

# Analysis and Application of A Novel Three-Dimensional Carbon Price Dynamic System

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**Abstract**—With the development of the carbon market in China, carbon market will play an important role in China's economic field. This paper introduces a novel three-dimensional carbon price dynamic system, which is established in corresponding with the complicated relationship among carbon price, energy consumption, and economic growth. As the evolution of Lyapunov exponents and bifurcation diagram, the dynamic behaviors of the system are analyzed. Statistical data of carbon price comes from China emissions exchange and the statistical data of energy consumption and economic growth come from China Statistical Yearbook. Genetic algorithm is used to identify the quantitative coefficients in this model. Through the empirical analysis, it is can be confirmed that this system fits the real situation. With the influence of different control strategies on the system, we can get appropriate control strategies to reduce energy intensity.

**Keywords**—carbon price; energy consumption; economic growth; dynamic system; energy intensity

## I. INTRODUCTION

Carbon market is an important way to control carbon emissions and alleviate environmental problems, in which carbon price plays an important role. With the development of China's economy, the consumption of energy is also growing. At the same time, due to the low efficiency of energy consumption and the large increase of carbon emissions, a large number of environmental problems have arisen. Carbon price and carbon market are getting more and more attention. However, China's current carbon prices fluctuate significantly. So the relationship among carbon price, energy consumption, and economic development, as well as how to reduce carbon emissions and energy intensity is the focus of this paper.

Previous studies have found that carbon price is affected by many factors. Li and Lei [1] use multi-time series model and ARCH model analysis method find that carbon price affected by industrial income, energy

price, government intervention, and the number of participating corporation. Maria et al. [2] use multivariate analysis to find that energy price is the main factor affecting carbon price, and extreme weather can also cause the price change of carbon emission rights. Alberola et al. [3] use structural change theory to prove energy prices have a positive impact on the price of carbon emission rights.

The influence of carbon price on energy consumption is not obvious. Bianco et al. [4] study the relationship between carbon price and energy consumption in Italy. They find that the change of carbon price will not have a great impact on energy consumption in a short period of time, and the rise of carbon price will just play a small role in curbing environmental pollution. Ren et al. [5] use non-cooperative game theory to study the impact of carbon price on renewable energy production. Rising carbon price will reduce carbon emissions, but will not significantly promote the development of renewable resources. Li et al. [6] study environment energy consumption in China through the gray relational analysis method, supply and demand theory. They find that carbon price is closely related to environmental problems.

Carbon prices mostly affect economic development by affecting the structure of energy use. Based on input-output simulation, Choi and Bakshi [7] conclude that carbon prices change economic development by changing energy consumption structure. Cao et al. [8] use a dynamic computable general equilibrium model of China find that increasing carbon price gradually from lower carbon price can better adjust energy structure and promote economic development. Tian et al. [9] use Multivariate Generalized Auto Regressive Conditional Heteroskedasticity (M-GARCH) to examine the relationship between the carbon price and stock price of electricity companies from EU-ETS. They find that rising carbon prices have a negative effect on the development of carbon-intensive enterprises and a positive effect on low-carbon enterprises.

Energy intensity is an important index to measure a country's energy use efficiency. It refers to the amount of energy consumed per unit of economic output. Under the condition of current energy structure in China, reducing energy intensity can effectively reduce carbon emissions. It is found that technological progress in energy utilization and energy structure

adjustment can effectively reduce energy intensity. Through path analysis and multiple regression analysis, Guo et al. [10] find that technological advances and related management improvements in energy treatment can change energy utilization structure and energy intensity. Liao et al. [11] use the divisor index method to find that technological progress played a major role in the decline of China's energy intensity, while the adjustment of energy structure played a relatively small role. Feng et al. [12] find that the change of energy consumption structure has little influence on energy intensity by Cointegration test. Only by improving energy utilization efficiency and upgrading industrial structure can we better reduce energy intensity. Voigt [13] find that technological advances in energy use in most countries of the world can reduce energy intensity better than energy structural adjustment based on logarithmic mean divisor index decomposition. Fang and Tian [14] establish a novel three-dimensional energy-saving and emission reduction chaotic system, and find that energy-saving and emission reduction had a positive correlation on energy intensity.

Previous studies on carbon price mainly focused on the influencing factors of carbon price. There are few studies on the relationship among carbon price, energy consumption, and economic development in China. And the models built by the research methods are often relatively simple and not enough image. Dynamic system equation can clarify the relationship between variables and clearly analyze the evolutionary relationship between variables. Sun and Tian [15] establish an energy resource demand-supply system to research the demand-supply in the East and the West of China. Wang and Tian [16] study the influence of different control strategies on energy intensity through a novel energy price-energy supply-economic growth dynamic system. So we set up a dynamic system model about carbon price, energy consumption, and economic development, which has not yet been studied in existing literature. We call it carbon price dynamic system.

This paper is mainly divided into the following parts. The carbon price system is set up and analyzed in section 2. Section 3 is used to study the complex behaviors of the carbon price dynamic system. Section 4 studies the impact of state change on China's energy intensity. In the last of the article, this paper draws the conclusion and discussion.

## II. ESTABLISHMENT OF THE CARBON PRICE DYNAMIC SYSTEM

Under the present condition of China, there is a positive relationship among energy consumption, economic development, and carbon price. Energy consumption will also be subject to various constraints and it is impossible to grow indefinitely. From the basis of the previous studies show that the relationship between the variables is nonlinear and forms complicated dynamic evolution system, which inspires

us to develop the system. The system can be divided into the following differential equations.

$$\begin{cases} \dot{x} = a_1(M - x) + a_2y + a_3z, \\ \dot{y} = -b_1x + b_2y(1 - y/N) + b_3xz, \\ \dot{z} = c_1x(1 - x/C) + c_2y - c_3z, \end{cases} \quad (1)$$

where  $x(t)$  is the time-dependent variable of carbon price of carbon market,  $y(t)$  is the time-dependent variable of energy consumption,  $z(t)$  is the time-dependent variable of economic growth (GDP).  $a_i, b_i, c_i, M, N, C$  ( $i = 1, 2, 3$ ) are positive constants,  $t \in I$ ,  $I$  is a given economic period.  $a_1$  is the development of carbon price,  $a_2$  is the influence coefficient of energy consumption to carbon price,  $a_3$  is the influence coefficient of economic growth to carbon price,  $M$  is the threshold of carbon price;  $b_1$  is the influence coefficient of carbon price to energy consumption,  $b_2$  is the speed of energy consumption,  $b_3$  is the influence coefficient of carbon price and economic growth to energy consumption,  $N$  is the peak value of energy consumption in a given period;  $c_1$  represents the influence coefficient of carbon price to economic growth,  $c_2$  represents the influence coefficient of energy consumption to economic growth,  $c_3$  stands for the speed of economic development,  $C$  is the inflexion of carbon price to economic growth.

The first formula of (1) indicates that the complicated relationship among the change rate of time-dependent carbon price  $dx/dt$ , carbon price  $x(t)$ , energy consumption  $y(t)$ , and economic growth  $z(t)$  in a given period.  $a_1$  represents the level of carbon price development. When  $x < M$ , i.e.  $(M - x) > 0$ , carbon price is smaller than the threshold of itself in the carbon market, carbon price has a positive influence on itself and the influence will weaken the increase of carbon price. Along with the development of economic growth, when  $x > M$ , i.e.  $(M - x) < 0$ , carbon price exceeds the threshold of itself in the carbon market, carbon price will have a negative influence on itself and the influence will strengthen the increase of carbon price. Energy consumption  $y(t)$  and economic growth  $z(t)$  have a positive influence on carbon price  $x(t)$ .

In the second formula of (1),  $-b_1(x)$  indicates that carbon price  $x(t)$  has a negative influence on the change rate of time-dependent energy consumption  $dy(t)/dt$ . The development of carbon price will

narrow the tendency of using energy.  $b_2y(1 - y/N)$  represents the self-detering effect of energy consumption. When  $y < N$ , i.e.  $1 - y/N > 0$ , energy consumption will have a positive influence on its development; when  $y > N$ , i.e.  $1 - y/N < 0$ , energy consumption will have a negative influence on its development.  $b_3xz$  represents carbon price and economic growth is positively proportional to energy consumption.

In the third formula of (1),  $c_1x(1 - x/C)$  represents the complicated relationship between carbon price and economic growth. When carbon price is at a lower level,  $x < C$ , i.e.  $1 - x/C > 0$ , carbon price is beneficial to economic development. With the progress of the economy, carbon prices will rise. When  $x > C$ , i.e.  $1 - x/C < 0$ , expensive carbon prices will inhibit the tendency of economic development.  $c_2y$  represents that energy consumption will boost the development of economics.

Energy intensity refers to the proportion of energy consumption and output, which can be used to measure the comprehensive utilization efficiency of energy in an economy.

In the dynamic system (1), we can count the amount of energy consumption in a given period as

$$Y = \int_0^T y(t)dt = \varphi_1(x, y, z, t). \quad (2)$$

GDP in this period can be described as

$$Z = \int_0^T z(t)dt = \varphi_2(x, y, z, t), \quad (3)$$

where  $T$  is the given period. So the energy intensity in this period can be described as:

$$U = \varphi_1(x, y, z, t) / \varphi_2(x, y, z, t). \quad (4)$$

### III. COMPLEX BEHAVIOR OF THE CARBON PRICE DYNAMIC SYSTEM

For a given dynamic system equation, different parameter changes will lead to different states of the system. This part mainly observes the dynamic behavior of the system under the condition of choosing certain parameters based on equilibrium point analysis and chaotic dynamics analysis.

#### A. Equilibrium points analysis

The carbon price dynamic system describes the complicated relationships among carbon price, energy consumption, and economic growth comprehensively. Based on MATLAB numerical simulation, when the parameters of (1) are given in different values, the system will show different dynamic behaviors. After amounts of numerical simulation, it is can be found that if we select the parameters as Table I, (1) will display chaotic behavior. Then (1) has real four

equilibrium points:  $S_0(0.156, -0.395, 0.352)$ ,  $S_1(11.186, 16.857, 0.219)$ ,  $S_2(0.001, -0.002, -0.333)$ ,  $S_3(-191.872, 13870.928, -15245.347)$ . We can get the eigenvalues at every equilibrium point as Table II. So  $S_0, S_1, S_2, S_3$  are saddle points according to Routh-Hurwitz criterion.

$$V = -a_1 + b_2 - 2b_2y/N - c_3 \quad (5)$$

TABLE I. SIMULATION PARAMETERS

Parameter	$a_1$	$a_2$	$a_3$	$b_1$	$b_2$	$b_3$
Value	1.320	0.850	0.790	0.010	0.060	0.480
Parameter	$c_1$	$c_2$	$c_3$	$M$	$N$	$C$
Value	1.920	0.733	0.010	0.200	8.217	7.100

TABLE II. EQUILIBRIUM POINTS AND THEIR CORRESPONDING EIGENVALUES

Equilibrium	$\lambda_1$	$\lambda_2$	$\lambda_3$
$S_0$	0.8927	-0.1028	-2.0542
$S_1$	-3.1268	$0.8053+1.974i$	$0.8053-1.974i$
$S_2$	0.1574	0.5993	-2.0267
$S_3$	0.6425	-40.2058	-164.2758

The dynamic system presented in (1) is a dissipative system if we select parameters as Table I.

#### B. Chaotic behavior of the carbon price system

When we take the parameters as Table I and let the initials are [0.6, 0.8, 1.2], a chaotic attractor could be found as shown in Fig. 1(a) and the corresponding time series of  $x(t)$ ,  $y(t)$ ,  $z(t)$  as shown in Fig.1 (b).

Fix parameters as Table I, and vary  $b_2$ . The Lyapunov exponent spectrum of parameter  $b_2$  is shown in Fig. 2.

The corresponding bifurcation diagram of state variable  $y$  with respect to parameter  $b_2$  is shown in Fig. 3.

From the Figs. 1-3, we can draw the conclusion that if we take the parameters as Table I and let the initial conditions as [0.6, 0.8, 1.2] the system presents complicated dynamic behavior. When  $b_2 : 0.06 \rightarrow 0.14$ , the max Lyapunov exponent of the system (1) is positive, which means the system is chaotic system. That is different from the previous chaotic system (Lorenz system, Chen system, Lü system). Although the system presents chaotic behavior when we take the above conditions, it is not to say (1) is chaotic all the time. Based on MATLAB numerical simulation, it may be stable or

chaotic when having different parameters or initial conditions. All in all, the system will present different states with different conditions.

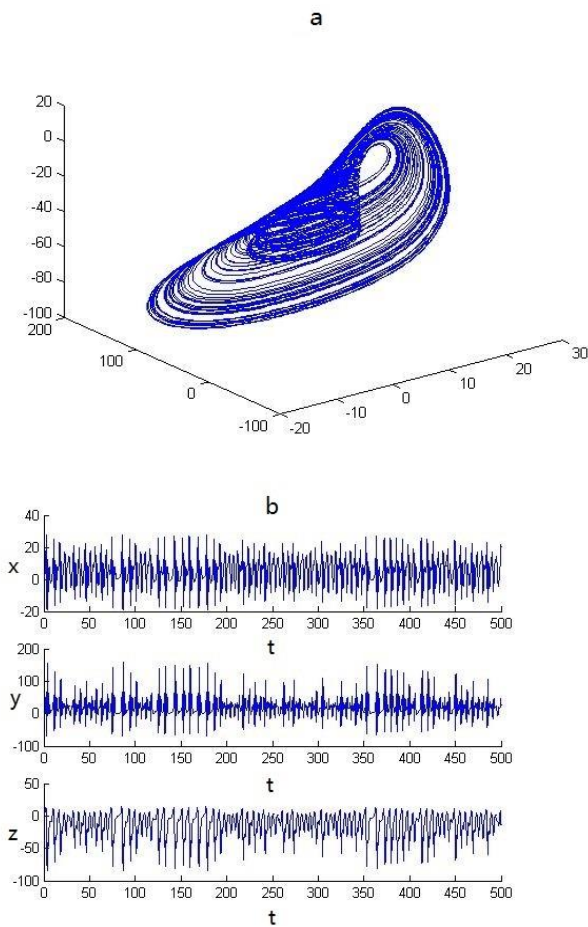


Fig. 1. (a) Carbon price attractor. (b) The time series of  $x(t)$ ,  $y(t)$ ,  $z(t)$

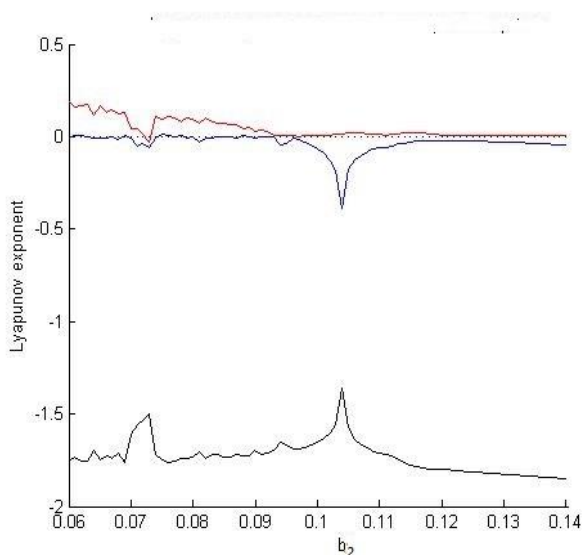


Fig. 2. Lyapunov exponent spectrum

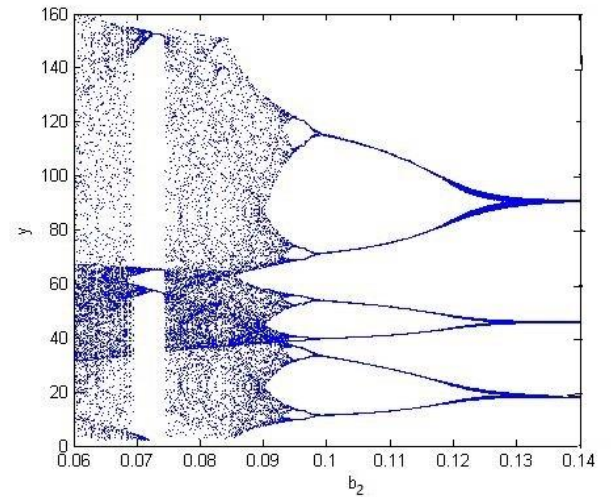


Fig. 3. Bifurcation diagram of  $y$

#### IV. EMPIRICAL STUDY OF THE CARBON PRICE DYNAMIC SYSTEM

This part mainly studies whether the carbon price dynamic system can meet the actual situation of China at present. Based on the statistical data of China, this section obtains the parameters of actual system with the genetic algorithm. Finally, the phase diagram proves that the system is in line with the actual situation.

##### A. Parameter identification

For a dynamic system with unknown parameters, the selection of parameters will have a significant impact on the dynamic behavior of the system. Because the genetic algorithm has the ability to learn and construct nonlinear complex relationships, we take the genetic algorithm to identify the coefficients of the chaotic system. In order to better demonstrate the relationship between carbon price, energy consumption, and economic growth, we denote the data of carbon prices, energy consumption, and the GDP as  $x$ ,  $y$ ,  $z$ , respectively. Carbon price data comes from China emissions exchange ([www.cerx.cn](http://www.cerx.cn)). Meanwhile, total energy consumption and indices of GDP data are obtained from the China Statistical Yearbook 2003-2017. As the data of total energy consumption and indices of GDP are collected annually, and the carbon price data are collected daily data, so we use weighted averages to integrate daily data into annual data in order to reach the unified standard. Table III shows the used carbon price, energy consumption and GDP. We can use the vector form instead of the (1) model:

$$\dot{X}(t) = f(X(t), \alpha), \quad (6)$$

where  $X$  represents the state of the system,  $\alpha$  represent the system parameter. Discrete it as:

$$X(k + 1) = X(k) + f(X(k)) = F(X(k), \alpha). \quad (7)$$

So, we can determine the system parameters by finding the minimum value of the following equation:

$$\min \frac{1}{2} \sum_{k=1}^T (\|X(k+1) - F(X(k), \alpha)\|)^2, s.t. \alpha_i > 0. \quad (8)$$

This paper uses the genetic algorithm (GA) to identify the system parameters. GA is a way of searching the optimal solution by simulating the natural evolution process, which can be used to determine the target through minimizes the obtained solution. Therefore, we can obtain the parameters of the actual system by GA.

Using the genetic algorithm to identify the coefficients of the system, the population size is set at 200, the crossover rate is 0.4, the mutation rate is 0.1, the error tolerance is  $10^{-6}$ , and the number of evolution is 200. Then, the parameters of the actual system are obtained as showed in Table IV.

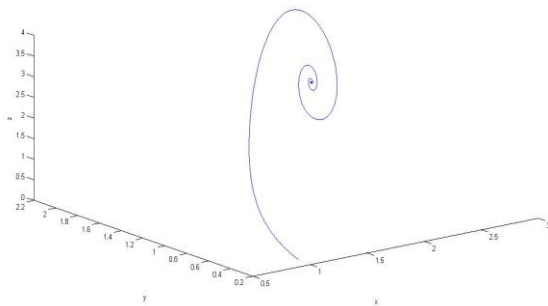


Fig. 4. The phase diagram of the actual system

TABLE III. CHINA CARBON PRICE, ENERGY CONSUMPTION, AND GDP STATISTICS DATA

Year	$x(\text{RMB/ton})$	$y(10^4\text{ton/standard coal})$	$z(10^8\text{RMB})$
2013	61.0986	416913	595244.4
2014	63.7547	425806	643974.0
2015	41.0064	429905.1	689052.1
2016	37.7519	436000	743585.5
2017	24.9944	449000	827121.7

TABLE IV. THE PARAMETERS OF THE ACTUAL SYSTEM

Parameter	$a_1$	$a_2$	$a_3$	$b_1$	$b_2$	$b_3$
Value	0.4147	0.1710	0.1672	0.0073	0.7868	0.4903
Parameter	$c_1$	$c_2$	$c_3$	$M$	$N$	$C$
Value	0.3277	0.8171	0.0929	0.7353	0.5670	0.9919

Taking the data of 2013 as the initial condition, standardization values are 0.9315, 0.2375 and 0.1277 respectively. Therefore, we select (0.9315, 0.2375, 0.1277) as the initial values, and the corresponding stable solutions are presented in Fig. 4. According to Fig. 4, the actual system is steady, which meets the real situation.

### B. The effects of control strategy on the energy intensity

As fossil energy is the main source of energy consumption in China, the large use of fossil fuels has led to higher and higher greenhouse gases in the air, causing serious environmental problems such as greenhouse effect and sea level rise. In order to reduce greenhouse gas emissions while ensuring energy utilization, reducing energy intensity is an effective method. At present, China's energy intensity is still relatively high. This part mainly studies whether we can reduce the energy intensity to a certain extent by adjusting and controlling the carbon price dynamic system we have established. The main way of this part is to regulate the government's policies and bring more industries into the carbon market.

China's current carbon price is still relatively low. Because  $a_1$  represents the level of carbon price development, raising carbon price will increase  $a_1$ . Fix the other parameters of the actual system as shown in Table IV, and let  $a_1$  be varied. Numerical simulation based on MATLAB, when  $a_1: 0.4147 \rightarrow 0.4762$ , the carbon price system always keeps stability. Fig.5 indicates the energy intensity when  $a_1 = 0.4147$  and  $a_1 = 0.4762$ . Contrast the curve of  $a_1 = 0.4147$  and  $a_1 = 0.4762$ , the curve of  $a_1 = 0.4762$  has a lower stable value, which indicated that increasing carbon price could effectively reduce energy intensity.

Currently, China's carbon market has not covered the whole industry, mainly in the high energy consumption and high emission industries such as power, cement, chemical industry. Since  $b_2$  represents the level of self-development of energy consumption, more industries covered by carbon market can adjust energy consumption structure and improve  $b_2$ . Fixing the other parameters of the actual system as shown in Table IV, and let  $b_2$  be varied. Numerical simulation based on MATLAB, when  $b_2: 0.7868 \rightarrow 0.9868$ , the carbon price system tends to stability. Fig.5 shows the energy intensity when  $b_2 = 0.7868$  and  $b_2 = 0.9868$ . Contrasting the curve of  $b_2 = 0.7868$  and  $b_2 = 0.9868$ , the curve of  $b_2 = 0.9868$  has a lower stable value, which indicated that building a carbon market including the more industry can effectively reduce energy intensity.

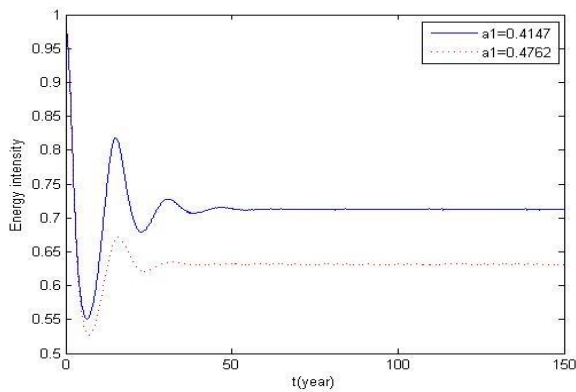


Fig. 5. The evolutionary trend of the energy intensity

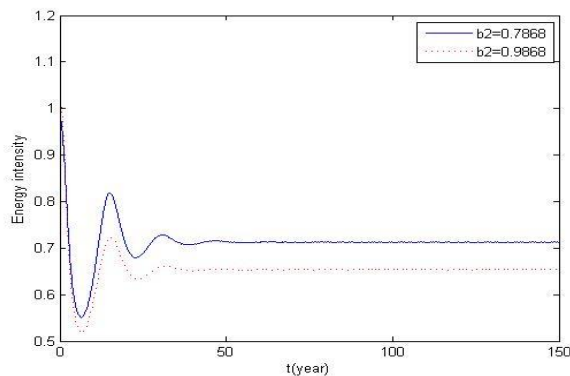


Fig 6. The evolutionary trend of the energy intensity

## V. CONCLUSIONS

This paper establishes a three-dimension dynamic system of carbon price, energy consumption, and economic growth (GDP), and analyse the dynamic behavior of the new system. Through analysis, we not only got the carbon price attractor, but also proved the chaos of the system. According to the data of carbon price, energy consumption, and economic development, the parameters of the actual system are simulated by genetic algorithm, and the parameters are verified to meet the actual situation. In the actual system, when some parameters are changed due to policy adjustment, the carbon price dynamic system presents different states. Based on this, this paper finds a way to control carbon emissions and reduce energy intensity. The results show that increasing carbon price or covering more industries in the carbon market can effectively control carbon emissions reduce energy intensity.

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