Analysis Of Cotton Bollworm-Cotton SIS With Disease

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***Abstract*—Helicoverpa armigera is an important borer pest in cotton buds and a dominant species in the budding period of cotton in China. In recent years, the damage has been very severe, which has brought huge losses to cotton production. Because the cotton bollworm is attached to the growth of cotton, it is of great significance for the prevention and control of cotton bollworm to study the relationship between cotton and bollworm. In this paper, the cotton-cotton bollworm ecosystem was taken as the starting point to study the relationship between susceptible cotton bollworm and infected cotton bollworm and cotton, and the disease-prone SIS model of "cotton bollworm-cotton" was constructed and simulated by numerical simulation R_0.The global asymptotic stability of the ill-posed equilibrium is simulated. The Hurwitz theorem is used to prove the asymptotic stability at the positive equilibrium point. The positive equilibrium point is illustrated by constructing the Liapunov function and Dulac theorem. The global stability. The results of the numerical simulation also illustrate the rationality of the conclusions obtained, and at the same time, we conciude that the amount of cotton bollworm and cotton reached equilibrium under the condition of $R_0 > 1$.**

Keywords—Cotton Ridgeworm; cotton; SIS model;Basic reproduction number R_0; stability

I. INTRODUCTION

Cotton is one of the most important cash crops in the world. In China, cotton is an agricultural product second only to food crops. It plays an important role in economic development. China is one of the world's major cotton producing countries. [1] With the continuous expansion of the cotton planting area, the impact of ecological disasters has also increased. Among them, the disaster caused by cotton bollworm is the most serious and representative. According to literature records, cotton bollworm is widely distributed in China and around the world, and it occurs in cotton and vegetable growing areas in China. The cotton areas in the Yellow River Basin and the Yangtze River Basin suffered much damage. Since the 1990s, cotton bollworm disasters have occurred in the cotton areas along the Yangtze River in Jiangsu, and there has been a tendency to grow into recurrent insect pests. The losses caused by cotton bollworms are increasing. [2] The direct economic losses caused by cotton bollworm in China each year 6 billion yuan. The cotton bollworm mainly feeds on cotton buds, flowers, and bolls, and also takes young leaves. Therefore, this article starts from the food supply side, and mainly studies the relationship between cotton growth and cotton bollworm feeding. In terms of cotton research: First, the growth characteristics of cotton plants. The growth and development characteristics of cotton are largely determined by the genetic characteristics of the specific variety itself, but environmental conditions and cultivation measures also have a certain effect on cotton growth [3-4]. Cotton sowing period directly affects the cotton growth process Proper early sowing can advance the cotton boll end period, thereby extending the effective flowering boll end period, and increasing the total number of bolls per unit area and increasing the yield [5]. Dong et al. (2006) research results show that in the Yellow River The combination of early sowing and low density in the cotton area of the watershed can extend the flowering period of cotton and increase the rate of boll formation, so that the output of lint is significantly increased and the cotton matures early [6]. The second is cotton planting density. Reasonable cotton planting density is the material basis for ensuring high and stable cotton production. A reasonable planting density can effectively coordinate the relationship between vegetative and reproductive growth of cotton, individual and group growth, and group and environmental conditions, so as to make fuller use of climate resources (light, heat, water, fertilizer, etc.) in order to achieve cotton High yields provide a material basis. At higher planting densities, the number of first fruit branch nodes increased, and the number of fruit branches and fruit branch nodes was significantly less than that of low-density fruit branches and fruit nodes. Cotton plant types were more compact (Li Hong et al., 2010). Zhou Yiqiang et al. (1999) observed the number of main stem leaves and stem thickness of cotton plants of different densities at different periods, and found that although the density was different at the seedling stage, the individuals were small and there was no sign of competition between the plants. There was no significant difference in stem thickness. However, with the delay of the fertility process, the enlargement of the individual led to the enlargement of the canopy of the population, and competition or interference between the plants; the true leaf number of the main stem and the thickness of the stem showed a downward trend. Thinning. At low density, the density of individual cotton plants is higher than that of crickets, and individual development is advantageous. In the research of cotton bollworm control, the first is induction control. According to the process of cotton bollworm growth and cotton growth and development, the feeding degree of cotton bollworm and its effect on cotton defense-related enzyme activity after feeding were systematically studied [7], and the lag period of insect resistance induced by a specific concentration of jasmonic acid was discussed. And its effect on the relative growth rate of cotton bollworm [8]. The second is comprehensive governance. At present, the research on the control of cotton bollworm has been comprehensively controlled from agricultural control, trap and kill control, biological control, chemical control, physical mechanical control and other means to minimize the cotton bollworm's loss to cotton in this season [9-13]. The third is to increase production. In [14], the quadratic orthogonal rotation combination design method was used to study the combined effects of four decision variables on high yield and quality, and a mathematical model was established with cotton yield as the objective function. Literature [15] also improved cotton yield from improving carbohydrate source-sink relationships. The fourth is the implementation of genetic engineering. Due to the amazing reproductive power of bollworms, many scholars have developed genetically modified cotton in China and eliminated cotton bollworms that caused serious damage to cotton [16-17]. Planting genetically modified insect-resistant cotton can have significant advantages in terms of production costs and environmental protection [18-19]. However, the current research literature only makes a qualitative analysis of cotton bollworm eating cotton, and lacks quantitative analysis and theoretical derivation. In this paper, the population of Helicoverpa armigera and its changes are taken as the research object. The theory of population ecology and infectious disease dynamics is used to comprehensively consider the spread of the disease and the interaction between the population. Among the factors of cotton, the effect of cotton density on cotton plant growth and bolling was researched, and the influence of cotton density on the food supply of cotton bollworm was further deduced, and the impact on the ecology of cotton bollworm was obtained. Combined with the statistical law of cotton-bottom bollworm development, the standard type [20-21] was adopted, and based on the similarity of infectious predator-prey model [22-23], the cotton bollworm-cotton Dynamic model. Under the assumption that other conditions do not affect the experimental results and further analysis of the model, the global asymptotic stability of the ill-conditioned equilibrium point is discussed. This shows that the boll regeneration rate is lower at this point, and the cotton disaster rate is correspondingly greatly reduced. This stable point is a brand-new relatively balanced ecosystem. Through data simulation, the error is within a reasonably controllable range. Based on system measurement and control of cotton bollworm migration and feeding, we can achieve sustainable ecosystem control of cotton bollworm, predict the time zone and planting area of pests, and take more accurate measures. Preventive measures, early preparation, and timely disposal provide scientific basis for the research on mitigating cotton bollworm disasters. In addition, we can control the density of cotton plants to a reasonable level based on the overall laws of the ecosystem. Within the scope, while the ecosystem is in balance, the cotton output is maximized to achieve the purpose of increasing the economic income of farmers.

II. MODEL ESTABLISHMENT

The cotton bollworm mainly feeds on cotton buds, flowers, and bolls, and also feeds on young leaves. Therefore, the population density of cotton bollworm is related to the number of cotton plants. That is, without the invasion of other species, the population density of cotton bollworm is closely related to the environmental capacity of cotton. Cotton provides energy and place for the growth and reproduction of cotton bollworm, while the cotton bollworm's own disease transmission and the systemic capacity of cotton adversely affect The increase in population density of cotton bollworm affects and restricts each other. In addition, cotton is the basis of the ecological environment. Different planting densities have a very close effect on cotton plant height, number of fruit branches, bolls and rotten bolls, and cotton bollworms use cotton bolls as their main food. Therefore, cotton density The boll population growth also has a great relationship, which is a factor that cannot be ignored in this ecosystem. Generally speaking, the plant height and the number of fruit branches of a cotton plant first increased and then decreased with the increase of planting density. The number of bolls per plant and the number of rotting bolls per plant decreased as the density increased, but the unit area As the density increases, the number of bolls first increases and then decreases, and the number of bad bells per unit area increases with the increase of density. The clothing fraction did not change significantly with cotton density, and the single boll weight decreased with increasing density. Therefore, we take the cotton growth situation, cotton environmental capacity, cotton density, predatory coefficients between cotton bollworms that are susceptible to infection, and cotton bollworms that are infected with cotton as elements to construct a cotton-cotton bollworm ecosystem model.

The model is as follows:

$$\begin{cases} \dfrac{dX(t)}{dt} = mX(t)\left(1-\dfrac{x(t)}{K}\right) - c_1 X(t)S(t) - c_2 X(t)I(t) \\ \dfrac{dS(t)}{dt} = \rho X(t)S(t) - d_1 S(t) - \beta S(t)I(t) + \theta I(t) \\ \dfrac{dI(t)}{dt} = \beta I(t)S(t) - d_2 I(t) - \theta I(t) \end{cases}$$

among them: $X(t)$ Populate density of cotton; $S(t), I(t)$ Populate densities of susceptible cotton worms and infected cotton worms, m express cotton growth rate, K Indicates the environmental capacity of cotton , c_1, c_2 prediction coefficient between susceptible cotton bollworm and infected cotton bollworm and cotton , $\rho = kc(0<\rho<1)$ is the conversion coefficient, which means that the cotton worm eats cotton and transforms it into energy needed for its own growth; d_1 expresses the natural mortality rate of susceptible cotton worms, d_2 Indicates the death rate of infected cotton worms, may wish to set $d_2 > d_1$, β Cotton infection rate, θ represents the recovery rate of infected cotton insects. In the model, the infected cotton insects are still susceptible. According to the actual biological significance, Considering above $R^3 = \{(X,S,I) | X \geq 0, S \geq 0, I \geq 0\}$, it is obvious from the definition of the forward invariant set that it is the positive invariant set of system (1), the population density of cotton bollworm and cotton at the moment $N(t) = X(t) + S(t) + I(t)$, Moment change rate t.

$$\frac{dN(t)}{dt}=\frac{dX(t)}{dt}+\frac{dS(t)}{dt}+\frac{dI(t)}{dt},$$

$$\frac{dN}{dt}\le -d_1(X+S+I)-\frac{mX^2}{K}+(m+d_1)X,$$ when

$$\limsup_{t\to+\infty} N\le \frac{C}{d_1}=\frac{K(m+d_1)^2}{4md_1}$$ 。 The system discussed in this article is defined on a closed set Ω, where

$$\Omega=\left\{(X,S,I)\in R^3 \middle| X+S+I\le \frac{K(m+d_1)^2}{4md_1}\right\}$$

$$\limsup_{t\to+\infty} N\le \frac{C}{d_1}=\frac{K(m+d_1)^2}{4md_1},$$

$$X+S+I\le \frac{K(m+d_1)^2}{4md_1},$$ Available to meet initial conditions $U_0\in R^3$,any solution of will enter and stay in it or tend to $\partial\Omega$.

III. MODEL ANALYSIS

A. Solving the equilibrium point

Solve a system of equations

$$\begin{cases} mX\left(1-\dfrac{X}{K}\right)-c_1XS-c_2XI=0 \\ \rho XS-d_1S-\beta SI+\theta I=0 \\ \beta SI-d_2I-\theta I=0 \end{cases}$$

when $I=0$, solve the equilibrium point

$E_0(0,0,0)$, $E_1(K,0,0)$,

$$E_2=\left(\frac{d_1}{\rho},\frac{m(\rho K-d_1)}{\rho Kc_1},0\right)$$

$I\neq 0$, $\beta S-d_2-\theta=0$, $S=\dfrac{d_2+\theta}{\beta}$ .Substituting into (2), we get

$$\rho XS-d_1S-d_2I=0,$$

then

$$\rho SX-d_2I=d_1S$$

$$m\left(1-\frac{X}{K}\right)-c_1S-c_2I=0,$$

then

$$mX+c_2KI=mK-c_1KS,$$

$$X=\frac{c_2Kd_1S+d_2(mK-c_1KS)}{\rho c_2SK+md_2}=\frac{c_2Kd_1S+d_2mK-c_1d_2KS}{\rho c_2SK+md_2}$$
$$=\frac{c_2Kd_1\dfrac{d_2+\theta}{\beta}+d_2mK-c_1d_2K\dfrac{d_2+\theta}{\beta}}{\rho c_2\dfrac{d_2+\theta}{\beta}K+md_2}$$
$$=\frac{c_2Kd_1(d_2+\theta)+\beta d_2mK-c_1d_2K(d_2+\theta)}{\rho c_2(d_2+\theta)K+m\beta d_2}$$

$$I=\frac{\rho S(mK-c_1KS)-md_1S}{\rho c_2SK+md_2}=\frac{\rho(mK-c_1KS)-md_1}{\rho c_2SK+md_2}S$$
$$=\frac{\rho\left(mK-c_1K\dfrac{d_2+\theta}{\beta}\right)-md_1}{\rho c_2\dfrac{d_2+\theta}{\beta}K+md_2}\frac{d_2+\theta}{\beta}=\frac{\rho\left[m\beta K-c_1K(d_2+\theta)\right]-m\beta d_1}{\rho c_2(d_2+\theta)K+m\beta d_2}\frac{d_2+\theta}{\beta}$$

Balance point

$$E_3\left(\frac{K(d_2+\theta)(c_2d_1-c_1d_2)+mK\beta d_2}{\rho c_2(d_2+\theta)K+m\beta d_2},\frac{d_2+\theta}{\beta},\frac{\rho\left[m\beta K-c_1K(d_2+\theta)\right]-m\beta d_1}{\rho c_2(d_2+\theta)K+m\beta d_2}\frac{d_2+\theta}{\beta}\right)$$

Conclusion: when $\rho K<d_1$, $E_0(0,0,0)$,

$E_1(K,0,0)$ exist,when $\rho K>d_1$

disease-free balance

$$E_2=\left(\frac{d_1}{\rho},\frac{m(\rho K-d_1)K\rho}{\rho Kc_1},0\right)$$ exist,take

$$M=\min\left\{\frac{(d_2+\theta)(c_1d_2-c_2d_1)}{\beta d_2},\frac{m\beta d_1+\rho c_1K(d_2+\theta)}{\rho K\beta}\right\}$$

when $m>M$,

Local equilibrium point $E_3=(X^*,S^*,I^*)$ appear.

IV.STABILITY OF THE EQULIIBRIUM POINTS

Solve the Jacobian matrix of $E_0=(0,0,0)$

$$A=J|(0,0,0)=\begin{pmatrix} m & 0 & 0\\ 0 & -d_1 & \theta\\ 0 & 0 & -d_2-\theta\end{pmatrix},$$

Due to characteristic roots of A, $\lambda_1=m>0,\lambda_2=-d_1<0,\lambda_3=-d_2-\theta$,

three negative roots of the equation, so $E_0(0,0,0)$ is unstable,

Solve the Jacobian matrix of $E_1(K,0,0)$

$$B=J|(K,0,0)=\begin{pmatrix} -m & -c_1K & -c_2K\\ 0 & \rho K-d_1 & \theta\\ 0 & 0 & -d_2-\theta\end{pmatrix}$$

Characteristic root of B, $\lambda_1=-m,\lambda_2=\rho K-d_1,\lambda_3=-d_2-\theta$, when $0<\rho K<d_1$

its characteristic roots are all negative real numbers, so the system（1）is Local asymptotic stability on $E_1(K,0,0)$

Solve the Jacobian matrix of $E_2=\left(\frac{d_1}{\rho},\frac{m(\rho K-d_1)}{\rho Kc_1},0\right)$

$$J\left|\left(\frac{d_1}{\rho},\frac{m(\rho K-d_1)}{\rho Kc_1},0\right)\right.=\begin{pmatrix} -\frac{md_1}{\rho K} & \frac{-c_1d_1}{\rho} & \frac{-c_2d_1}{\rho}\\ \frac{m(\rho K-d_1)}{Kc_1} & 0 & \theta-\beta\frac{m(\rho K-d_1)}{\rho Kc_1}\\ 0 & 0 & \beta\frac{m(\rho K-d_1)}{\rho Kc_1}-d_2-\theta\end{pmatrix}$$

Solve the eigenvalue equation:

$$\left(\lambda-\beta\frac{m(\rho K-d_1)}{\rho Kc_1}+d_2+\theta\right)\left[\left(\lambda+\frac{md_1}{\rho K}\right)\lambda-\frac{c_1d_1}{\rho}\frac{-m\rho K+md_1}{Kc_1}\right]=0$$

when $\lambda_1=\beta\frac{m(\rho K-d_1)}{\rho Kc_1}-d_2-\theta<0$,then $\frac{m\beta}{c_1}-\frac{md_1\beta}{\rho Kc_1}<d_2+\theta$

take $g(\lambda)=\left(\lambda+\frac{md_1}{\rho K}\right)\lambda-\frac{c_1d_1}{\rho}\frac{-m\rho K+md_1}{Kc_1}$,

$\lambda_2+\lambda_3=-\frac{md_1}{\rho K}<0$, So long as $\rho K>d_1$, then $\lambda_2,\lambda_3<0$,when $R_0=\frac{m\beta(\rho K-d_1)}{\rho Kc_1(d_2+\theta)}<1$, then $R_0<1$, Then we have

$E_2=\left(\frac{d_1}{\rho},\frac{m(\rho K-d_1)}{\rho Kc_1},0\right)$ is local asymptotic stability.

Solve the Jacobian matrix of $E_3=(X^+,S^+,I^+)$

$$D=J|(X^+,S^+,I^+)=\begin{pmatrix} m-\frac{2m}{K}X^+-c_1S^+-c_2I^+ & -c_1X^+ & -c_2X^+\\ \rho S^+ & \rho X^+-d_1-\beta X^+ & -d_2\\ 0 & \beta I^+ & 0\end{pmatrix}$$

Solve the eigenvalue equation: $f_0\lambda^3+f_1\lambda^2+f_2\lambda+f_3=0$, among then

$f_0 = 1$, $f_1 = x_1 + x_2$,

$f_2 = \beta d_2 I^+ + x_1 x_2 + \rho c_1 S^+ X^+$

$f_3 = \beta d_2 I^+ x_1 + \rho c_2 \beta X^+ S^+ I^+$,

$x_1 = \frac{2m}{K} X^+ + c_1 S^+ + c_2 I^+ + m, x_2 = d_1 + \beta X^+ - \rho X^+$

Due to *Hurwitz* theorem, just construct

$$\Delta_1 = f_1, \Delta_2 = \begin{vmatrix} f_1 & f_3 \\ 1 & f_2 \end{vmatrix}, \Delta_3 = \begin{vmatrix} f_1 & f_3 & 0 \\ 1 & f_2 & 0 \\ 0 & f_1 & f_3 \end{vmatrix}$$

and guarantee $\Delta_1 > 0_1, \Delta_2 > 0, \Delta_3 > 0$, The system (1) is at equilibrium

$E_3 = (X^*, S^*, I^*)$ is local asymptotic stability.when

$$x_1 + x_2 = \frac{2m}{K} X^+ + c_1 S^+ + c_2 I^+ + \beta X^+ + d_1 + m - \rho X^+$$

$$m - \rho X^+ = \frac{\rho K (d_2 + \theta)\left[c_2 (m - qE) + c_1 d_2 - c_2 d_1\right] + m(m\beta d_2 + \rho K \beta d_2)}{\rho c_2 (d_2 + \theta) K + m \beta d_2}$$

therefore $c_1 d_2 - c_2 d_1 > 0$, so $f_1 = x_1 + x_2 > 0$

$$\Delta_2 = (x_1 + x_2)\left(\beta d_2 I^+ + x_1 x_2 + \rho c_1 S^+ X^+\right) - \beta d_2 I^+ x_1 - \rho c_2 \beta X^+ S^+ I^+$$

$$> (x_1 + x_2)\left(\beta d_2 I^+ + \rho c_1 S^+ X^+\right) - \beta d_2 I^+ x_1 = x_2 \beta d_2 I^+ + \rho c_1 x_1 S^+ X^+ + \rho c_1 S^+ X^+ x_2 > 0$$

$\Delta_3 = f_3 \Delta_2$, $f_3 = \beta d_2 I^+ x_1 + \rho c_2 \beta X^+ S^+ I^+ > 0$

Due to *Hurwitz* theorem, The system (1)is at equilibrium $E_3 = (X^*, S^*, I^*)$

is local asymptotic stability.

V. JUDGMENT OF GLOBAL STABILITY

Theorem 1: when $R_0 < 1$, No pathological equilibrium $E_2 = \left(\frac{d_1}{\rho}, \frac{m(\rho K - d_1)}{\rho K c_1}, 0\right)$ is global asymptotic stability.

Proof:take

$$E_2 = \left(\frac{d_1}{\rho}, \frac{m(\rho K - d_1)}{\rho K c_1}, 0\right) = (X_2, S_2, 0)$$

Take *Liapunov* function

$$V = X - X_2 - X_2 \ln \frac{X}{X_2} + c_1\left(S - S_2 - S_2 \ln \frac{S}{S_2}\right)$$

$\frac{dV}{dt} \le -\frac{m}{K}(X - X_2)^2 - (c_1 + c_2)\rho X < 0$,therefore $\frac{dV}{dt} \le 0$, so $R_0 < 1$,

No pathological equilibrium $E_2 = \left(\frac{d_1}{\rho}, \frac{m(\rho K - d_1)}{\rho K c_1}, 0\right)$ is global asymptotic stability.

Theorem 2:when $R_0 > 1$, Global Asymptotic Stability of Local Equilibrium $E_3 = (X^+, S^+, I^+)$

Proof: Construct the Lyapulov function as follows

$$V(t) = \frac{\rho}{c_1} X^+\left(\frac{X}{X^+} - \ln \frac{X}{X^+} - 1\right) + S^+\left(\frac{S}{S^+} - \ln \frac{S}{S^+} - 1\right) + I^+\left(\frac{I}{I^+} - \ln \frac{I}{I^+} - 1\right)$$

Obviously $V(t)$ is a positive definite function,take

$$x = \ln \frac{X}{X^+}, s = \ln \frac{S}{S^+}, i = \ln \frac{I}{I^+}$$

Considered global asymptotic stability E_3 on $\Omega_0 = \{s(t) > i(t) \geq 0, x(t) \geq 0\}$

$$\frac{dV(t)}{dt} = -\frac{mX^+}{K}\frac{\rho}{c_1}\left(X - X^+\right)^2 + \theta I^+ \frac{S - S^+}{S}\left(\frac{I}{I^+} - \frac{S}{S^+}\right) - \frac{\rho c_2}{c_1}\left(I - I^+\right)\left(X - X^+\right) \leq 0$$

Therefore, $R_0 > 1$, E_3 is the global asymptotic stability

VI.NUMERICAL SIMULATION

Numerical simulations verify global asymptotic stability of local equilibrium points $E_3 = \left(X^*, S^*, I^*\right)$, The parameters of model (1) are selected as follows:

$$m = 0.04, b = 2, f = 0.01, k = 10000, d_1 = 0.01$$

$$d_2 = 0.02, \theta = 0.05, c_1 = 0.02, c_2 = 0.01,$$

Any initial value $E_0 = (10,10,10)$, at this time $R_0 = 14.283 > 1$, The global asymptotic stability numerical simulation diagram (Figure 1) is shown by local equilibrium point $E_3 = \left(X^*, S^*, I^*\right)$, and the local equilibrium points under different parameters are obtained $E_3 = \left(X^*, S^*, I^*\right)$ and basic regeneration number $R_0 > 1$, The trend E_3 is shown in Figure 2:

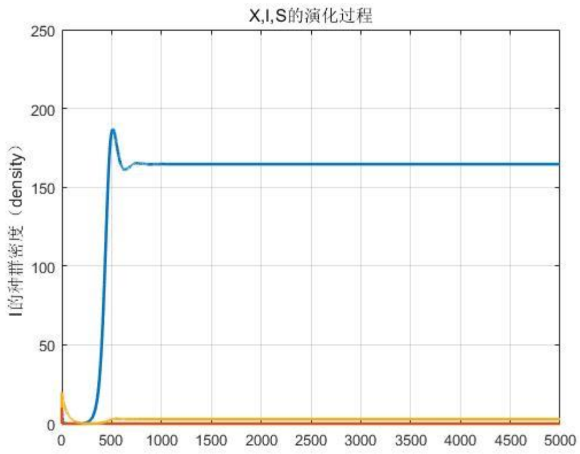

Figure 1 Numerical simulation diagram of the global asymptotic stability of the local equilibrium point

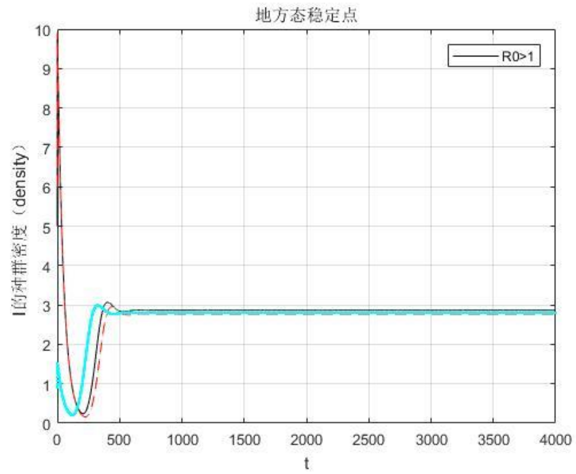

Figure 2 Trend of local equilibrium points under different parameters