Effects Of Heat Generation And Thermal Radiation On Micropolar Fluid Flow Over An Exponentially Permeable Shrinking Sheet

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***Abstract*— In this paper, the problem of two-dimensional incompressible flow of boundary layer for micropolar fluid over an exponentially permeable shrinking sheet with heat generation and thermal radiation is considered. The governing equations are first transformed into a system of non-dimensional equations via the non-dimensional variables, and then into self-similar ordinary differential equations before they are solved numerically using the shooting method. Numerical results are obtained for the skin friction coefficient, couple stress coefficient and heat transfer coefficient as well as the velocity, micro rotation and temperature profiles are presented for different values of the governing parameters. It is found that the solutions for a shrinking sheet are non-unique. The results indicate that the heat tranfer coeeficient decreases with heat generation parameter.**

Keywords—Heat generation, Thermal radiation, Boundary layer, Micropolar fluid, Shooting method

I. Introduction

The study of boundary layer flow and heat transfer in the field of fluid dynamics have a huge number of applications in industry and engineering. Various problems are investigated related to the flow and heat transfer of a fluid past a sheet in Newtonian and Non-Newtonian fluids.

Al-Odat et al. (2006) investigated the thermal boundary layer on an exponentially stretching continuous surface with an exponential temperature distribution in the presence of the magnetic field effect. Miklavčič and Wang (2006), studied the properties of the flow due to a shrinking sheet with suction. Suction occurs when the fluid condenses on the surface, such as in chemical vapour deposition. They prove existence and discuss (non) uniqueness of exact solutions.

Sajid and Hayat (2008) analyzed the effect of radiation on the boundary layer flow and heat transfer of a viscous fluid over an exponentially stretching sheet. They employed the homotopy analysis method (HAM) to determine the convergent series expressions of velocity and temperature.

Ishak and Nazar (2008) discovered the heat transfer over a stretching surface with uniform or variable heat flux in micropolar fluids. They transformed the boundary layer equations into ordinary differential equations, and then they are solved numerically by a finite-difference method. Bidin and Nazar (2009) investigated on steady laminar two-dimensional boundary layer flow and heat transfer of an incompressible viscous fluid with a presence of thermal radiation over an exponentially stretching sheet

Ishak et al. (2010) studied the effects of radiation on the thermal boundary layer flow induced by a linearly stretching sheet immerse in an incompressible micropolar fluid with constant surface temperature. An analysis is made by Bhattacharyya (2011) continue studied in this field by presented the boundary layer flow and heat transfer over an exponentially shrinking sheet.

Yacob and Ishak (2012) proposed on steady two-dimensional flow of a micropolar fluid over a shrinking sheet in its own plane. The shrinking velocity is assumed to vary linearly with the distance from a fixed point on the sheet. They found that the solution exists only if adequate suction through the permeable sheet is introduced. Moreover, stronger suction is necessary for the solution to exist for a micropolar fluid compared to a classical Newtonian fluid.

The research is then further by Bhattacharyya (2012). He studied the steady two-dimensional boundary layer flow and reactive mass transfer past an exponentially stretching sheet in an exponentially moving free stream.

Bhattacharyya et al. (2012) discovered the effects of thermal radiation on the flow of micropolar fluid and heat transfer past a porous shrinking sheet. They obtained dual solutions of velocity and temperature for several values of the each parameter.

Later micropolar fluid study has been extended by Turkyilmazoglu (2014). He investigated the flow of micropolar fluid and heat transfer past a porous shrinking sheet and determine mathematically the bounds of multiple existing solutions of purely exponential kind.

Recently, Hussanan et al. (2018) analyzed heat and mass transfer phenomenon in a micropolar fluid and the expressions for velocity, microrotation, temperature and concentration.

The present work has been undertaken in order to investigate the boundary layer for micropolar fluid over an exponentially permeable shrinking sheet with heat generation and thermal radiation.

The present study extends the idea of Bhattacharyya et al. (2012) to include heat generation. The reported results are new and original.

II. MATERIAL AND METHODS

Mathematical Formulation : Consider a steady two-dimensional incompressible flow and heat transfer on a micropolar fluid over an exponentially permeable shrinking sheet with the usual boundary approximations, the governing equations for the micropolar fluid and heat transfer is written in the following form :

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\left(v+\frac{\kappa}{\rho}\right)\frac{\partial^2 u}{\partial y^2}+\frac{\kappa}{\rho}\frac{\partial \mathrm{N}}{\partial y} \qquad (2)$$

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}=\frac{\gamma}{\rho j}\frac{\partial^2 N}{\partial y^2}-\frac{\kappa}{\rho j} \qquad (3)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa^*}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}+\frac{Q_0}{\rho c_p}(T-T_\infty) \qquad (4)$$

Subject to boundary conditions:

$$u=U_w==-ae^{x}/_{L}, \qquad v=v_w, \quad N=-n\frac{\partial u}{\partial y},$$
$$T=T_w\, at\; y=0,\; u\to 0,\; N\to 0, \qquad (5)$$
$$T\to T_\infty \;\; as\; y\to\infty$$

where u and v are velocity components in x and y directions, $\nu(=\mu/\rho)-$ the kinematic fluid viscosity, $\rho-$ the fluid density, $\mu-$ the dynamic viscosity, $N-$ the microrotation or angular velocity whose direction is normal to the $xy-$ plane, $j-$ microinertia per unit mass, $\gamma-$ spin gradient viscosity, $\kappa-$ the vortex viscosity (gyro-viscosity), $T-$ the temperature, κ^*- the thermal conductivity of the fluid, c_p- the specific heat, q_r- the radiative heat flux, T_w- the temperature at the sheet, $T_\infty-$ the free stream temperature both assumed to be constant, and Q_0- the heat generation constant. Here, v_w is the wall mass transfer velocity with $v_w<0$ for mass suction and $v_w>0$ for mass injection. We note that n is a constant such that $0\le n\le 1$. The case $n=0$ indicates $N=0$ at the surface. It represents flow of concentrated particles in which the microelements closed to the wall surface are unable to rotate. This case is also known as strong concentration of microelements. The case $n=0.5$ indicates the vanishing of the anti-symmetric part of the stress tensor and denotes weak concentration of microelements. Whereas, the case $n=1$ is used for the modeling of turbulent boundary layer flows. We assumed that spin gradient viscosity γ is given by:

$$\gamma=(\mu+\kappa/2)j=\mu(1+K/2) \qquad (6)$$

where $K=\kappa/2$ is the material parameter and
$j=2Lve^{-x/L}/a$ is the microinertia per unit mass.

The governing equations (1)-(4) subject to the boundary conditions (5) can be expressed in a simpler form by introducing the following transformation:

$$u=ae^{x/L}f'(\eta) \qquad v=-\sqrt{\frac{av}{2L}}e^{x/2L}[f(\eta)+\eta f'(\eta)]$$
$$N=\left(\frac{a}{2Lv}\right)\sqrt{2Lva}\,e^{3x/2L}h(\eta) \qquad (7)$$
$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \qquad \eta=\sqrt{\frac{a}{2Lv}}e^{x/2L}y$$

where η is the similarity variable and ψ is the stream function which is defined in usual notation as $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$, which identically satisfies equation (1) . The ordinary differential equation is obtained by applying the introduce similarity transform, equation (7).

Substituting Eq(7) into Eq.1-3, we get the following ordinary differential equations:

$$(1+K)f''+ff''-2f'^2+Kh'=0 \qquad (8)$$

$$(1+K/2)h''+fh'-3f'h-K(2h+f'')=0 \qquad (9)$$

$$(1+R)\theta''+P_rf\theta'+P_rQ\theta'=0 \qquad (10)$$

Subject to the boundary conditions:

$$f(0) = S,\ f'(0) = -1, h(0) = -nf''(0), \theta(0) = 1 \quad (11)$$
$$f'(\infty) \to 0, h(\infty) \to 0, \theta(\infty) \to 0$$

Quantities for physical interest in this study are the local skin friction coefficient C_f , local couple stress M_x , and local Nusselt number Nu_x , which are defined as :

$$C_f = \frac{\left[(\mu+\kappa)\frac{\partial u}{\partial y}+\kappa N\right]_{y=0}}{\rho U^2 w}, \quad M_x = \frac{\gamma\left(\frac{\partial N}{\partial y}\right)_{y=0}}{\rho x U^2 w},$$

$$NU_x = -\frac{x}{(T_w-T_\infty)}\left(\frac{\partial T}{\partial y}\right)_{y=0} \quad (12)$$

where the local Reynolds number is given by

$$Re_x = \frac{xU_w}{\nu} \quad (13)$$

III. Results and Discussion

The ordinary differential equations (8), (9) and (10) subjected to the boundary conditions (11) has been solved numerically using Maple 2015. It have been solved numerically by applying in shooting method command in Maple. The purpose of this study is to obtain the effect of variation parameter such as K, R and Q on the velocity profiles f'(η), and microrotation profiles h(η), and temperature profiles θ(η), as well as numerical results on skin friction and couple stress and heat transfer coefficient.

Miklavčič (2006) and Wang (2006) and Bhattacharyya (2012) showed that for Newtonian fluids $(K = 0)$, the steady similarity solution of boundary layer flow due to a linearly shrinking sheet with wall mass transfer is possible to obtain if the wall mass suction parameter S is greater than or equal to 2 and the flow due to exponentially shrinking sheet is possible when $S \geq 2.2667$.

For micropolar fluid, the present study shows that for $K = 0.1$, the similarity solutions exist when $S \geq 2.3213$ and no similarity solution exists for $S < 2.3213$. Further, the increment in material parameter K causes more reduction in the solution suction domain.For $K = 0.2$, the similarity solution exists when $S \geq 2.3754$ and consequently no solution exists for $S < 2.3754$. More numerical results for skin friction, couple stress and heat transfer coefficients and dimensionless velocity, microrotation and temperature profiles are computed for various values of material parameter $K(0 \leq K \leq 0.2)$, mass suction parameter $S(S \leq 2.6)$, heat generation parameter $Q(Q \leq 1.0)$, radiation parameter $R(R \leq 1.0)$ and $n(0 \leq n \leq 0.5)$.

The variations of quantities f"(0) which is related to skin friction coefficient with suction S for several values of material parameter K is shown in Figure 1. The dual similarity solutions for micropolar fluid similar to that of Newtonian fluid are obtained. Also, from Figure 1 it is observed that the skin friction coefficient decreases with increasing values of K for the first solution and for the second solution it increases. It is also observed from these figures that the values of skin friction coefficient, f"(0) is always positive, which implies that the fluid exerts a drag force on the sheet and the heat is transferred from the hot sheet to the cold fluid, respectively.

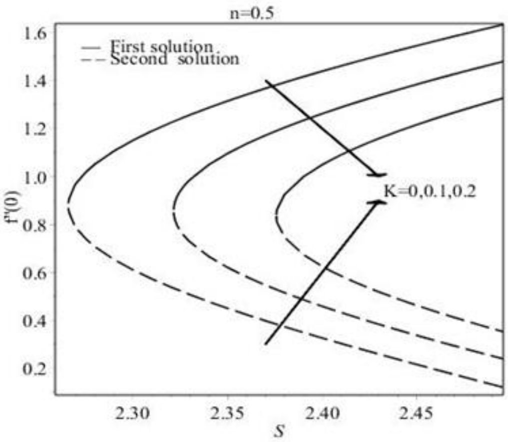

Figure 1 Skin friction coefficient f"(0) with S for different values of K.

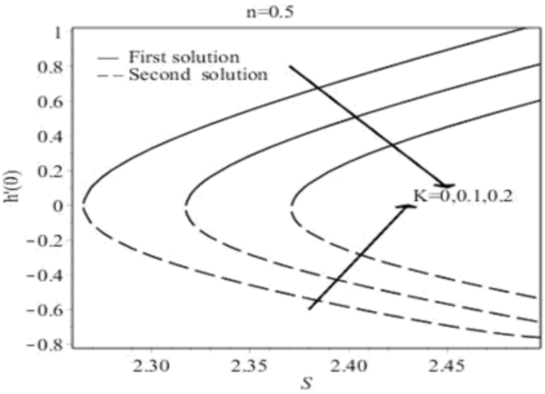

Figure 2 Couple stress coefficient h'(0) with S for different values of K.

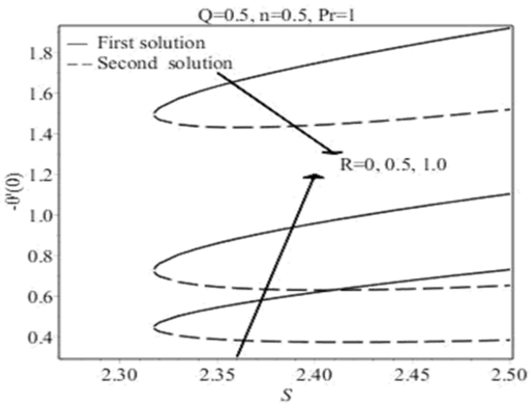

Figure 3 Heat transfer coefficient - θ'(0) with S for different values of R

The variations of quantities $h'(0)$ which is related to couple stress coefficient with suction S for several values of material parameter K is shown in Figure 2. Also, from Figure 2 it is observed that the couple stress coefficient decreases with increasing values of K for the first solution and for the second solution it increases. It is also observed from these figures that the values of couple stress coefficient, $h'(0)$ is always positive, which implies that the fluid exerts a drag force on the sheet and the heat is transferred from the hot sheet to the cold fluid, respectively.

The variations of quantities $-\theta'(0)$ which is related to heat transfer coefficient with suction S for several values of radiation parameter R is shown in Figure 3. Also, from Figure 3 it is observed the heat transfer coefficient decreases with increasing values of R for the first solution and for the second solution it increases. It is also observed from these figures that the values of heat transfer coefficient, $-\theta'(0)$ is always positive, which implies that the fluid exerts a drag force on the sheet and the heat is transferred from the hot sheet to the cold fluid, respectively.

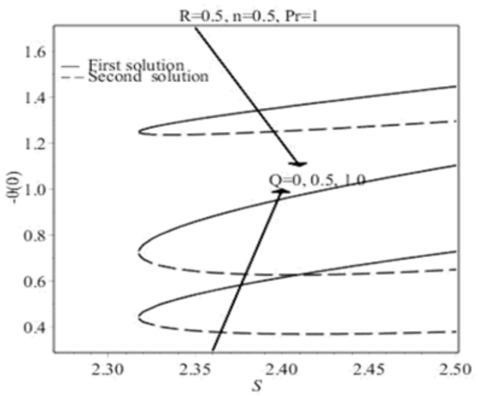

Figure 4 Heat transfer coefficient - θ'(0) with S for different values of Q.

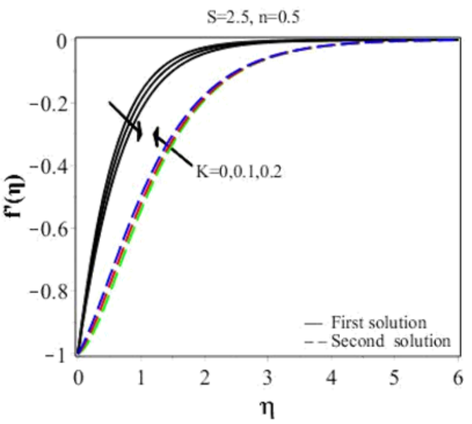

Figure 5 Velocity profiles for different values of K

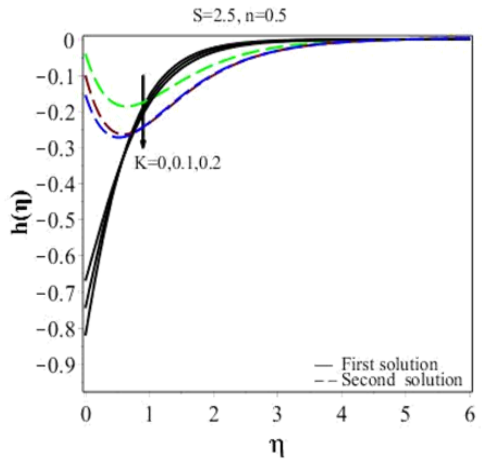

Figure 6 Microrotation profiles for different values of K.

The variations of quantities $-\theta'(0)$ which is related to heat transfer coefficient with suction S for several values of heat generation parameter Q is shown in Figure 4. The dual similarity solutions for micropolar fluid similar to that of Newtonian fluid are obtained. Also, from Figure 4 it is observed the heat transfer coefficient decreases with increasing values of Q for the first solution and for the second solution it increases.The velocity profiles for different values of material parameter K is plotted in Figure 5. For the first solution, the velocity and thermal boundary layer thicknesses increase, while opposite effect is observed in case of second solution as shown in Figure 5.

The microrotation profiles for different values of material parameter K is plotted in Figure 6. Figure 6 shows that for large values of η the microrotation decreases for both solutions. It is also seen from these figures that far field boundary conditions are asymptotically satisfied for both first and second solutions, supporting the validity of the obtained numerical results. The temperature profiles for different values of radiation parameter R is plotted in Figure 7. The radiation parameter R defines the relative contribution of conduction heat transfer to thermal radiation transfer. It is obvious that an increase in the radiation parameter results in decreasing temperature within the boundary layer.

The effect of heat generation parameter Q on the temperature is shown in Figure 8. From this figure, we observe that when the value of heat generation parameter increases, the temperature distribution decreases along the boundary layer. Figure 9 exhibit the velocity profiles for different.

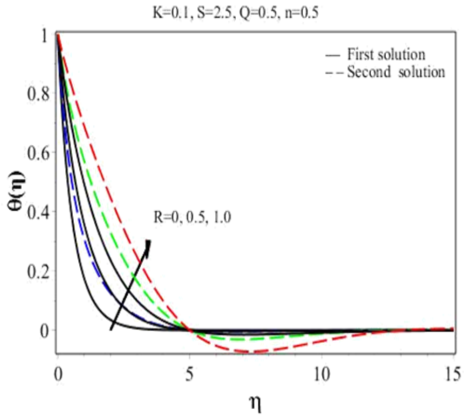

Figure 7 Temperature profiles for different values of R.

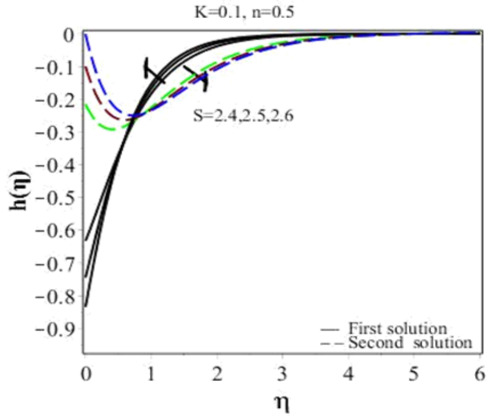

Figure 10 Microrotation profiles for different values of S

Figure 10 exhibit the microrotation profiles for different values of mass suction parameter. From the figures, it can be seen that for first solution the velocity and microrotation boundary layer thicknesses decrease with increasing values of suction, while in case of second solution, the opposite effect is observed. The effect of on the dimensionless velocity profiles is demonstrated in Figure 11. For the first solution the velocity of fluid increases and consequently the velocity boundary layer thickness decreases, whereas for second solution the velocity of fluid decreases.

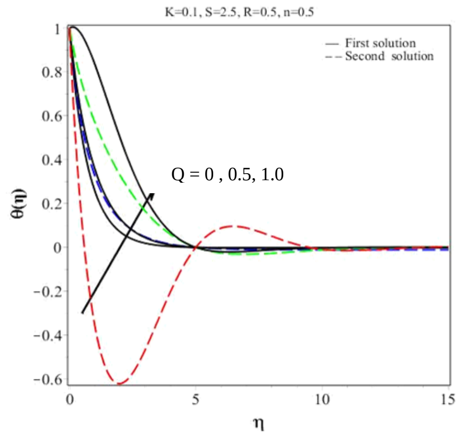

Figure 8 Temperature profiles for different values of Q.

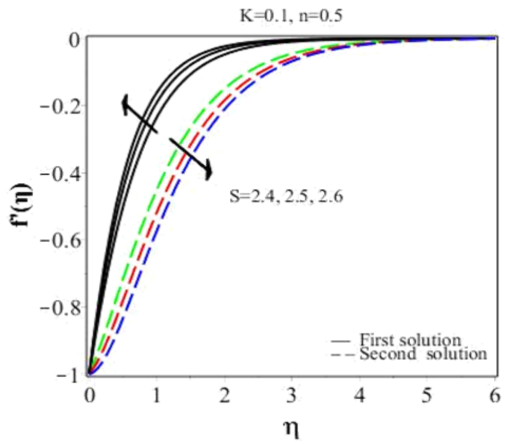

Figure 9 Velocity profiles for different values of S

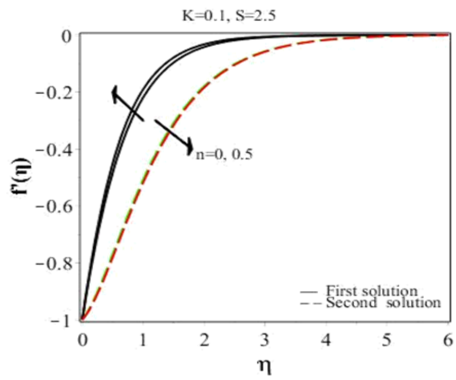

Figure 11 Velocity profiles for different values of n.

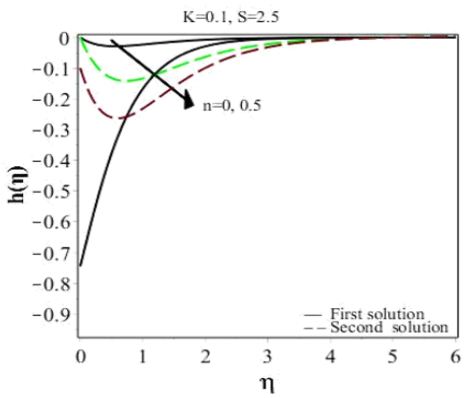

Figure 12 Microrotation profiles for different values of n.

Figure 9 exhibit the velocity profiles for different values of mass suction parameter S . From the figures, it can be seen that for first solution the velocity and microrotation boundary layer thicknesses decrease with increasing values of suction, while in case of second solution, the opposite effect is observed.

The effect of on the dimensionless velocity profiles is demonstrated in Figure 12. For the first solution the velocity of fluid decreases and consequently the velocity boundary layer thickness decreases, whereas for second solution the velocity of fluid decreases. On the other hand, the microrotation profiles decreases for both solutions as shown in Figure 12.

IV . CONCLUSION

In this study, we have studied the steady boundary layer flow of micropolar fluid and heat transfer due to an exponentially permeable shrinking sheet in presence of heat generation and thermal radiation. The problem was solved using Maple 2015 and the obtained self-similar ordinary differential equations are numerically solved. From the investigation, it can be concluded that:

- Dual solutions for velocity, microrotation and temperature were found when the solution exists.
- The velocity decrease for first solution and increase for second solution with increasing values of the material parameter and opposite effects were found for increment of mass suction.
- Microrotation decreases with material parameter for both solutions.
- The skin friction coefficient, couple stress coefficient and heat transfer coefficient decrease for first solution and increase for second solution with material parameter.
- The fluid temperature reduces with the increase in values of radiation parameter and heat generation parameter for both solutions.

V. ACKNOWLEDGEMENT

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