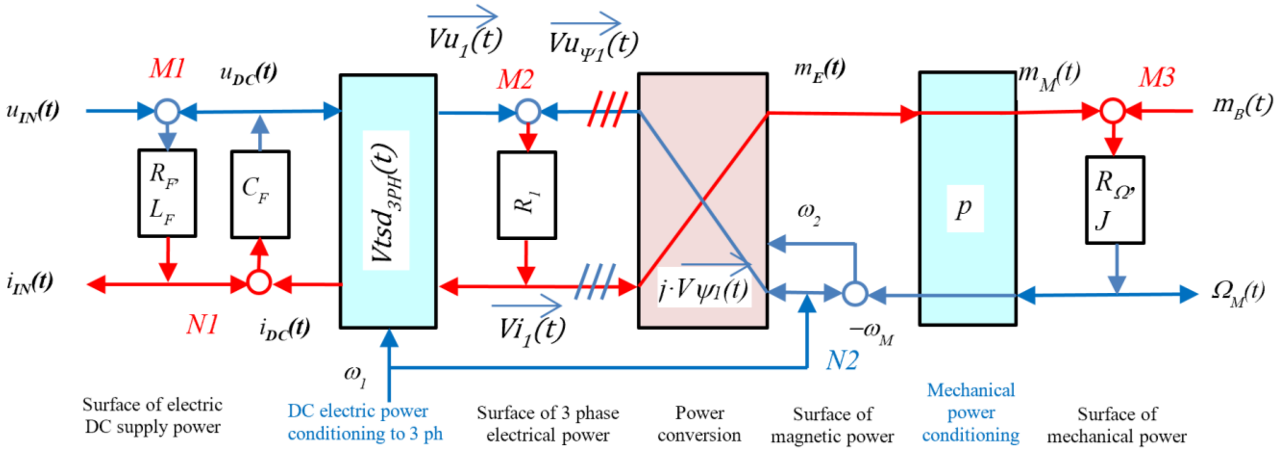

Figure 4. Block diagram of modified analytical description of induction machine as a part of DC fed traction drive. Two types of black boxes have to be used in correct description of power conversion taking place inside the traction drive. First blue box represents the input power conditioning. Electric energy stored in DC link is changed in alternating electric 3 phase system by modulation with rotating vector of control distribution $tsd_{3PH}(t)$. The second (orange) box presents the change of the energy from electrical into mechanical type. The induction machine converts the 3-phase rotating vector of stator current $Vi_1(t)$ into torque in rotating magnetic field $m_E(t)$. The electrical "speed" $Vi_1(t)$ coordinates are converted into the "force" coordinate $m_E(t)$. The rotating vector of stator electrical "force" coordinate $Vu_{\psi 1}(t)$ is divided in two parts: mechanical part of speed coordinate $\omega_M(t)$ and rotor slip angular frequency part. Second blue box inside of the machine model is necessary to describe the conditioning of coordinates from mechanical in magnetic field into the coordinates in mechanical power at the shaft of the machine. The conditioning of mechanical coordinates can be described as simple multiplication with "p" representing the pole pair of stator windings.

II. CONTROL METHOD OF CONVERTER – FED INDUCTION MACHINE IN ADHESION TRACTION APPLICATION

In the most cases only two basic control strategies of the converter – fed induction machine are discussed.

1. Strategy $\Psi_1 = const.$
   $U_1 = var.$, $\omega_1 = var.$, $\omega_2 = var.$ *(U control) or*
   $I_1 = var.$, $\omega_1 = var.$, $\omega_2 = var.$ *(I control)*
2. Strategy $U_1 = const.$, $\omega_1 = var.$, $\omega_2 = var.$

Between the both methods exist a substantial difference. The difference can be shown on the equation (16) as a difference between U_1 and $U_{\psi 1}$. The difference is equal to the voltage drop on stator winding resistance.

$$U_1(f) = R_1 \cdot I_1(f) + U_{\psi 1}(f) \qquad (16)$$

In the $\Psi_1 = const.$ method the value of converter voltage fundamental U_1 have to be controlled in a way ensuring the same constant value of stator flux linkage independent of operation point of the drive. The control method of $\Psi_1 = const.$ is currently used for frequency range lower than nominal operating point of the machine.

In $U_1 = const.$ method implemented in the drives with 2 levels converter the fast controllability of the stator flux linkage value is practically lost and the value of $U_{\psi 1}$ depends mainly on stator frequency ω_1 and additionally on rotor frequency ω_2. The $U_1 = const.$ is currently used in frequency range higher than the nominal value of the machine stator frequency. This causes partially the loss of full controllability of the drive.

The instability occurring in the DC link voltage in $U_1 = const.$ control method is transferred to the stator winding of induction machine. Convolution of converter harmonics with DC link harmonics is exclaiming different consequences for the stability of the traction drive. Only the use of 3 level converters allow suitable control of the stator voltage fundamental harmonic value U_1 in this frequency range.

The best suitability for adhesion traction drives seems to have a method ATD (Adhesion Traction Drive) being a mix of both methods. The mix method can be described in following way:

3. Strategy ATD, $U_{\psi 1} = f(\omega_M + \omega_2)$, $\omega_M = var.$, $\omega_2 = var.$, e.g. $I_1 = const.$ or $\omega_2 = const$

In strategy ATD the amplitude of voltage fundamental characteristic will not be directly controlled by the stator frequency (as usually) but in very special way by vehicle velocity. In multi drives traction systems the amplitude of voltage fundamental will be given by the drive with the lowest mechanical speed. In states of good adhesion, the vehicle velocity and stator frequency are in stabile relation. In the slip and slide states without acceleration of vehicle the phase voltage does not change with the stator frequency and stator flux change place resulting in an operation at mechanical characteristic of series excited machine. The advantages of such type of characteristic applicated to adhesion traction drives are very well known.

III. THREE BASIC MECHANICAL CHARACTERISTIC OF CONVERTER – FED INDUCTION MACHINE

In Figure 5, the difference between U_1 and Ψ_1 characteristic is shown. In normal operating conditions without dynamic states, the vehicle velocity and the stator frequency are in constant relation and the amplitude of voltage follows the stator frequency with appropriate time shifting and damping. The course of the ATD characteristic crossing the same operating point is very different from U_1 and Ψ_1 characteristic. The reaction of the drive in sudden occurring of slip and slide is different too. The vehicle wheels will cause the change of the converter output frequency without change of stator voltage.

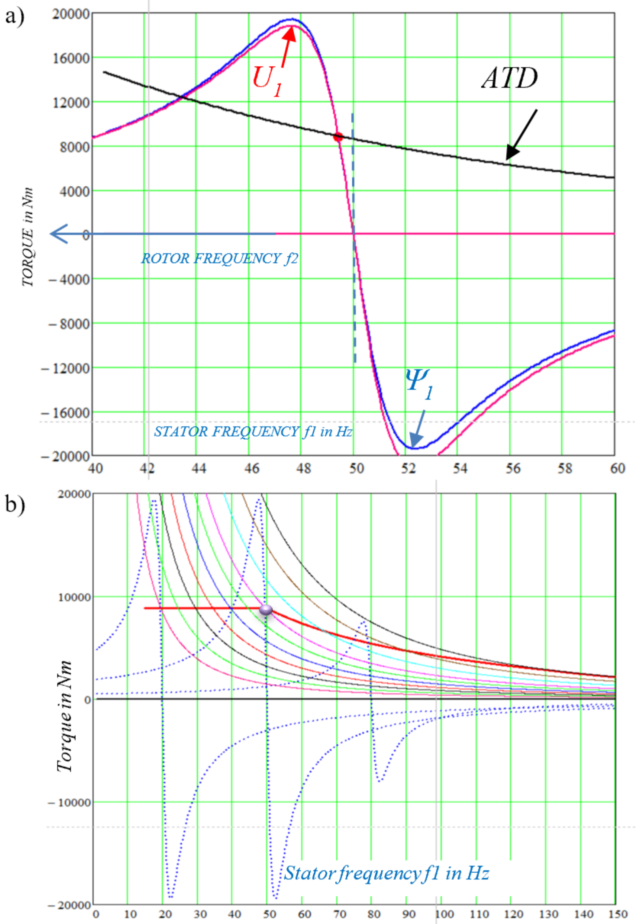

Figure 5. a) Three basic mechanical characteristic of converter – fed induction machines in the nominal load operating point. Only characteristics Ψ_1 = const. and U_1 = const. are actually used in the traction system drives. The characteristics Ψ_1 = const. are used in frequency range lower than the nominal frequency. The characteristics U_1 = const. are used in frequency range higher than the nominal frequency of machine. In the practical load range, which is located between nominal operating point in motoring and nominal operating point in braking, the mechanical characteristics Ψ_1 and U_1 are almost equal and similar to the properties of shunt excited direct current machines. The derivate dM/df1 of these characteristics has very large values. Such behaviour of the traction machine is very unfavourable for traction drive characteristics. b) examples of the family of characteristic ADT curves at mechanical power surface. The red line shows a traction characteristic

Reduction of flux, increase of stator frequency moves the operating point of traction machine along the series – excited characteristic ATD. In normal adhesion conditions the operation point follow the U_1 or Ψ_1 characteristic. The gradient angle of the serial characteristic can be simple controlled by additional parameters. For example, with $\omega_2=f(I_1)$, ω_2= const., ω_2= f($\Delta\omega_M$) e.t.c.

IV. BASIC MECHANICAL CHARACTERISTIC IN PARAMETRICAL FORM

Mechanical characteristics, defined as relation between the average value of torque M and average value of angle frequency ω_M of machine shaft shown at Figure 5 can be represented in analytical way as equation (17), (18) and (19). Equation (17) and (18) describe the characteristic of induction machine similar to the characteristic of shunt excited DC machine. Characteristic (17) is implemented in frequency range *0* Hz to *fn*. Characteristic (18) is implemented in frequencies higher than nominal frequency *fn*.

The equation of mechanical characteristic controlled with Ψ_1 = const. is given in parametric form:

$$\begin{cases} M(\Psi_1,\omega_1,\omega_2) = \dfrac{3}{2}\cdot p\cdot(1-\sigma)\cdot\dfrac{|\Psi_1|^2}{L_1}\cdot\dfrac{\omega_2\cdot T_2}{1+\sigma^2\cdot\omega_2^2} \\ \Omega_M = \dfrac{\omega_1-\omega_2}{p} \end{cases} \tag{17}$$

The equation of characteristic U_1 = const. in parametric form:

$$\begin{cases} M(U_1,\omega_1,\omega_2) = \frac{3}{2}\cdot p\cdot(1-\sigma)\cdot L_1\cdot\frac{|U_1|^2}{{R_1}^2}\cdot \\ \qquad\cdot\frac{\omega_2\cdot T_2}{(1-\sigma\cdot\omega_1\cdot T_1\cdot\omega_2\cdot T_2)^2+(\omega_1\cdot T_1+\omega_2\cdot T_2)^2} \\ \Omega_M = \frac{\omega_1-\omega_2}{p} \end{cases} \tag{18}$$

For the almost linear part of both characteristics enclosed between nominal load in motoring and braking operation a simplified characteristic of induction machine in the explicit form usually used only by shunt excited DC machine can be derived. Normally only this part of mechanical characteristic is used in drives with converter – fed induction machine.

$$\Omega_M \approx \frac{\omega_1}{p} - \frac{R_1\cdot M_\Psi}{3\cdot(1-\sigma)\cdot\frac{T_2}{T_1}\cdot p^2\cdot\left(\frac{|\Psi_1|}{\sqrt{2}}\right)^2} \approx \frac{\omega_1}{p} - \frac{R_1\cdot M_\Psi}{C_A\cdot p^2\cdot|\Psi_1|^2} \approx \frac{|U_1|}{p\cdot|\Psi_1|} - \frac{R_1\cdot M_\Psi}{C_A\cdot p^2\cdot|\Psi_1|^2} \tag{19}$$

The ATD characteristic of converter fed induction machine can be made very similar to the characteristic of series excited DC machine. The amplitude of stator voltage is in that case controlled independent from the fundamental of stator frequency. This is the basis of the drive properties developed in slip and slide dynamic states where the amplitude of stator voltage depends not on stator frequency but on the vehicle velocity. The separation of amplitude and frequency control in dynamic states allows shifting the operating point of the machine along a series characteristic and asymptotic reducing the torque to zero without change of torque sign.

The equation of series characteristic realised in drive with induction machine is as follow:

$$\begin{cases} M(v,\omega_1,\omega_2) = \frac{3}{2}\cdot p\cdot(1-\sigma)\cdot L_1\cdot\frac{|U_{1H}(v)|^2}{{R_1}^2}\cdot \\ \qquad \cdot\frac{\omega_2\cdot T_2}{(1-\sigma\cdot\omega_1\cdot T_1\cdot\omega_2\cdot T_2)^2+(\omega_1\cdot T_1+\omega_2\cdot T_2)^2} \\ \Omega_M = \frac{\omega_1-\omega_2}{p} \end{cases} \quad (20)$$

$|U_{1H}(v)|$ represents the amplitude of the converter voltage fundamental depending on vehicle speed valid in both operating states: in normal operating conditions and in slip and slide dynamic states.

References

[1] Shepherd, A., 1990. Higher Electrical Engineering, Longman Scientific & Technical, Essex CM20 2JE

[2] Dreszer, J., 1975 Mathematik Handbuch für Technik und Naturwissenschaft, Harri Deutsch Verlag, Thun

[3] Seely, S., 1962, Electromechanical Energy Conversion, MCGRAV-HILL BC 1962 NEW YORK

[4] Skarpetowski, G.; 1974, Przekładnia prądu przemiennego do lokomotyw spalinowo-elektrycznych, PW

[5] Skarpetowski, G., 1997, Uogólniona teoria przekształtników statycznych, ISSN 0137-2319 WPW 199

[6] Skarpetowski, G., 2016, Przetworniki i Przekształtniki Energii, ISBN 978-83-86219-86-5

[7] Skarpetowski, G., 1994, Method of Controlling Electric Vales of a converter, US Patent Nr. 5,331,537