Fluid Flow of A Rotating Rectangular Straight Duct in Darcian Porous Medium

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***Abstract*— This paper mainly discusses the fluid flow of a straight duct which rotated counterclockwise along the y-axis in the porous. Darcy is used to defining the porous ($Da \leq 500$) and non-Darcy ($Da \geq 500$) media. Three different cases that have been studied such as Dean number $Dn = 100, 500,$ and 1000, where rotation number $R = 500$ is treated as constant in the range of Da between 0.1 and 1000. Only significant results have been discussed in this study. Steady solution has been obtained by using Spectral method where the Chebyshev polynomial is used in horizontal and vertical directions. It has been observed that flow is rapidly converged and there is no significant effect of Darcy number in the flow behavior due to the large Dean number and Rotation parameter. Furthermore, the anomalous behavior of vortex is observed in the porous medium, although vortex towards the upper wall in the axial flow has been vanished at large rotation parameter while the vortex at the center in the duct become weakened. It also found that elliptical ring shape in the vicinity to the left wall in the case of non-porous media and we observed that the flow energy decreased as increasing the rotation parameter at constant Dean number in both porous and non-porous medium.**

Keywords—Dean number (Dn); Rotation parameter (R); Darcy number(Da); Spectral method; Porous medium;

I. INTRODUCTION

Flow through a rotating rectangular straight duct in the porous and non-porous media has been magnificent and enormous application found in chemical, mechanical engineering, and medical science. rotating systems penetrate with engineering applications such as gas turbines, cooling system, electric generators, and heat exchangers. Flow in a rotating straight duct is of interest to study because the secondary flows, in this case, are qualitatively similar to those in the stationary curved system (e.g. [1]). Many researchers are also interested to show their capability to study the porous and non-porous media due to its complexity in the industrial fields. For these reasons, scientist, applied mathematician and physicists pay crucial attention to understand the flow behaviour along the center of the rectangular duct in presence of Dean number (Dn) in both porous and non-porous media. The asymptotic limit of weak and strong rotations for the rotating straight pipe has carried out by [2]. In the contemporary period, [3] also used a perturbation expansion to observe the Hagen-Poiseuille flow. [4], and [5] were famous for providing the profound concept on the fluid flow in a pipe in presence of the rotating parameter. [5] examined that the bifurcation structure of two-dimensional, pressure-driven flows through a rectangular duct was rotating about an axis perpendicular to its own at a fixed Ekman number of 0.01 in the different values of Rossby number at aspect ratio $\gamma = 1$ by using Arc-length continuation method. In a recent time [6] has studied extensively on the fluid flow in a rotating rectangular straight duct for different aspect ratio in the presence of magnetic field. They found that flow is symmetric and fully developed at high Magnetic field although the flow strength becoming weakened gradually as increasing the rotation parameter. Therefore, delving into the topics of fluid flow through the rotating rectangular duct in the presence of the

Darcy number is also interested in many areas such as petroleum and resource engineering, geothermal energy extraction, ground water hydrology, medical research as well as in the environment.

Darcy (1856) proposed fluid flow application in a porous medium is based on the experimental law. Later**,** [7] has studied the flow of fluids through a porous medium and speculated about the importance of the porous medium by implementing the classical hydrodynamics theory. He concluded that "*the whole field of ground water hydrology including the provision and maintenance of water supplies, the scientific solution of irrigation problems, and efficient development of drainage system is only an application of the theory of the flow of liquids through sands*". He also mentioned that the efficient construction of filter beds and the predictions of their properties by the sanitary engineer requires an understanding of the same basic principles. Furthermore, ceramic engineers must be familiar with hydrodynamics theory when studying the diffusion and the flow of fluids through ceramic materials such as bricks and porous earthenware. In recent time, [8] mentioned that the applicability of Darcy law in science has a fundamental assumption in large scale flow studies. For example, **[9]** have studied the steady two-dimensional non-isothermal boundary layer flow for a heated horizontal surface in presence of a porous medium and they found that the wall temperature and nanoparticle volume fraction vary non-linearity with the axial distance. [10] have studied the non-Newtonian power-law fluid flow and heat transfer from a non-isothermal stretching sheet in a porous medium with the internal heat generation and absorption by using the finite difference numerical method. [11] have observed the double diffusion from a plate in a porous medium. [12] examined the onset of convection in a layer of a porous medium which is filled with the non-Newtonian Nano-fluids of power-law type. [13] have studied generalized Newtonian fluid flow through a porous medium in which they presented a model the dependence of the fluid viscosity on the velocity is replaced by a dependence on a smoothed velocity. [14] investigated dissipative free convective flow from a vertical cone in a non-Newtonian power-law fluid in a porous medium by using Chebyshev Spectral numerical method. In very recent time, [15] observed the effect of dissipation on free convective flow of a non-Newtonian Nano-fluid in a porous medium with a micro-organism. Many applications such as heat pipes or heat exchangers chemical combustions/reactors often use porous media to transport and store the energy and control the flow. Therefore, it is felt that the fluid flow through a rotating rectangular straight duct in the porous medium also plays an important role in the engineering and industrial fields. [16] have studied fluid flow through a porous medium using a distinct element based numerical method. They developed a numerical model to simulate the fluid flow through heterogeneous porous media in the presence of natural fractures interacted with hydraulic fracture by considering laminar and radial fluid flow conditions. **[17]** described the mechanics of fluid flow through the porous medium. They illustrated the equation for the permeability for the case of an isotropic medium as $\Delta P = -\frac{\mu}{K\vartheta}$, where μ is the dynamic viscosity of the fluid, ΔP is the pressure difference of the flow, and K is the permeability which is dependent of the mature of the fluid, however it also depends on the geometry of the medium. The values of K varies widely based on the natural materials. [16] also found that the typical values of K for soils, in terms of the unit m^2, are : clean gravel $10^{-7}-10^{-9}$, clean sand $10^{-9}-10^{-12}$, peat $10^{-11}-10^{13}$, stratified clay $10^{-13}-10^{-16}$ and unwaethered clay $10^{-16}-10^{-20}$. Many other researchers are also highly concerned about the study of turbulent flow in porous media for example [18], [19], [20], [21], [22], [23] [24], and so on.

This present paper described the fluid flow in the rotating rectangular duct in the presence of the porous media due to its significant role in the engineering field, especially in medical science. The Chebyshev-Spectral method will be used as a main tool for the numerical solution.

II. GOVERNING EQUATION

We considered the laminar flow of an incompressible viscous fluid in a rotating straight pipe in the porous and non-porous media. The flow-configuration and coordinate system are shown in figure (1).

$$\frac{\partial q'}{\partial t} + q'(q'.\overline{\nabla}) = \bar{F} - \frac{1}{\rho}(\overline{\nabla}.p') + \nu\overline{\nabla^2}.q' + 2(\bar{\Omega}\wedge q') - \frac{\nu}{K}q' \quad (1)$$

where q' is the velocity vector, ν is the kinematic viscosity, $\bar{\Omega}$ is an angular velocity which rotates about $y-$ axis. There is no body force, so the Eq. (1) can be written as:

$$\frac{\partial q'}{\partial t} + \frac{1}{2}(\nabla q') - q'\wedge(\nabla\wedge q') = -\frac{1}{\rho}(\overline{\nabla}.p') + \nu\overline{\nabla^2}.q' + 2(\bar{\Omega}\wedge q') - \frac{\nu}{K}q' \quad (2)$$

We know that $\xi = \nabla\wedge q'$. Then the Eq.(2) can be expressed as:

$$\frac{\partial q'}{\partial t} - (q'\wedge\xi) = -\nabla\left(\frac{p'}{\rho} + \frac{q'}{2}\right) + \nu\overline{\nabla^2}.q' + 2(\bar{\Omega}\wedge q') - \frac{\nu}{K}q' \quad (3)$$

Furthermore, the continuity equation for a viscous compressible fluid also can be written as

$$\frac{\partial p}{\partial t} + \nabla(\rho q') = 0 \quad (4)$$

where ρ is the density of the fluid. For the incompressible fluid, the Eq. (4) is $\nabla.q' = 0$.

Then the fluid moves in a rotating straight duct with a rectangular cross-section along the center line in the porous and non-porous medium can be exposed as:

$$\frac{\partial q'}{\partial t} + q'(q'.\overline{\nabla}) = -\frac{1}{\rho}(\overline{\nabla}.p') + \nu\overline{\nabla^2}.q' + 2(\bar{\Omega}\wedge q') - \frac{\nu}{K}q' \quad (5)$$

Here, angular velocity $\bar{\Omega} = (\Omega_{x'}, \Omega_{y'}, \Omega_{z'}) = (0, -\omega, 0)$ and the velocity vector $q' = (u', v', w')$. The boundary conditions are: $u' = v' = w' = 0$ on the wall of the straight duct. The assumption of fully developed flow mean that the pressure derivatives are set to zero except z' derivatives. For the steady flow, $\partial u'/\partial t = \partial v'/\partial t = \partial w'/\partial t = 0$ and there is no body force inside the straight duct. a Here the axis of the rotation is perpendicular to the span of the pipe and the axial pressure gradient $-\partial p'/\partial z' = G$ is the constant and it is maintained by external means i.e. p' is the modified pressure which includes the gravitational and centrifugal force potentials.

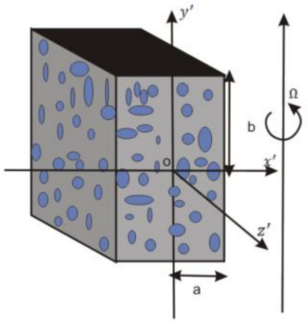

Figure 1: Geometrical configuration of the rectangular straight duct in the Darcian porous medium.

Therefore, we have from the Eq.(1-5)

$$u'\frac{\partial u'}{\partial x'} + v'\frac{\partial u'}{\partial y'} = -\frac{1}{\rho}\frac{\partial p'}{\partial x'} + \nu\left(\frac{\partial^2 u'}{\partial x'^2} + \frac{\partial^2 u'}{\partial y'^2}\right) - 2\Omega w' - \frac{\nu}{K}\frac{\partial u'}{\partial x'} \quad (6)$$

$$u'\frac{\partial v'}{\partial x'} + v'\frac{\partial v'}{\partial y'} = -\frac{1}{\rho}\frac{\partial p'}{\partial y'} + \nu\left(\frac{\partial^2 v'}{\partial x'^2} + \frac{\partial^2 v'}{\partial y'^2}\right) - \frac{\nu}{K}\frac{\partial v'}{\partial y'} \quad (7)$$

$$u'\frac{\partial w'}{\partial x'} + v'\frac{\partial w'}{\partial y'} = -\frac{1}{\rho}\frac{\partial p'}{\partial z'} + \nu\left(\frac{\partial^2 w'}{\partial x'^2} + \frac{\partial^2 w'}{\partial y'^2}\right) + 2\Omega u' - \frac{\nu}{K}\frac{\partial u'}{\partial x'} \quad (8)$$

$$\frac{\partial u'}{\partial x'} + \frac{\partial u'}{\partial y'} = 0 \quad (9)$$

Now, we normalized the velocity, pressure and length scales as following way:

$$u' = \frac{\nu}{a}u;\ x' = xa;\ p' = \frac{\nu^2}{a^2}\rho p;\ v' = \frac{\nu}{a}v;\ y' = ya;\ w' = \frac{w}{a}w;\ z' = 0 \quad (10)$$

where the variables with a prime are the dimensional quantities and "a" be the half width of the cross section of the duct. The boundary condition is that the velocities are zero at $x = \pm 1$ and $y = \pm\left(\frac{b}{a}\right) = \pm\gamma$ (aspect ratio). Also, "b" is the half height of the cross section. Therefore, we have introduced new variables $\bar{y} = \left(\frac{y}{\gamma}\right)$, $u = \left(\frac{\partial\psi}{\partial y}\right)$ and $v = \left(\frac{\partial\omega}{\partial x}\right)$ which staisfies the continuity equation.

Finally the basic equation for ψ and ω as

$$\frac{\partial^4\psi}{\partial x^4} + \frac{2}{\gamma}\frac{\partial^4\psi}{\partial\bar{y}\partial x^2} + \frac{1}{\gamma^4}\frac{\partial^4\psi}{\partial y^4} = -\frac{1}{\gamma^3}\frac{\partial\psi}{\partial\bar{y}}\frac{\partial^3\psi}{\partial\bar{y}^2\partial x} + \frac{1}{\gamma^3}\frac{\partial\psi}{\partial x}\frac{\partial^3\psi}{\partial\bar{y}^4} - \frac{1}{\gamma}\frac{\partial\psi}{\partial y}\frac{\partial^3\psi}{\partial\bar{x}^4} + \frac{1}{\gamma}\frac{\partial\psi}{\partial x}\frac{\partial^3\psi}{\partial x^2\partial\bar{y}} - \frac{1}{\gamma}\frac{\partial\psi}{\partial\bar{y}}R - \left(\frac{1}{\gamma}\frac{\partial^2\psi}{\partial\bar{y}^2} + \frac{\partial^2\psi}{\partial x^2}\right).\frac{1}{Da} \quad (11)$$

and

$$\frac{\partial^2\omega}{\partial x^2} + \frac{1}{\gamma^2}\frac{\partial^2\omega}{\partial\bar{y}^2} = -\frac{1}{\gamma}\frac{\partial\psi}{\partial x}\frac{\partial\omega}{\partial x} - Dn + \frac{1}{\nu}\frac{\partial\psi}{\partial\bar{y}}R - \frac{1}{\gamma}\frac{\partial\omega}{\partial\bar{y}}\frac{1}{Da} \quad (12)$$

In this study, R denotes the rotating parameter and it is defined as $2\left(\frac{\Omega a^2}{\nu}\right)$. Rotating parameter is often called Taylor numbers. Darcy number (Da) is also defined as $\frac{a^2}{K}$ where K is the permeability. In addition, we also defined Dean number $\left(\frac{Ga^3}{\rho\nu^2}\right)$. The boundary conditions for ψ and ω are given by

$$\omega(\pm 1, \bar{y}) = \omega(x, \pm 1) = \psi(\pm 1, \bar{y})$$

and $$\left(\frac{\partial\psi}{\partial x}\right)(\pm 1, \bar{y}) = (x, \pm 1) = \left(\frac{\partial\psi}{\partial y}\right)(x, \pm 1)$$

Therefore, it is important to determine and observe the flux through the straight duct. Then the dimensional total flux Q' through the duct can ben defined as

$$Q' = \int_{-b}^{b}\int_{-a}^{a}\omega\, dx'\, dxy' = \nu aQ \quad (13)$$

where $Q = \int_{-\gamma}^{\gamma}\int_{-1}^{1}\omega\, dx'\, dy'$ is the non-dimensional flux.

III. CALCULATION TECHNIQUES

The current study is based on numerical methods. The spectral method is used mainly as a numerical technique in order to achieve the solution (see, [6]). This method is powerful and efficient, has been used by many authors such as [15], [25] to simulate complex transport problems. Therefore, it is essential to discuss the method in details. Hence, the basic ideas of the Spectral and the Collocation methods are given below.

The expansion by the polynomial function is utilized to obtain a steady solution. the series of the Chebyshev polynomial is used in the $\bar{x}$ and $\bar{y}$ direction respectively where x and y are variables. Assuming the flow is symmetric along the axial direction. Then the expansion function $\phi_n(x)$ and $\psi_n(x)$ are expressed as:

$$\phi_n(x) = (1 - x^2)T_n(x) \quad (14)$$

$$\psi_n(x) = (1 - x^2)^2 T_n(x) \quad (15)$$

where, $T_n = \cos(n\cos^{-1}(x))$ is the n-th order first kind Chebyshev polynomial. $\omega(x, \bar{y})$ and $\psi(x, \bar{y})$ are expanded in terms of the function $\phi_n(x)$ and $\psi_n(x)$.

$$\omega(x, \bar{y}) = \sum_{m=0}^{M}\sum_{n=0}^{N}\omega_{mn}\phi_m(x)\phi_n(y) \quad (16)$$

$$\psi_n(x,\bar{y}) = \sum_{m=0}^{M}\sum_{n=0}^{N} \psi_{mn}\psi_m(x)\psi_n(y) \qquad (17)$$

where M and N are the truncation numbers in the x and $\bar{y}$ directions respectively. In this study, $M = 10$ and $N = 20$. To obtain the solution for $\omega(x,\bar{y})$ and $\psi_n(x,\bar{y})$, the expansion series are substituted into the basic Eq. (11) and (12). The Collocation method [26] applied in x and $\bar{y}$ directions yield a set of non-linear differential equation for ω_{mn} and ψ_{mn}. The Collocation points as taken as $(x_i, \bar{y}_j$:

$$x_i = cos\left[\pi\left(1-\frac{i}{M+2}\right)\right], \qquad (i = 1,2, \ldots. M+1)$$
(18)

$$\bar{y}_j = cos\left[\pi\left(1-\frac{j}{N+2}\right)\right], \qquad (i = 1,2, \ldots. N+1)$$
(19)

and the non-linear differential equations are expanded symbolically as:

$$A_1\omega + B_1\omega + C_1\omega = N_1(\omega_{mn}, \psi_{mn}) \qquad (20)$$

$$A_2\psi + B_2\psi + C_2\psi = N_2(\omega_{mn}, \psi_{mn}) \qquad (21)$$

where A_1, B_1, C_1 and A_2, B_2, C_2 are the square matrices with $(M+1)(N+1)$ dimension. $\omega = (W_{00}, W_{0M}, \ldots W_{0N}, \ldots W_{MN})$ and $\psi = (\psi_{00}, \psi_{0M}, \ldots. \psi_{0N}, \ldots \psi_{MN})$, N_1, N_2 are the non-linear operators. The obtained non-linear algebraic equations are solved by the Newton-Raphson iteration method as follows:

$$\omega^{(p+1)} = C_1^{-1}N_1(\omega_{mn}{}^{(p)}, \psi_{mn}{}^{(p)}) \qquad (22)$$

$$\psi^{(p+1)} = C_2^{-1}2(\omega_{mn}{}^{(p)}, \psi_{mn}{}^{(p)}) \qquad (23)$$

where p denotes the iteration number. To avoid the difficulty near the point of inflection for the steady solution. We use the arc-length method. The arc-length
equation is:

$$\sum_{m=0}^{M}\sum_{n=0}^{N}\left[\left(\frac{d\omega_{mn}}{dS}\right)^2 + \left(\frac{d\psi_{mn}}{dS}\right)^2\right] = 1 \qquad (24)$$

In Eq. (24), the arc-length 'S' plays a central role in the formulation and Eq. (24) is solved simultaneously with Eq. (20) and (21) by using the Newton-Raphson iteration method. An initial guess at a point $S + \Delta S$ is considered starting from point 'S' as follows:

$$\omega_{mn}(S + \Delta S) = \omega_{mn}(S) + \frac{d\omega_{mn}(S)}{dS} \qquad (25)$$

$$\psi_{mn}(S + \Delta S) = \psi_{mn}(S) + \frac{d\psi_{mn}(S)}{dS} \qquad (26)$$

Therefore, an iteration is carried out to obtain a correct solution. The convergence is assumed by taking sufficiently small $\varepsilon_p(\varepsilon_p < 10^{-8})$ which is defined as

$$\varepsilon_p = \sum_{m=0}^{M}\sum_{n=0}^{N}\left[\left(\omega_{mn}{}^{(p+1)} - \omega_{mn}{}^{(p)}\right)^2 + \left(\psi_{mn}{}^{(p+1)} - \psi_{mn}{}^{(p)}\right)^2\right] \qquad (27)$$

The basic equations and the boundary conditions allow us to get a symmetric solution with respect to the horizontal line passing through the axial direction.

IV. RESULTS

Two different cases were studied attentively to observe the flow behavior in the porous and non-porous media in presence of Dean number and rotation parameter. Firstly, we have shown the effects of the Darcy number and then discussed how rotation parameter contributes in the porous and non-porous media.

A. Effects of the Darcy number (D_a):

Numerical simulation conducted according to the above-mentioned techniques over the range of Darcy number between 0.1 and 1000 in which rotation parameter is considered as constant ($R = 500$) while the Dean number D_n are 100, 500 and 1000 used. The result shows that the flow is becoming rapidly converged after $D_a = 20$ and the similar results have obtained for all cases (See figure 2), however, the behavior has changed due to the pressure gradient. Figure (3-5) illustrated the results of the flow behaviours inside the duct. Secondary flow contains two clear contours which have formed in the centre of the duct for the stream lines while three contours have observed for the vector plot for the small Dean number.

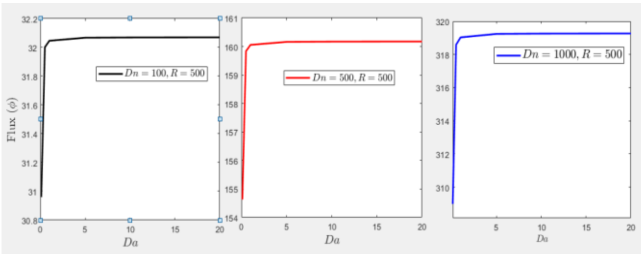

Figure 2: Direct numerical simulation result of the flux in terms of Darcy number at $Da = 100, 500, and\ 1000$

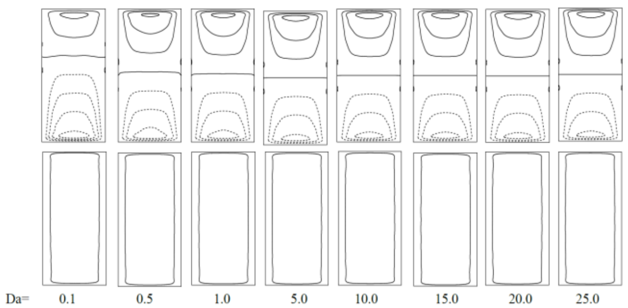

Figure 3: Streamlines of the secondary flow (top) and axial flow (bottom) in each row at different Darcy number where the Dean number and rotation parameter is treated as constant. ($D_n = 100\ and\ R = 500$).

We consider that the number of the contour may increase with the increase of the Darcy number. In this case, we have found three contours in the streamline where plot by increasing of the Darcy number and vice-versa for the vector plot.

Secondary flows have been shown at the top in the figure 4 while the axial flow described in the bottom. The results show that the contours in the secondary flow are increasing by decreasing the Darcy number

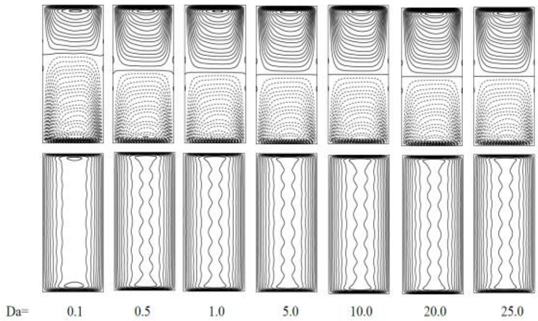

Figure 4: Streamlines of the secondary flow (top) and axial flow (bottom) in each row at different Darcy number where $D_n = 500, R = 500$.

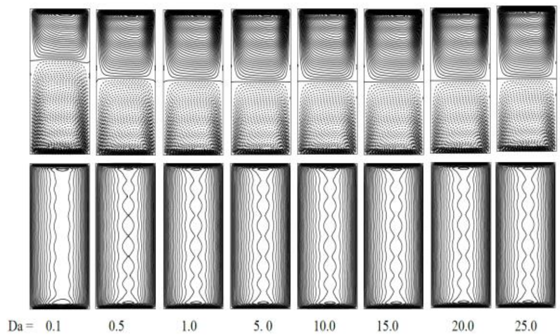

Figure 5: Streamlines of the secondary flow (top) and axial flow (bottom) in each row at different Darcy number where $D_n = 1000, R = 500$.

(D_a). It has also observed that the centre of the flow remains same this mean that the secondary flows turn into a steady state. One can be called this phenomenon as “self-preservation of the flow”. However, axial flows are gradually breaking its streamline and becoming ring shape. Furthermore, we increased the value of Dean number, $\mathrm{D_n} = 1000$ and $R = 500$. We observed that the centre of the secondary flow in the duct has shifted to right wall in the duct and strong effect of pressure also noticed towards the wall due to the large Dean number. The streamline of the axial flow splits into more ring in comparison with the result of $\mathrm{D_n} = 500, R = 500$ and D_a= 0.1 -1000.

B. Effects of the Rotation parameter (R):

The effects of the rotation parameter have been observed in figure 6 where D_n and D_a treated as constant number. Therefore, it has found that the vortex of the centre shifted from centre to the left wall side in the duct. This indicated that flow is weakening as increasing the rotation parameter. Strong vortex appeared in the right side of the wall as gradually increasing of the rotation parameter (R). Also, table 1 shows numerical data to support these plots. Therefore, it is crystal clear that the energy is decreasing by increasing of rotation parameter We have also computed our model by considering $D_a = 1000$ and $D_n = 1000$ where as R is varied gradually towards to higher positive integer. In this case, we found completely different phenomenon in comparison to above case for $D_a = 0.1$. We found two vortices in the left and right corner of the bottom wall in the secondary flows. On the other hand, there is a single and large vortex appeared in the bottom wall of the axial flow at $R = 10$. We also noticed that a little bit less concentrated and big vortex than bottom wall vortex in the upper wall corner of the wall. In addition. this vortex gradually becoming small and concentrated by increasing Rotation parameter. Furthermore, two strong vortices also found at just immediately centre towards the upper wall of the duct. One can consider that there two vortices i.e. one is clockwise and other is anticlockwise. Anticlockwise counter vortex gradually becoming small following to the rotation parameter (R) and shifted to the right-side wall of the duct. Clockwise vortex also shifted towards to the upper wall. This vortex moves to the upper corner of the left wall in the duct and becoming smaller by increasing rotation parameter (R).

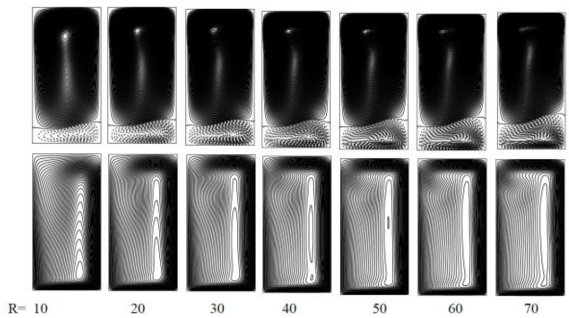

Figure 6: Streamlines of the secondary flow (top) and axial flow (bottom) in each row at different Rotation parameter where $D_n = 1000, D_a = 0.1$.

Rotation Parameter $(\boldsymbol{R})$	Flux × $\mathbf{10^2}$
10	5.20433
20	5.14699
30	5.12338
40	5.05309
50	4.96844
60	4.87969
70	4.79153

Table 1: Effect of Rotation parameter where $D_n = 1000$ and $D_a = 0.1$.

In the second case, we observed that an annular vortex is found in the centre of the duct while a single vortex is visible in the bottom wall. Other vortex is revealed in the upper side of the duct wall. It is very clear to understand that the vortex of the centre is also occupied more place of the duct and shifted to the left side of the wall. Ring shape is also observed in the other side of the wall at $R = 70$. It is assuming that vortex will be disappeared, and the ring shape will be appeared at high value of R.

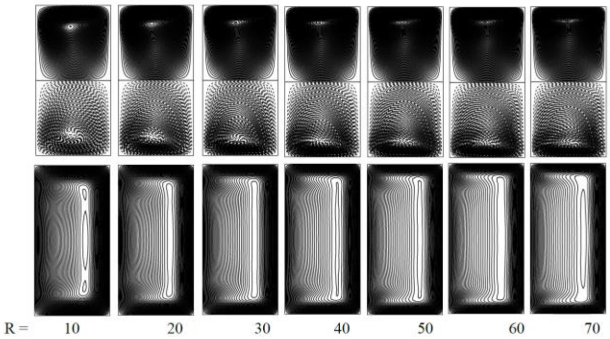

Figure 7: Streamlines of the secondary flow (top) and axial flow (bottom) in each row at different Rotation parameter where $D_n = 1000, D_a = 1000$.

Rotation Parameter ($\boldsymbol{R}$)	Flux × $\mathbf{10^2}$
10	7.15421
20	6.59999
30	6.24584
40	5.97775
50	5.75971
60	5.57534
70	5.41546

Table 2: Effect of Rotation parameter where $D_n = 1000$ and $D_a = 1000$.

V. Conclusion

The fluid flow through a rotating rectangular straight duct in the porous medium has been studied for the constant aspect ratio ($\gamma = 2$) which has been revealed many interesting characteristics of the flow with the physics. The well-constructed mathematical and numerical calculation techniques have been taken to analyses the equations.

According to the results, we have obtained the following important points:

1. Flow is rapidly converged i.e. small effect has been observed at various Darcy numbers where we have treated constant D_n and R. It is also concluded that the change of the flow behaviours mainly are contingent on the Dean number rather than Darcy number D_a. Therefore, it thought that the tendency of the flow in the porous rotated rectangular duct is towards to the steady i.e. self-preservation state.
2. In the case of strong porous media, we found that the anomalous behavior of the vortex. It is also found that vortex towards the upper wall in the axial flow almost vanished at large rotation parameter (R). In addition, we observed that ring shape appeared towards the right wall of the duct due to the rotation parameter.
3. In the case of non-porous media, we found that the main annulus vortex at the centre in the duct became weakened and exhibits like an elliptical ring towards the left wall at large rotation parameter R.
4. Tendency of the vortex at the center in the secondary flow is moved to the right wall of the duct at high Darcy number D_a. It is also found that the vortex of the vector plot is tendency to become more stronger by increasing of rotation parameter R in the strong porous medium.

Finally, we can conclude that the flow energy weakened with the increasing of rotation parameter at constant Darcy number in both porous and non-porous media respectively. Furthermore, we also observed that Darcy number D_a effects provide us to chance to find the self-preservation state inside the duct. This means that the fluid flow could be reached a steady state in the duct with the presence of the Dean number and rotation parameter. Therefore, it also important to study further at low Dean number and rotation parameter to investigate the effect of porous and non-porous media.

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