Heat Currents At The Transtion Temperature Of Mercury Doped Cuprate Superconductors

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***Abstract*—We explore heat transport as a thermodynamic property of mercury doped cuprate superconductors at the transition temperature. Mercury doped cuprate superconductors have higher transition temperature and hence they could be the best materials for making room temperature superconductors. It was observed that for higher critical temperatures to be achieved, the heat currents flow in these materials should be reduced since heat currents reduce the critical temperature of the superconductors. It is projected that, for the critical temperature of 300K to be achieved, the heat current flow in Hg1201, Hg1212 and Hg1223 should be 95J/s, 242J/s and 414J/s respectively. Higher values of upper critical magnetic fields promotes heat current flow in the superconductors, therefore higher critical temperatures needs low upper critical fields. At low critical temperatures the thermal diffusivity in the superconductors is very low indicating that, heat diffuses on the surface of the material very slowly. However, when the critical temperatures of the materials is increased towards the room temperature thermal diffusivity increases exponentially therefore higher critical temperature can only be achieved if thermal diffusivity in the superconducting materials is higher and the heat current is lowered.**

Keywords—Thermal diffusivity, Heat currents, Heat flux, Transition temperature, Thermal conductivity

Introduction

Although a superconductor is a perfect conductor of charge, it is a poor conductor of heat. Thermal conductivity in the superconducting state is much less than the electrical conductivity. In fact, in the limit of very low temperatures (T→0), electronic heat conduction in a superconductor with a full energy gap goes to zero since there are no thermally excited quasi-particles to carry heat, the Cooper pairs in the condensate carry no heat (Tu & Lee, 2016). Heat transport has been found to be highly sensitive to temperature, magnetic fields and disorders, and being used in the probe of superconducting energy gap.

Therefore, there is deep theoretical desire to understand the electronic and vibrational properties of high-T_C superconductors. Cuprate superconductors have been found to be better thermal conductors than low-T_C superconductors because of higher critical temperatures hence raised thermal energy in the superconducting state. This has raised technological interest on how efficiently and by what means the heat flows in these materials. Richard & Vorontsov (2016) studied heat conductivity in the superconducting state of non-uniform superconductors using Boltzmann transport theory and greens function techniques. They found out that, thermal conductivity increases with increase in temperature in the superconducting state, but very few studies have focused on the heat transport at the critical temperature of high-T_C superconductors. This theoretical research will focus on the effect of heat currents on the critical temperature of mercury doped cuprate superconducting materials.

2. Theoretical Formulation

2.1 Heat current in the superconducting state using Fourier's law.

The heat flux Q is defined as;

$$Q = -kA\frac{dT}{dl} \qquad \text{.................................1.0}$$

Where Q is normal to the isothermal surface and positive in the direction of decreasing temperature and k is the thermal conductivity of the material. According to the second law of thermodynamics, heat always flow from a hotter region to a colder region because of the temperature gradient. Thus, negative sign indicates that the heat flow is in the direction of negative temperature gradient and that serves to make heat flow positive. Thermal conductivity, k provides an indication of the rate at which heat energy is transferred through a medium by conduction process.

Dividing equation 1.0 by A, where A cross-sectional area, it reduces to;

$$\vec{q} = \frac{Q}{A} = \frac{\text{kA}}{A}\frac{dT}{dl}$$

$$\vec{q} = -k\frac{dT}{dl} \qquad \text{..............................1.1}$$

$\vec{q}$ is the quantity of heat passing through a unit area of the surface of the superconductor normal to the direction of flow of heat per second. Considering the heat flux in three dimension, the equation 1.1 becomes,

$\vec{q}= -k\nabla T$ ………………………………. 1.2

Where, ∇T is temperature gradient vector.

The thermal conductivity k, in the superconducting state of a superconductor is given by (Mbiye *et al*,. 2018);

$$k=\frac{1}{3m}n_e\pi^2K_B^2T\tau - \frac{2}{3}\left(\frac{\pi^3\mu_0^2 k_B^2 T C\hbar^2 E_F^2\xi^2 \mathrm{H0}\left\{1-\left(\frac{T}{T_C}\right)^2\right\}J_Z^2 exp\frac{2X}{\lambda_L}}{\emptyset_0 B e^7 B_y^2(0)}\right)\sum_J \frac{1+\exp\left(\frac{2\varepsilon_J}{k_B T}\right)}{\sqrt{\Delta_{(0)}^2-\varepsilon_J^2}}+ \frac{K_B}{2\pi^2 v}\left(\frac{K_B T}{\hbar}\right)^3\int_0^{\frac{\Theta_D}{T}}\tau(x)\frac{x^4\exp x}{(\exp x-1)^2}dx \quad …….. 1.3$$

Substituting equation 1.3 into 1.2 gives

$$\vec{q}=-\left\{\frac{1}{3m}n_e\pi^2K_B^2T\tau - \frac{2}{3}\left(\frac{\pi^3\mu_0^2 k_B^2 T C\hbar^2 E_F^2\xi^2 \mathrm{H0}\left\{1-\left(\frac{T}{T_C}\right)^2\right\}J_Z^2 exp\frac{2X}{\lambda_L}}{\emptyset_0 B e^7 B_y^2(0)}\right)\sum_J \frac{1+\exp\left(\frac{2\varepsilon_J}{k_B T}\right)}{\sqrt{\Delta_{(0)}^2-\varepsilon_J^2}}+ \frac{K_B}{2\pi^2 v}\left(\frac{K_B T}{\hbar}\right)^3\int_0^{\frac{\Theta_D}{T}}\tau(x)\frac{x^4\exp x}{(\exp x-1)^2}dx\right\}\nabla T$$

……..…………………………….…… 1.4

Equation 1.4 gives the heat flux in the superconducting state.

The net amount of heat flowing through the superconductor's boundary surface S per unit time due to conduction can be defined as (Mihir, 2017):

$\frac{\partial H}{\partial t}=\iint \vec{q}.\hat{n}dS$ ………………………… 1.5

Where, H is the total amount of heat inside the volume of the superconductor.

To consider the heat transport in the entire superconductor, the vector calculus divergence theorem is used, which states that, the closed surface integral of a vector equals to the volume integral of the divergence of the same vector field. i.e

$\oint \vec{q}\,.\hat{n}dS=\iiint(\nabla.\vec{q})dV$ ………………….. 1.6

Applying equation 1.5 to 1.6 gives;

$\frac{\partial H}{\partial t}=\iiint(\nabla.\vec{q})dV$ ……………………………1.7

Substituting equation 1.4 into 1.7 and simplifying gives;

$$\frac{\partial H}{\partial t}= -V\left\{\frac{1}{3m}n_e\pi^2K_B^2T\tau - \frac{2}{3}\left(\frac{\pi^3\mu_0^2 k_B^2 T C\hbar^2 E_F^2\xi^2 \mathrm{H0}\left\{1-\left(\frac{T}{T_C}\right)^2\right\}J_Z^2 exp\frac{2X}{\lambda_L}}{\emptyset_0 B e^7 B_y^2(0)}\right)\sum_J \frac{1+\exp\left(\frac{2\varepsilon_J}{k_B T}\right)}{\sqrt{\Delta_{(0)}^2-\varepsilon_J^2}}+ \frac{K_B}{2\pi^2 v}\left(\frac{K_B T}{\hbar}\right)^3\int_0^{\frac{\Theta_D}{T}}\tau(x)\frac{x^4\exp x}{(\exp x-1)^2}dx\right\}\nabla^2 T$$

………………………………. … 1.8

Equation 1.8 gives the net heat current flow in the superconducting state

2.2 Heat current at the Transition temperature of a superconductor

At the critical temperature, the superconducting material turns to normal. This also occurs at critical field H_C and critical current density J_C. The thermal diffusivity in a material is defined as (Sachdera, 2014);

$\gamma = \frac{k}{\rho c}$ ……………………………………… 1.9

The heat flux can be obtained by substituting for k in equation 1.9 into 1.2 to give;

$\vec{q} = -\rho\gamma c\nabla T$ …………………… 1.10

Substituting equation 1.10 into 1.7 and simplifying gives heat current at the transition point.

$\frac{\partial H}{\partial t} = -\left[V\rho\gamma\frac{C}{M}\right]\nabla^2 T$……………………… 1.11 Where, $\frac{C}{M} = c$ and C is the heat capacity of the material and M is the mass of the material.

The heat capacity C at the critical temperature of the material is given by; (Odhiambo, *et al.,* 2016):

$C=\left(\frac{dQ}{dT}\right)_{T=T_C}$ …………………………….. 1.12

Where Q is the latent heat absorbed when superconductivity is destroyed by the application of magnetic field above the critical field is given by;

$Q=-\frac{T}{4\pi}H(T)\frac{dH_{C(T)}}{dT}$……………………1.13

H (T) is the applied magnetic field at the temperature T.

To obtain the heat capacity C just before the transition from superconducting to normal state, the first derivative of equation 1.13 is obtained with respect to temperature T.

$C= \frac{dQ}{dT} =-\frac{d}{dT}\left[\frac{T}{4\pi}H(T)\frac{dH_C(T)}{dT}\right]$

$C=-\frac{1}{4\pi}\left\{TH(T)\frac{d^2H_C(T)}{dT^2}+T\frac{dH(T)}{dT}\frac{dH_C(T)}{dT}+ H(T)\frac{dH_C(T)}{dT}\right\}$……………………………. 1.14

The thermal diffusivity of material at the critical temperature is given by (Mbiye, *et al.*,2018);

$$\gamma=-\frac{V}{3H_0^2}\left(\frac{n_e\pi^3K_B^2T_C^2\tau}{m}+\frac{3K_B^4}{2\pi v\hbar^3}T_C^4\int_0^{\frac{\Theta_D}{T_C}}\tau(x)\frac{x^4\exp x}{(\exp x-1)^2}dx\right)$$

………1.15

where V is the volume of the superconducting substrate, H_0 is the critical field at T=0 and m is the electron reduced mass.

To obtain heat current at the critical temperature, equation 1.14 and 1.15 is substituted into 1.11, on simplification it gives;

$$\frac{\partial H}{\partial t}=\left\{-\frac{\rho V^2}{12\pi M H_0^2}\left(TH(T)\frac{d^2H_C(T)}{dT^2}+T\frac{dH(T)}{dT}\frac{dH_C(T)}{dT}+ H(T)\frac{dH_C(T)}{dT}\right).\left(\frac{n_e\pi^3K_B^2T_C^2\tau}{m}+ \frac{3K_B^4}{2\pi v\hbar^3}T_C^4\int_0^{\frac{\Theta_D}{T_C}}\tau(x)\frac{x^4\exp x}{(\exp x-1)^2}dx\right)\right\}\nabla^2 T$$

………………………………......... 1.16

At the transition from normal to superconducting state, $T=T_C$ and H=0, equation 1.16 reduces to;

$$\frac{\partial H}{\partial t}= \left\{-\frac{\rho V^2}{12\pi M H_0^2}\left(T_C\left[\frac{dH_C(T)}{dT}\right]^2\right).\left(\frac{n_e\pi^3K_B^2T_C^2\tau}{m}+\frac{3K_B^4}{2\pi v\hbar^3}T_C^4\int_0^{\frac{\Theta_D}{T_C}}\tau(x)\frac{x^4\exp x}{(\exp x-1)^2}dx\right)\right\}\nabla^2T.. \quad 1.17$$

The critical magnetic field relates with temperature T, in the superconducting state as (Andrei, 2004),

$H_C(T)$=H$_0${1-$(\frac{T}{T_C})^2$}…………………1.18

Substituting equation 1.18 for H_C in 1.17 and simplifying gives;

$$\frac{\partial H}{\partial t}=-\frac{\rho V^2}{3\pi M T_C}\left(\frac{n_e\pi^3K_B^2T_C^2\tau}{m}+\frac{3K_B^4}{2\pi v\hbar^3}T_C^4\int_0^{\frac{\Theta_D}{T_C}}\tau(x)\frac{x^4\exp x}{(\exp x-1)^2}dx\right)\nabla^2T\text{….. }1.19$$

Mass M = ρV ……………………………1.20

Substituting equation 1.20 into 1.19 and simplifying gives,

$$\frac{\partial H}{\partial t}=-\frac{V}{3\pi T_C}\left(\frac{n_e\pi^3K_B^2T_C^2\tau}{m}+\frac{3K_B^4}{2\pi v\hbar^3}T_C^4\int_0^{\frac{\Theta_D}{T_C}}\tau(x)\frac{x^4\exp x}{(\exp x-1)^2}dx\right)\nabla^2T\text{ …}1.21$$

Equation 1.21 gives the net heat current flow at the transition temperature of the superconductor.

In the absence of any external heat generation or release of heat energy in the superconductor, Fourier steady state diffusion heat equation is used which states that

$$\nabla^2T=\frac{1}{\gamma}\frac{\partial T}{\partial t} \quad \text{……………………….. }1.22$$

Therefore, equation 1.21 can be written as,

$$\frac{\partial H}{\partial t}=-\frac{V}{3\pi T_C}\left(\frac{n_e\pi^3K_B^2T_C^2\tau}{m}+\frac{3K_B^4}{2\pi v\hbar^3}T_C^4\int_0^{\frac{\Theta_D}{T_C}}\tau(x)\frac{x^4\exp x}{(\exp x-1)^2}dx\right)\frac{1}{\gamma}\frac{\partial T}{\partial t}\text{ ….}1.23$$

Substituting equation 1.15 into 1.23 gives;

$$\frac{\partial H}{\partial t}=\frac{-\frac{V}{3\pi T_C}\left(\frac{n_e\pi^3K_B^2T_C^2\tau}{m}+\frac{3K_B^4}{2\pi v\hbar^3}T_C^4\int_0^{\frac{\Theta_D}{T_C}}\tau(x)\frac{x^4\exp x}{(\exp x-1)^2}dx\right)}{-\frac{V}{3H_0^2}\left(\frac{n_e\pi^3K_B^2T_C^2\tau}{m}+\frac{3K_B^4}{2\pi v\hbar^3}T_C^4\int_0^{\frac{\Theta_D}{T_C}}\tau(x)\frac{x^4\exp x}{(\exp x-1)^2}dx\right)}\frac{\partial T}{\partial t}\quad 1.24$$

Simplifying equation 1.24 gives:

$$\frac{\partial H}{\partial t}=\frac{H_0^2}{\pi T_C}\frac{\partial T}{\partial t}\text{ …………………… }1.25$$

Equation 1.25 gives net heat current flow at $T=T_C$, which can also be written as:

$$\dot{H}=\frac{H_0^2}{\pi T_C}\frac{\partial T}{\partial t}\quad\text{………………………}1.26$$

3. RESULTS AND DISCUSSION

3.1 Essential parameters for mercury doped cuprate superconductors.

The Table 1.1 shows the critical temperature, critical fields, Debye frequency and temperature of some mercury doped Cuprate superconductors that have been used in obtaining the results (Rapando, *et al*,. 2013 & Grisonnanche, *et al*,. 2014).

Table 1.1. The critical temperatures, critical fields, Debye frequency and Debye temperature of some mercury doped Cuprate Superconductors.

Cuprate formula	Short hand notation	Transition temp. T_C (K)	Upper critical magnetic field at T=0 (Tesla)	Debye freq. ω_D (Hz) X 10^{13}	Debye Temp. Θ_D (K)
$HgBa_2CuO_4$	Hg1201	93	30.5	6.8	520
$HgBa_2CaCu_2O_8$	Hg1212	128	41.5	4.0	306
$HgBa_2Ca_2Cu_3O_{10}$	Hg1223	135	52.8	3.6	275

3.2 Heat currents at various critical temperatures of Superconductors

Equation (1.24) was used to determine the heat current at various critical temperatures of the selected mercury doped cuprate superconductors and the results presented in figure 1.1

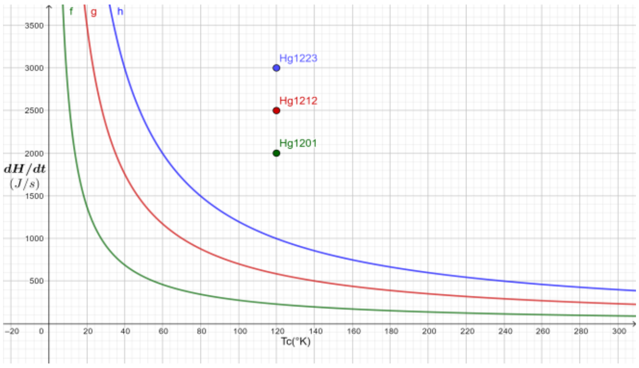

Figure 1.1: Heat current at various critical temperature in some selected mercury doped Cuprate Superconductors

An increse in heat current in the superconductor leads to a corresponding decrease in the critical temperature. In other words, the heat current is nversely proportional to the critical temperature. The heat current flow in Hg1223 at its critical temperature T_C=135k is 887.28 J/s , while in Hg 1212 at its critical temperature T_C=128K is 548.14 J/s and in Hg1201 with the critical temperature T_C =93K is 296.07 J/s. The study shows that materials with higher critical temperatures have higher heat current flow than those with lower critical temperatures i.e. The rate of heat flow in Hg1223 is higher at all the critical temperatures. However, for the critical temperature to be increased to room temperature, the study predicts that the heat current in the material should be reduced to 95J/s, in Hg1201, 242J/s in Hg1212 and 414J/s in Hg1223, at 300K. In other words, higher critical temperature can only be achieved when the heat current flow in the superconductor is reduced to certain values characteristic of a superconductor. From the study, it is evident that, an increase in heat current flow in the superconductor, lowers its critical temperature and also destroys the superconductivity. This is because when the heat current flow exceeds a certain value, characteristic of a superconductor, Cooper pairs are split hence superconductivity in the material is destroyed.

Heat current flow is inversely proportion to the critical temperatures of mercury doped cuprate superconductors. Higher values of transition temperature can achieved if heat current is highly reduced in these materials since heat current breaks the Cooper pairs responsible for superconductivity.

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