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Model Reference Adaptive Control and Fuzzy Optimal Controller for Mobile Robot

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***Abstract*—Mobile robot motion accuracy has not been yet achieved to full satisfaction using classical feedback controllers and may be not perform well online because of the variation in process dynamics due to changes in environmental conditions. To overcome the above problems, this paper presents a different control methodology for two-wheeled mobile robot (TWMR) to achieve higher accuracy for balancing and trajectory tracking control problems. Firstly, design adaptive PD controller based model reference adaptive control, where the adaptive adjusting law is derived by using lyapunov method. Secondly, coupling a fuzzy logic technique and optimal control theory to construct Fuzzy optimal controller based Linear Quadratic Regulator with Integral Control (FLQRIC). For performance analysis, the simulation results show that the proposed FLQRIC can respond smoothly to the desired trajectory and the controller can adapt quickly and correctly to the desired performance. In addition, FLQRIC has better performance compared to the adaptive PD controllers in the sense of robustness against disturbances. Finally, the fuzzy optimal control yields significant improvements in tracking performance by interacting with its environment and generating the command inputs to the nonlinear plant utilizing feedback information from the linearized plant.**

Keywords— model reference adaptive control; optimal control; integral control; fuzzy logic control.

I. Introduction

Control of mechanical systems is a general problem in various research areas with many applications. Wheeled mobile robots have attracted a great deal of attention in research and in industry as well due to the simple mechanical construction, and the big variety of potential application prospects in many areas [1]. Most researchers have shown interest in mobile robots and focused on stability and exact path following performance to control the robot system [2]. Modeling and controllers of the TWMR have been widely investigated in both academia and industry. The robot may be controlled by system decoupling and LQR for each subsystem, where the state feedback matrix is chosen by trial and error, and this relationship has not been calculated from Algebraic Riccati Equation (ARE), which is drawback of this method [3],[4]. Proportional-Derivative (PD) controller is used to control the robot but the disadvantages of this controller it is not working properly against the external disturbances [5]. In addition, an Adaptive Radial Basis Function Neural Network controller used to guarantee the stability of the robot in presence of external disturbances, where this method has a good performance, but the controller design has a lot of complexity [6]. This paper deals with the modeling of two-wheeled self-balancing robot and design of an adaptive controller and fuzzy optimal controller for the system based on Matlab simulation. This paper is organized as follows: in section II, the dynamic model of the mobile robot is provided, section III deals with the problem of designing controller based on Model Reference Adaptive Control (MRAC) and FLQRIC techniques, and section IV deals with several results to show that the proposed controller is effective. Finally, the conclusion is presented in section V.

II. Dynamics of TWMR

TWMR is an important branch of a mobile robot, because it is high orders, instability, multi-variables, nonlinearity and heavy coupling. Mathematical dynamic modelling of TWMR is rather important in terms of stability analysis and system simulation, it is also very important that control algorithms are created according to these model parameters [7]. The mechanical structure of the TWMR consists from three main parts [8]:

- The Wheels: Moves the system backward or forward to balance the body of the system.
- The Chassis: Holds the motors, circuits and any parts required for the system.
- The Pendulum: The parts of the system that causes the instability and need to be stabilized.

The dynamic model of the system is derived based on the Newton-Euler equations of motion. The pendulum and wheel dynamics are initially analyzed separately, and this will eventually lead to two non-linear equations of motion, which completely describe the behavior of the system [9]. The tilt angle acceleration and vehicle acceleration, describing the main two dynamic behaviors of the TWRM, are obtained as [10].

$$(I_p + M_p l^2)\ddot{\theta} - \frac{2k_e k_m}{R\, r}\dot{x} + \frac{2k_m}{R}V_a + M_p g l \sin\theta + F\, Z\cos\theta = -M_p l\, \ddot{x}\cos\theta \qquad (1)$$

$$(2M_w + M_p + \frac{2I_w}{r^2})\ddot{x} + \frac{2k_e k_m}{R\, r^2}\dot{x} + M_p l\ddot{\theta}\cos\theta - M_p l\,\dot{\theta}^2 \sin\theta + F = \frac{2k_m}{R\, r} V_a \quad (2)$$

The definition parameters of TWMR and the simulation of the nonlinear model given by equation of motion is carried out using the masking simulink model as shown by Fig. 1.

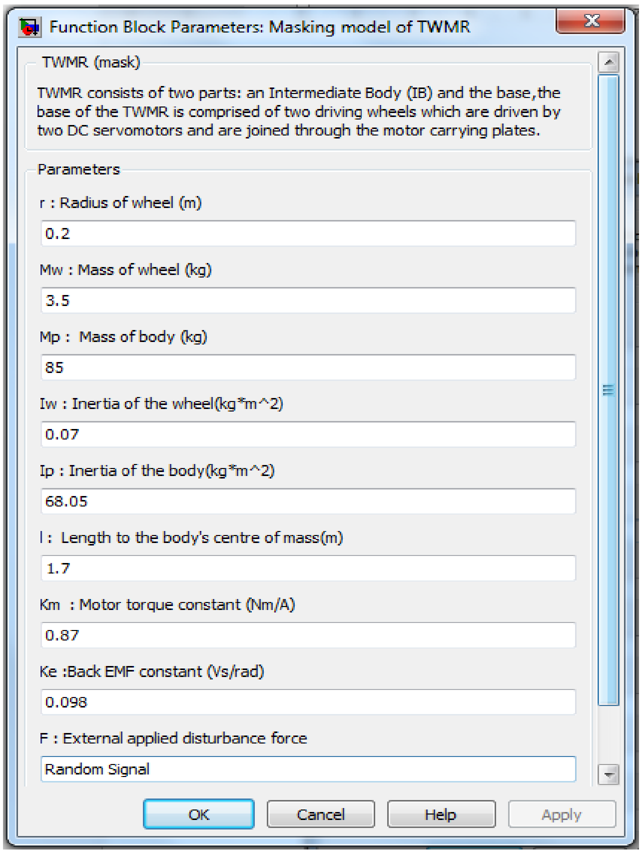

Fig.1. Masking simulink model of the TWMR

III. Methodology of Controller Design

A. *Adaptive PD Based MRAC*

MRAC is used to design the adaptive controller that works on the principle of adjusting the controller parameters so that the output of the actual plant tracks the output of a reference model having the same reference input. Mathematical approach like Lyapunov theory can be used to develop the adjusting mechanism. The basic block diagram of MRAC system is shown in the Fig. 2. There are two loops an inner loop (regulator loop) that is an ordinary control loop consisting of the plant and the controller, and an outer (adaptation) loop that adjusts the parameters of the controller in such a way as to eliminate the error between the model and plant outputs [11],[12].

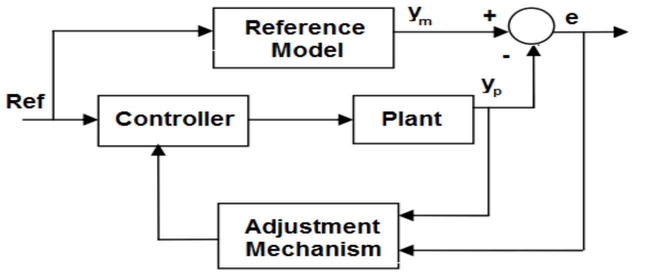

Fig. 2. Model reference adaptive controller

The structure depicted in Fig 3 can be used as an adaptive PD controlled system. The parameters of this controller is Kp and Kd. Variations in the process parameters bp and ap can be compensated for by variations in Kp and Kd. We are going to find the form of the adjustment laws for Kp and Kd [13].

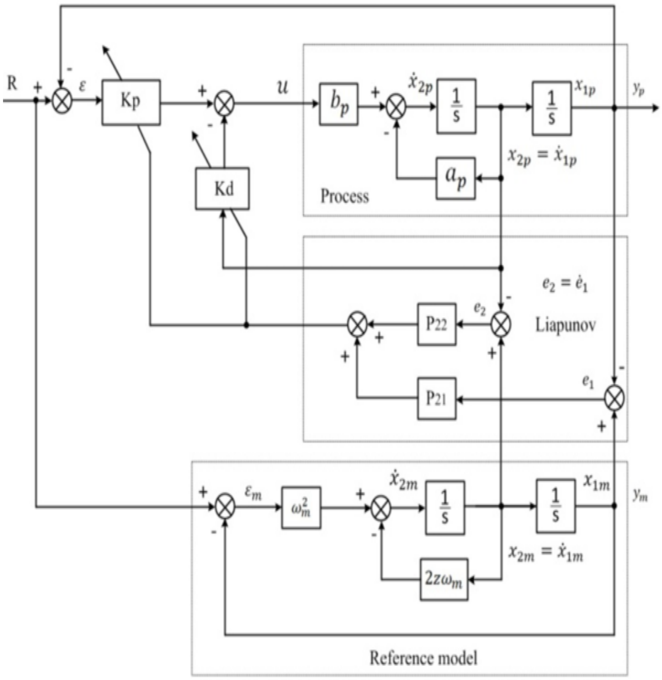

Fig. 3. Adaptive system designed with liapunov [13]

The following steps are thus necessary to design an adaptive controller with the method of Liapunov [14],[15].

Step 1: Determine the differential equation for e. The state space of the process is:

$$\begin{bmatrix}\dot{x}_{1p}\\ \dot{x}_{2p}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -b_p.K_p & -(a_p + b_p.K_d)\end{bmatrix}\begin{bmatrix}x_{1p}\\ x_{2p}\end{bmatrix} + \begin{bmatrix}0\\ b_p.K_p\end{bmatrix}.[R] \quad (3)$$

The new state variable ε is introduced as

$\varepsilon = R - x_{1p} \rightarrow \dot{\varepsilon} = \dot{R} - \dot{x}_{1p} = -x_{2p}$

The state space model of the process can be rewritten as

$$\begin{bmatrix}\dot{\varepsilon}\\ \dot{x}_{2p}\end{bmatrix} = \begin{bmatrix}0 & -1\\ -b_p.K_p & -(a_p + b_p.K_d)\end{bmatrix}\begin{bmatrix}\varepsilon\\ x_{2p}\end{bmatrix} + \begin{bmatrix}0\\ 0\end{bmatrix}u \quad (4)$$

The state space of model reference can be rewritten as

$$\begin{bmatrix}\dot{x}_{1m}\\ \dot{x}_{2m}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\omega_m^2 & -2z\omega_m\end{bmatrix}\begin{bmatrix}x_{1m}\\ x_{2m}\end{bmatrix} + \begin{bmatrix}0\\ \omega_m^2\end{bmatrix}R \qquad (5)$$

Where ω_m is the natural frequency and z is the damping ratio. By the same way,

$\varepsilon_m = R - x_{1m} \rightarrow \dot{\varepsilon}_m = \dot{R} - \dot{x}_{1m} = -x_{2m}$

It can be rewritten as

$$\begin{bmatrix}\dot{\varepsilon}_m\\ \dot{x}_{2m}\end{bmatrix} = \begin{bmatrix}0 & -1\\ \omega_m^2 & -2z\omega_m\end{bmatrix}\begin{bmatrix}\varepsilon_m\\ x_{2p}\end{bmatrix} + \begin{bmatrix}0\\ 0\end{bmatrix}R \qquad (6)$$

We can be defined the error vector as

$e = x_m - x_p \rightarrow \dot{e} = \dot{x}_m - \dot{x}_p$

$$\dot{e} = A_m e + (A_m - A_p)x_p + (B_m - B_p)u \qquad (7)$$

Where

$A = A_m - A_p$

$$A = \begin{bmatrix}0 & 0\\ \omega_m^2 - b_p.K_p & -2z\omega_m + (a_p + b_p.K_d)\end{bmatrix}$$

$$= \begin{bmatrix}0 & 0\\ a_{21} & a_{22}\end{bmatrix},$$

$B = B_m - B_p = \begin{bmatrix}0\\ 0\end{bmatrix},$

$e^T = [e_1 \quad e_2]\,, e_1 = x_{1m} - x_{1p}\,,$

$e_2 = x_{2m} - x_{2p}$

Step 2: Choose a Liapunov function V(e)

$$V(e) = e^T P e + a^T \alpha\, a \qquad (8)$$

P is an 'arbitrary' definite positive symmetrical matrix, a is vector which contain the non-zero elements of the A. α is diagonal matrices with positive elements which determine the speed of adaptation.

Step 3: Determine the conditions under which $\dot{\boldsymbol{V}}(\boldsymbol{e})$ is definite negative,

$$\dot{V} = \dot{e}^T P e + e^T P \dot{e} + 2\dot{a}^T \alpha\, a\ \dot{V} = e^T(A_m^T P + P A_m)e + 2e^T P(Ax_p) + 2e^T PBu + 2\dot{a}^T \alpha\, a \qquad (9)$$

Let: $A_m^T P + A_m P = -Q$

Because the matrix A_m belongs to a stable system (reference model), it follows from the theorem of Malkin that Q is a definite positive matrix [11].

Based on that, $e^T(A_m^T P + P A_m)e = -e^T Q e$

Stability of the system can be guaranteed if the last part of $\dot{V}(e)$ is set equal to zero.

$$e^T P(Ax_p) + \dot{a}^T \alpha\, a + e^T PBu = 0 \qquad (10)$$

Where

$P = \begin{bmatrix}P_{11} & P_{21}\\ P_{21} & P_{22}\end{bmatrix},$

$\alpha = \begin{bmatrix}\alpha_{11} & 0\\ 0 & \alpha_{22}\end{bmatrix},$

a = $[a_{21} \quad a_{22}]$,

$x_p = \begin{bmatrix}\varepsilon\\ x_{2p}\end{bmatrix}$

After some mathematical manipulations, this yields:

$$K_p = \frac{1}{\alpha_{11}\, b_p}\int (P_{21}.e_1 + P_{22}.e_2)\varepsilon\, dt \qquad (11)$$

$$K_d = -\frac{1}{\alpha_{22}\, b_p}\int (P_{21}.e_1 + P_{22}.e_2)x_{2p}\, dt \qquad (12)$$

Step 4: Solve P from $A_m^T P + A_m P = -Q$,

$$\begin{bmatrix}0 & \omega^2{}_m\\ -1 & -2z\omega_m\end{bmatrix}\begin{bmatrix}P_{11} & P_{21}\\ P_{21} & P_{22}\end{bmatrix} + \begin{bmatrix}P_{11} & P_{12}\\ P_{21} & P_{22}\end{bmatrix}\begin{bmatrix}0 & -1\\ \omega^2{}_m & -2z\omega_m\end{bmatrix} = -\begin{bmatrix}q_{11} & q_{21}\\ q_{21} & q_{22}\end{bmatrix}$$

(13)

This yield

$P_{21} = \frac{-q_{11}}{2\omega_m^2} \quad and \quad P_{22} = \frac{q_{11} + q_{22}.\omega_m^2}{4z\omega_m^3}$

So,

$$K_p = \frac{1}{\alpha_{11}\, b_p}\int \left(\frac{-q_{11}}{2\omega_m^2}.e_1 + \left(\frac{q_{11} + q_{22}.\omega_m^2}{4z\omega_m^3}\right).e_2\right)\varepsilon\, dt$$

(14)

$$K_d = -\frac{1}{\alpha_{22}\, b_p}\int \left(\frac{-q_{11}}{2\omega_m^2}.e_1 + \left(\frac{q_{11} + q_{22}.\omega_m^2}{4z\omega_m^3}\right).e_2\right)x_{2p}\, dt$$

(15)

The above equations and the adaptive control system designed in Fig. 3 is redrawn as simulink model as shown in Fig. 4.

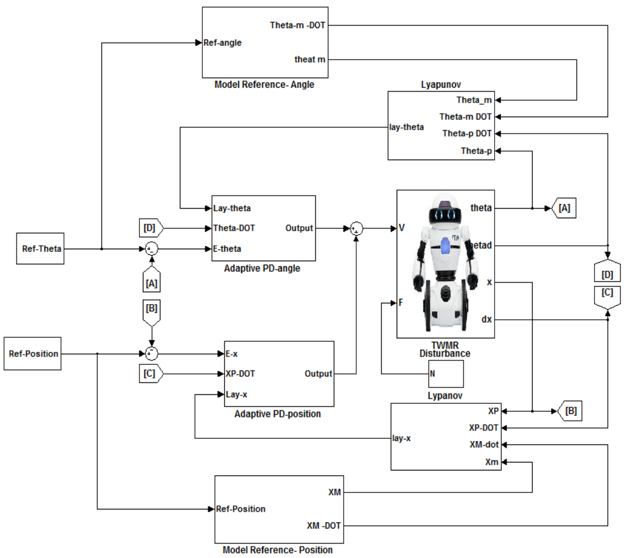

Fig. 4. Simulink model of adaptive controllers with TWMR

B. Fuzzy Optimal Controller Based Integral Action

Sometimes it is seen that alone state feedback control is not sufficient, it requires additional integral controller along with the state-space model. In such cases, the combined scheme also uses output feedback to regulate the error explicitly and thus ensures better control and reference tracking. The state space model of linearized TWMR constructed using matlab function called trim and linmod functions for the masking simulink model of nonlinear TWMR in Fig. 1. We now combine a Linear Quadratic Regulator with Integral Controller to construct new controller called LQRIC for linearized model of TWMR. So that the error signal will approach to zero as t tends to zero. After that we will design fuzzy control system using the error signal which constructed from the difference between the output of linearized which controlled by LQRIC and actual nonlinear system as shown in Fig. 5.

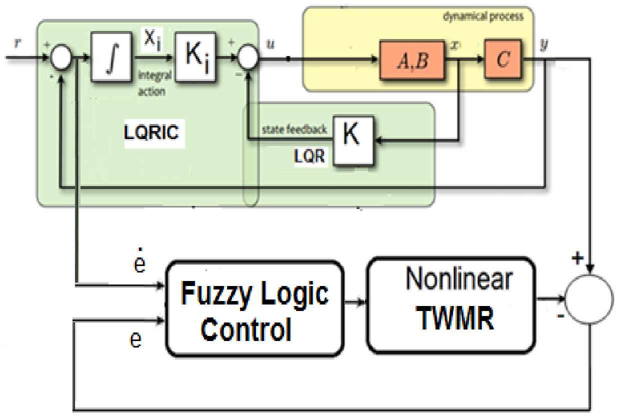

Fig. 5. Fuzzy state feedback with integral controller for TWMR

The state space model of the system is constructed as, $\dot{x} = Ax + Bu$. The first step of LQRIC controller design is to define a new augmented state vector given by

$$x_a = \begin{bmatrix} x \\ x_i \end{bmatrix} = \begin{bmatrix} x \\ e_i \end{bmatrix} \quad (16)$$

$$\dot{x}_i = \dot{e}_i = r - y = r - Cx \quad (17)$$

The new state space form is,

$$\begin{bmatrix} \dot{x} \\ \dot{x}_i \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x \\ x_i \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r \quad (18)$$

$$y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x \\ x_i \end{bmatrix}$$

The new matrices are

$A_a = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}$,

$B_a = \begin{bmatrix} B \\ 0 \end{bmatrix}$,

$C_a = \begin{bmatrix} C & 0 \end{bmatrix}$

Now, the state feedback can be written as:

$$u = -\begin{bmatrix} K & -K_i \end{bmatrix} \begin{bmatrix} x \\ x_i \end{bmatrix} = -K_a x_a \quad (19)$$

Where

$K_a = \begin{bmatrix} K & -K_i \end{bmatrix}$

K_a : Gain of LQRIC

K: Gain of LQR.

Ki : Integral gain.

The closed-loop state equation with the state feedback control u is

$$\begin{bmatrix} \dot{x} \\ \dot{x}_i \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x \\ x_i \end{bmatrix} - \begin{bmatrix} B \\ 0 \end{bmatrix} \begin{bmatrix} K & -K_i \end{bmatrix} \begin{bmatrix} x \\ x_i \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r \quad (20)$$

$$\begin{bmatrix} \dot{x} \\ \dot{x}_i \end{bmatrix} = (A_a - B_a K_a) \begin{bmatrix} x \\ X_i \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r \quad (21)$$

The goal is to determine the value of the matrixes to minimize the performance index which is assumed by the following cost function:

$$J = \int_0^{\infty} (x_a^T Q \, x_a + u^T R \, u) \, dt \quad (22)$$

Where Q is a positive semi-definite symmetric matrix and R is a positive definite symmetric matrix. In which the solution of the following ARE [17]:

$$A_a^T \mathrm{P} + \mathrm{P} A_a - \mathrm{P} B_a \mathrm{R}^{-1} B_a^T \mathrm{P} + \mathrm{Q} = 0 \quad (23)$$

Where P is the solution of ARE and the feedback control gains K_a represented as

$$K_a = \mathrm{R}^{-1} B_a^T \mathrm{P} \quad (24)$$

After designed LQRIC we can construct the fuzzy optimal control for the nonlinear model of TWMR, where the linearized model of TWMR with LQRIC operate as model reference for the nonlinear system which will be controlled using FLQRIC. The fuzzy controller has two inputs, the error $(e = y_{\mathrm{Linear}} - y_{\mathrm{Nonlinear}})$, the error rate $(\dot{e} = r_{Reference} - y_{\mathrm{Linear}})$ and one output which represent the control signal. The fuzzy set of each input variable is represented by three linguistic variables which as N (Negative), Z (Zero) and P (Positive). In addition, the fuzzy set of the output variable is represented by three linguistic variables which as S (Small), M (Medium) and B (Big). The proposed fuzzy logic controller uses conventional triangular membership functions for the inputs and Gaussian membership functions for the output as shown in Fig. 6. Finally, the fuzzy optimal controller with integral control in Fig. 5 is redrawn as simulink model as shown in Fig. 7.

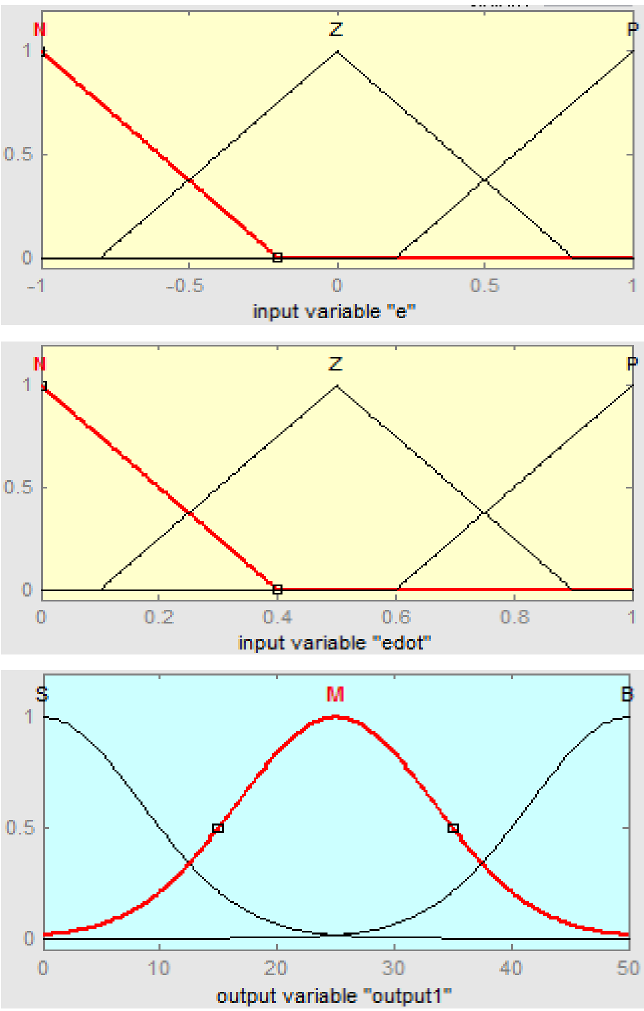

Fig. 6. Input and output membership functions for FLQRIC

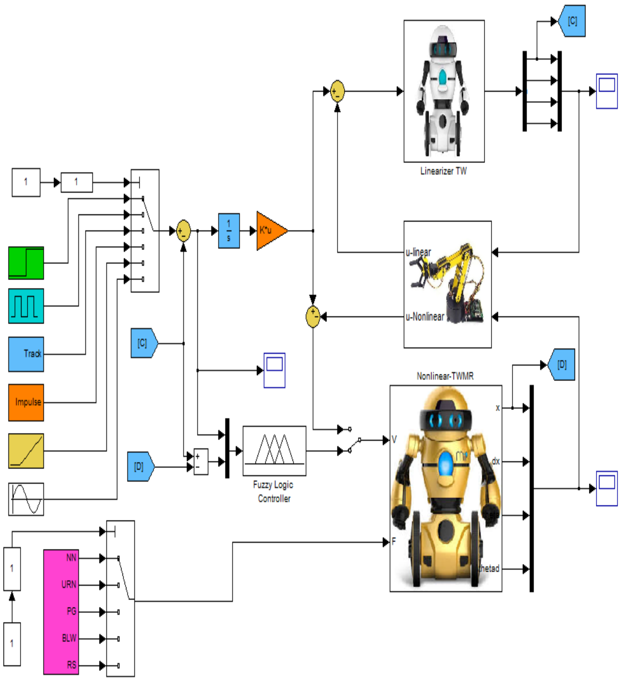

Fig. 7. FLQRIC for nonlinear TWMR

IV. SIMULATION RESULT

The MATLAB Simulink environment is used to evaluate the influence each control method on balancing and trajectory tracking of TWMR. The parameters of the model reference for balancing and tracking are chosen as, z =1 and $\omega_m = \omega_\theta = \omega_x = 10$ rad/sec. The adaptive PD controller for balancing angle is achieved by setting $\alpha_{11-\theta} = 700$, $\alpha_{22-\theta} = 1500$ and $Q_{\theta=}[500\ 300; 300\ 400]$ while the adaptive PD controller for position with $\alpha_{11-x} = 1000$, $\alpha_{22-x} = 2000$ and $Q_{x=}[300\ 100; 100\ 400]$.

From the solution of ARE, the optimal control gain, K_a= [-1008 -2569 1575 1188] and integral gain, $K_a = 316$. Responses of the system for different setpoints with FLQRIC have been compared with the responses obtained using adaptive PD controllesr as shown in Fig. 8 and Fig. 9. The simulation result show that the FLQRIC has very good dynamic response, faster settling time, good stabilization and accurate tracking for the desired trajectory which satisfies the design criteria very much. However, as shown in Fig. 10, due to a large enough disturbance, it is clearly that the tracking errors for the both controller are reduced to zero, but FLQRIC controller reduces the amplitude of oscillation rapidly. Performance of adaptive controller show that the robot runs well and the presented control technique is useful to realize our intention.

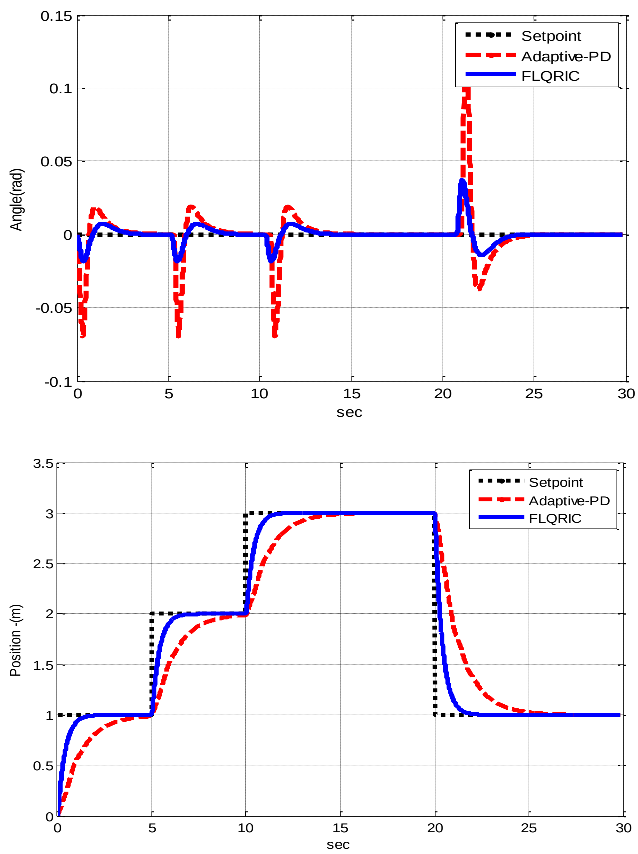

Fig. 8. Response of TWMR using an adaptive PD and FLQRIC for different level setpoint

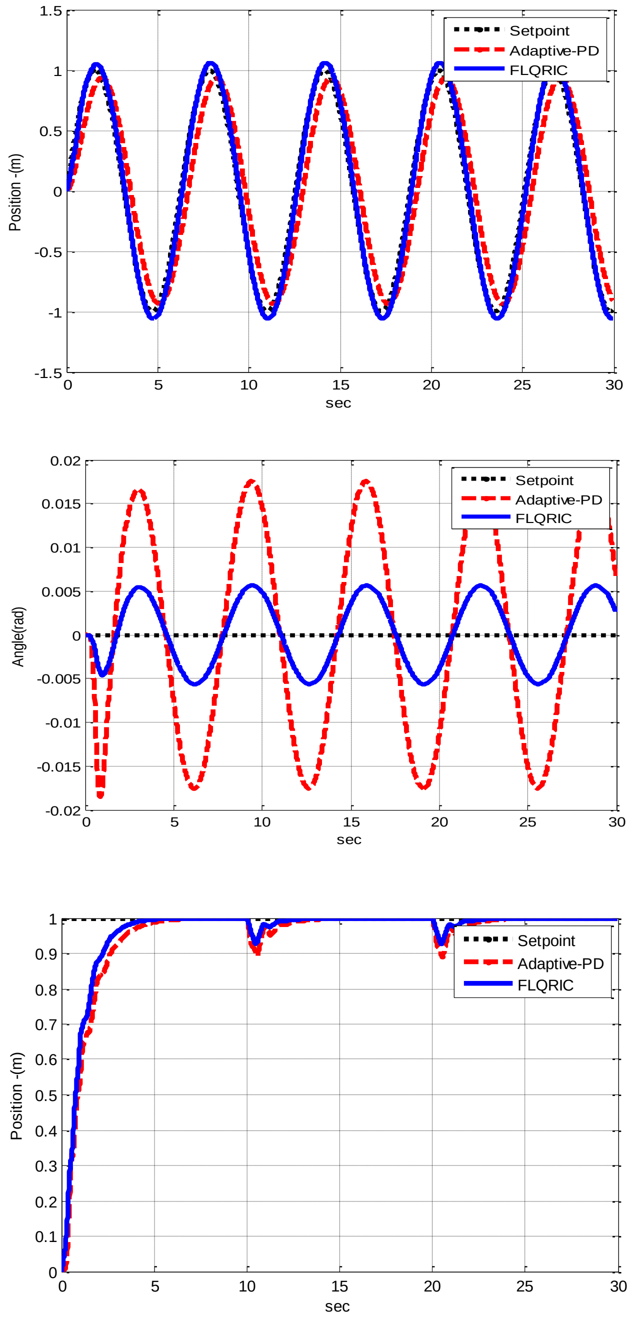

Fig. 9. Response of TWMR using an adaptive PD and FLQRIC for sine wave setpoint

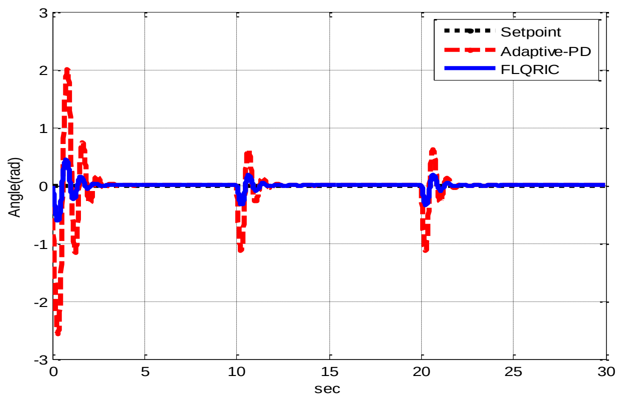

Fig. 10. Response of TWMR using an adaptive PD and FLQRIC with noise signal

V. Conclusion

The paper has presented adaptive controller using second order MRAC method and intelligent controller using combining fuzzy logic and optimal control theory for tilt angle and trajectory tracking control of the TWMR in presence of parameter variations and model uncertainties. The performance evaluation is carried out by means of simulations on matlab and simulink. Different input reference signals have been applied to test the effectiveness of the controller design and it is demonstrated that an acceptable tracking accuracy can be achieved. It is concluded that, under the influence of different references the FLQRIC is successful to achieve a high tracking performance in transient and steady state time. Performance of FLQRIC show that the robot runs well and the presented control technique is useful to realize our intention.

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