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State Feedback Plus Integral Error Controller Approach For Robot Arm Control Design

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***Abstract*—In this study, it is demonstrated that, state feedback plus integral error controller offers significant advantages for robot manipulator design. Dynamic model of the entire system is obtained as third order differential equation by taking the dynamic of direct current motor. Linearization process was carried out by finding equations that define minor deviations around the nominal trajectory of the manipulator. In the system, pole placement process was utilized as control design tool. Since linearization coefficients are updated along the trajectory of the manipulator, controller gains are also updated due to pole placement process. Due to interaction between the joints of the manipulator, torque applied by one joint to the other is processed as disturbing input for each joint. Thus, it was possible to apply independent joint control. System performance is demonstrated by making simulations for manipulator with two degrees of freedom.**

Keywords—Robot manipulator control, linearization technique, independent joint control, modeling, simulation

1. INTRODUCTION

In all of the studies carried out on robot manipulator control, it aimed that robot arm follows the required trajectory as close as possible. Also, high resistance to modeling errors and external disturbing inputs are among the leading desired features. Performance of these objectives with the lowest cost is also among the significant design requirements.

Manipulator system is a multi input and multi output system. However, independent input signals are required for each joint position control. Dynamic model of the manipulator system obtained through different methods is not linear and includes related terms defining

interaction between joints. It is known that manipulator dynamic model plays an important role in establishing control strategies [1]. Joints are in interaction with each other due to inertia, centrifugal, Coriolis and gravity loads. Due to these reasons, manipulator control expresses itself as a significant problem which is always studied [2].

In many previous studies on robot control, dynamics of the direct current motor is not taken into consideration in control algorithms. However, particularly in high speed motions, it is known that motor dynamics plays an important role on the system performance. Due to these reasons, manipulator control approaches designed without taking motor dynamic into consideration can be insufficient in real applications. In another common application that is carried out, motor armature inductance is ignored and total model is obtained as second degree again [3].

The most common application to apply independent joint control and to linearize the dynamic model is to eliminate non-linear terms through feedback rings in the system [4]. In other words, by applying non-linear control, the system is separated into linear sub-systems. In order to apply this method efficiently, dynamic model of the system shall be known fully. Many modeling errors and external disturbing forces, dynamic model of the system cannot be determined fully. Therefore, this approach can be insufficient under some circumstances.

A study showing that state feedback plus integral error controller in multiple input - multiple output systems is more advantageous compared to conventional derivative controller [5]. In this study, the controlled process is the transfer function given by Niederlinski.

In the ideal state, when torques which are calculated through reverse dynamic are applied to the joints, the manipulator follows the nominal trajectory. However, due to several disturbing factors, the manipulator shows deviations from the nominal trajectory. Therefore, in order to rectify the deviations, a controller shall be designed and integrated to the system. In a source, introduction of torques which are calculated through reverse dynamic to joints is named as first controller and the controller which is designed to rectify the deviations from the trajectory is named as second controller [6].

In this study, initially the dynamic model of the manipulator system is established as second order vectoral differential equation by using Lanrange equations. This equation is linearized by using Jacobian matrices which define minor deviations around the nominal trajectory of the manipulator.

Therefore, linearization coefficients are updated throughout the trajectory. Then, dynamic model of the entire system is found by combining linear dynamic model of the manipulator system with dynamic equations of the direct current motor. The obtained third order differential equation is applicable for minor deviations from the nominal trajectory required to be followed by the manipulator. Control strategy used in this study can be separated into two stages. The first stage is to calculate the torques required for enabling the manipulator to follow the nominal trajectory through reverse dynamic and apply them on the joints. The manipulator deviates from the nominal trajectory due to various disturbing internal and external factors. Linearized dynamic model of the system which is applicable for such minor deviations from the trajectory, applies corrective torques to the joints to correct deviations using state feedback plus integral error control strategy. This function of the controller can be considered as the second stage. Pole placement process was used as the controller design tool. Torques calculated for joints are updated throughout the trajectory followed by the manipulator through linearization coefficients, controller gains and reverse dynamic in small time periods. Due to interaction between joints, torque applied by one joint to another is processed as disturbing input signal for each joint. The controller that is used is resistant to major disturbing forces due to sequential feedback network. Thus, by applying independent joint control, high system performance was realized.

2. MANIPULATOR KINEMATICS

Manipulator system with two degrees of freedom is seen on Fig. 1. O_2 point on the figure is assumed to be stationary. Direct current motors are located at O and O_1 points. The manipulator consists of two rotating joints and two series link. At first, basic and local coordinate systems are placed on the manipulator for kinematics analysis. Basic coordinate system is located at O point and it is stationary. Local coordinate systems are located at O_1 and O_2 points. Joints rotate around z axis perpendicular to the paper plane.

Position and direction of the hand on the robot arm can be determined according to the basic coordinate system. Transformation matrices define the relationship between neighbor links.

Position and direction of the hand can be calculated according to the basic coordinate system by using these matrices. Similarly, by using the reverse of the transformation matrices, values of the joint variables can be found. In other words, when the robot arm is at any position, angle values to be achieved by θ_1 and θ_2. This process is referred to as reverse kinematics. Kinematics parameters and transformation matrices are determined by using 'Denavit Hartenberk' demonstration.

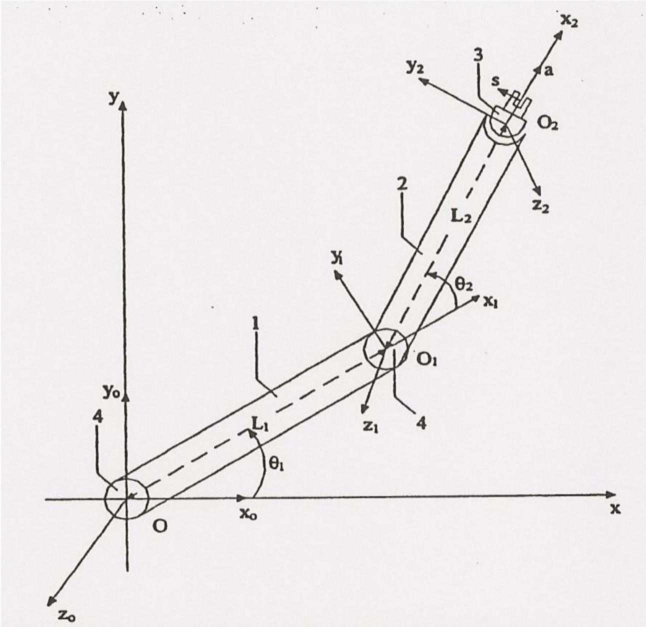

Figure 1. Manipulator with two rotating joints
1. First link 2. Second link 3. Hand 4. Motor

Equality 1.1 demonstrates the numerical values of the structural parameters of the manipulator and gravitational acceleration. Here, M_1 and M_2 define the mass of first and second links respectively, L_1 and L_2 defines lengths of the links.

$$M_1=M_2= 5 \text{ kg},\ L_1=L_2=L= 0.5 \text{ m} \quad g=9.81 \text{ m/sec}^2 \qquad (1.1)$$

3. MANIPULATOR DYNAMICS

A manipulator system with n freedom degree can be defined as follows as second order vectoral differential equation through Langrange equations.

$$T= D(q)\ddot{q} + h(q, \dot{q})\dot{q} + c(q) \qquad (2.1)$$

Here, T matrix at the dimension of nx1, defines generalized torque acting on the joints. D matrix is at the dimension of nxn and demonstrates the effect created by the acceleration of inertia masses. Matrix h is at nx1 dimension and defines the torque acting on joints due to gravity. q symbol defined as generalized coordinate is at nx1 dimension and represents angular displacement of joints. Accordingly, $\ddot{q}$ and $\dot{q}$ represents angular acceleration ($\ddot{q}$) and angular speed ($\dot{q}$) respectively.

$$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$
(2.2)

Equality 2.2 demonstrates the dynamic model of the manipulator.

$$D_{11} = (1/3)L^2(M_1+4M_2+3M_2\cos(\theta_2)) \quad (2.3)$$
$$D_{12} = (1/3)M_2L^2 + (1/2)M_2L^2\cos(\theta_2) \quad (2.4)$$
$$D_{21} = (1/3)M_2L^2 + (1/2)M_2L^2\cos(\theta_2) \quad (2.5)$$
$$D_{22} = (1/3)M_2L^2 \quad (2.6)$$

$$h_2 = (1/2)L^2\sin(\theta_2)M_2\dot{\theta}_1\dot{\theta}_1 \quad (2.7)$$
$$h_1 = -(1/2)L^2\sin(\theta_2)M_2\dot{\theta}_2(\dot{\theta}_2+2\dot{\theta}_1) \quad (2.8)$$

$$c_1 = (1/2)gL(M_1\cos(\theta_1)+M_2\cos(\theta_1+\theta_2)+2M_2\cos(\theta_1)) \quad (2.9)$$
$$c_2 = (1/2)gLM_2\cos(\theta_1+\theta_2) \quad (2.10)$$

3. LINEARIZATION OF THE DYNAMIC MODEL

Dynamic model of the manipulator can be defined in first order vectoral differential equation form as follows.

$$\dot{x} = f[x(t),u(t)] \quad (3.1)$$

In this equation, x vector which is at the dimension of nx2 are state variables which define angular displacements and derivatives for manipulator system. Vector u is at n dimension and define torques acting on the joints due to motion. Currently, control input for manipulator system without taking motor equations into consideration, can be seen as revealing torques (u) suitable for joints for the required trajectory to be followed by the manipulator. Nominal trajectory (x_n) is obtained through application of nominal torques (u_n) on the joints. Dynamic model of the system for this case is described as follows. δx(t) and δu(t) express the deviations from the nominal trajectory.

$$\dot{x}_n = f[x_n(t),u_n(t)] \quad (3.2)$$

$$\delta x(t) = x(t) - x_n(t) \quad (3.3)$$

$$\delta u(t) = u(t) - u_n(t) \quad (3.4)$$

If equations which define the deviations are placed to its position in system state variables equation and if the right part of the obtained equation is opened with Taylor series, linear dynamic model of the manipulator system is obtained for minor deviations. (∂f/∂x) and (∂f/∂u)refer to the Jacobian matrices.

(3.5)

$$d/dt(x_n+\delta x) = f[x_n(t)+\delta x(t), u_n(t)+\delta u(t)]$$

$$f[x_n(t)+\delta x(t),u_n(t)+\delta u(t)] = f[x_n(t),u_n(t)]+[(\partial f/\partial x)_{xn,un}]\delta x + [(\partial f/\partial u)_{xn,un}]\delta u + \ldots \quad (3.6)$$

$$\delta\dot{x} = [(\partial f/\partial x)_{xn,un}]\delta x + [(\partial f/\partial u)_{xn,un}]\delta u \quad (3.7)$$

$$\delta\dot{x} = A(x_n,u_n)\delta x + B(x_n,u_n)\delta u \quad (3.8)$$

Dynamic model linearized for a manipulator with two freedom degrees is shown below to constitute an example. a and b coefficients shown in the following equations represent the elements of Jacobian matrices. Therefore, these coefficients are updated along the trajectory followed by the manipulator system.

$$x_1 = \theta_1,\ x_2 = \theta_2,\ x_3 = \dot{\theta}_1,\ x_4 = \dot{\theta}_2,\ T_1 = u_1,\ T_2 = u_2 \quad (3.9)$$

$$\delta\dot{x}_1 = \delta x_3 \quad (3.10)$$
$$\delta\dot{x}_2 = \delta x_4 \quad (3.11)$$
$$\delta\dot{x}_3 = a_{31}\delta x_1 + a_{32}\delta x_2 + a_{33}\delta x_3 + a_{34}\delta x_4 + b_3\delta u_1 \quad (3.12)$$
$$\delta\dot{x}_4 = a_{41}\delta x_1 + a_{42}\delta x_2 + a_{43}\delta x_3 + a_{44}\delta x_4 + b_4\delta u_2 \quad (3.13)$$
$$\delta\ddot{\theta}_1 = a_{31}\delta\theta_1 + a_{32}\delta\theta_2 + a_{33}\delta\dot{\theta}_1 + a_{34}\delta\dot{\theta}_2 + b_3\delta T_1 \quad (3.14)$$
$$\delta\ddot{\theta}_2 = a_{41}\delta\theta_1 + a_{42}\delta\theta_2 + a_{43}\delta\dot{\theta}_1 + a_{44}\delta\dot{\theta}_2 + b_4\delta T_2 \quad (3.15)$$

$$\delta x = x - x_n \quad (3.16)$$

$$\ddot{\theta}_1 = a_{31}\theta_1 + a_{32}\theta_2 + a_{33}\dot{\theta}_1 + a_{34}\dot{\theta}_2 + b_3u_1 \quad (3.17)$$
$$-a_{31}\theta_{n1} - a_{32}\theta_{n2} - a_{33}\dot{\theta}_{n3} - a_{34}\dot{\theta}_{n4} b_3u_{n1} + \ddot{\theta}_{n1} \quad (3.18)$$
$$c = -a_{31}\theta_{n1} - a_{32}\theta_{n2} - a_{33}\dot{\theta}_{n3} - a_{34}\dot{\theta}_{n4} - b_3u_{n1} + \ddot{\theta}_{n1} \quad (3.19)$$
$$\ddot{\theta}_1 = a_{31}\theta_1 + a_{32}\theta_2 + a_{33}\dot{\theta}_1 + a_{34}\dot{\theta}_2 + b_3u_1 + c \quad (3.20)$$

$$\ddot{\theta}_2 = a_{41}\theta_1 + a_{42}\theta_2 + a_{43}\dot{\theta}_1 + a_{44}\dot{\theta}_2 + b_4u_2$$
$$-a_{41}\theta_{n1} - a_{42}\theta_{n2} - a_{43}\dot{\theta}_{n3} - a_{44}\dot{\theta}_{n4} - b_4u_{n2} + \ddot{\theta}_{n2} \quad (3.21)$$
$$d = -a_{41}\theta_{n1} - a_{42}\theta_{n2} - a_{43}\dot{\theta}_{n3} - a_{44}\dot{\theta}_{n4} - b_4u_{n2} + \ddot{\theta}_{n2} \quad (3.22)$$
$$\ddot{\theta}_2 = a_{41}\theta_1 + a_{42}\theta_2 + a_{43}\dot{\theta}_1 + a_{44}\dot{\theta}_2 + b_4u_2 + d \quad (3.23)$$
(3.24)

4. MODELING OF DIRECT CURRENT MOTOR

Dynamic equations of the direct current motor are defined with the following equations.

$$L_a(di_a/dt) = -R_ai_a + e_a - e_b,$$

$$d\theta/dt = \omega,$$

$$e_b = K_b\omega,$$

$$T_m = K_i i_a \quad (4.1)$$

L_a=Armature inductance = 0.005 Henry
i_a = Armature current (amp)
R_a = Armature resistance = 1 ohm
e_a = Armature voltage (V)

e_b = Feedback voltage (V)
K_b = Feedback voltage coefficient = 0.1 V sec/rad
K_i = Current-torque coefficient = 10 N.m/amp
T_m = Torque produced by the motor (N.m)

Θ = Angular displacement of the motor shaft (rad)
Ω = Angular speed of the motor shaft (rad/sec)

Dynamic equations of the direct current motor and dynamic model of the manipulator system can be combined and the equality defining the total torque acting on the joints can be presented as follows.

$$T_m + T_D = J\ddot{\theta} + B_t\dot{\theta} + T \qquad (4.2)$$

J = 0.1 N.m.sec^2/rad
B_t = 0.01 N.m.sec/rad
This presented equality defines the total torque balance for any joint in the manipulator system. Here, T_D defines disturbing torque acting on the joint, J defines polar inertia moment of the motor, B_t (N.m.sec/rad) defines total viscous friction coefficient, T is the torque determined by using linearized dynamic model of the manipulator. Θ defines the angular displacement of the motor and the joint. In this equality teeth number rate between motor and manipulator joint is taken 1. T symbol which is determined by using linearized equation (equality 3.8) and which defines the torque acting on the joint due to manipulator motion, includes linearization coefficients, dynamics of other joints and torques calculated through reverse dynamic. For instance, by using linear dynamic equation determined in Equality 3.20 for the first joint of the manipulator with two freedom degrees, torque acting on this joint due to motion can be presented as follows.

$$[\ddot{\theta}_1 - (a_{31}\theta_1 + a_{32}\theta_2 + a_{33}\dot{\theta}_1 + a_{34}\dot{\theta}_2 + c)]/\, b_3 = u_1 = T_1 \qquad (4.3)$$

In this equality, c coefficient defines the torque which includes its dynamic in two joints and which shall be applied on the first joint thorough reverse dynamic. This function represents the first stage of the control defined in the introduction section. This torque and linearization coefficients are updated along the trajectory of the manipulator. Thus, joints are partially separated from each other. However, second stage of the control system that corrects the deviations from the trajectory is interdependent due to linear equation. This case can be seen in Equality 4.3. This interaction is processed as disturbing input for each joint in the designed control system. This way, independent joint control can be applied. Thus, characteristic function of each joint is calculated separately in the system and pole placement process is carried out. Poles are placed on negative real axis on s plane and repetitive pole is used. Each joint has sixth degree characteristic function. Therefore, six repetitive poles of each joint are placed on negative real axis. Block diagram of the control system for manipulator with two freedom degrees is shown in Figure 2.

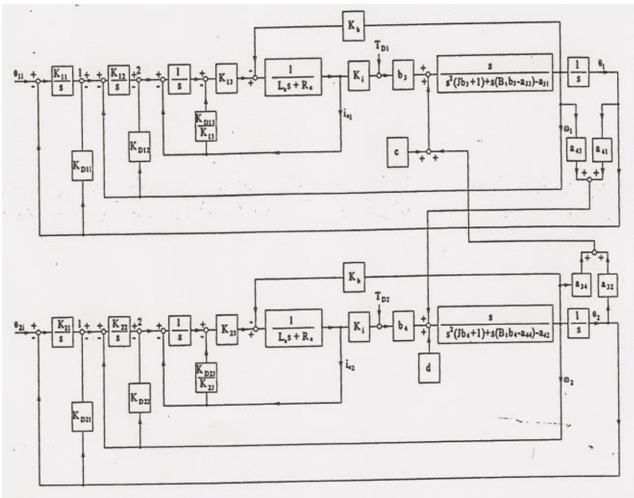

Figure 2. Control block diagram of the manipulator system

5. SIMULATIONS

Performance of the designed control system is evaluated by making simulations of the responses given by the manipulator with two freedom degrees to trajectory inputs. A software updating the linearization coefficients and controller gains along the trajectory followed by the manipulator was designed in Matlap program. In the simulations, six repetitive poles of each two joints were placed on -500 point on the negative real axis. Parameter updating count is realized 100 times throughout the trajectory. Maximum accelerations which the first and second joint of the manipulator are required to reach in the trajectory plan are determined as 70rad/sec^4 and 40rad/sec^4. In Fig. 3, angular acceleration-time diagram which is established in the trajectory plan and the manipulator joints are required to follow, are shown. In Fig. 4, acceleration-time graphics followed by the manipulator joints are shown. In graphics which are defined with dotted lines in Fig. 5 and 6 are angular positions and speeds determined in the trajectory plan. Graphic defined with continuous line shows the angular position followed by the manipulator and speeds.

To evaluate the resistance of the manipulator against the disturbing inputs, 1000 Nm ramp disturbing input is applied to each joint and simulations for the same trajectory plan and parameters are created. In Fig. 7, position-time responses for both joints are shown.

Acceleration (rad/sec2) Time (sec) a. Acceleration-time graphic of the first joint	Acceleration (rad/sec2) Time (sec) b. Acceleration-time graphic of the second joint

Figure 3. Acceleration-time diagrams established in the trajectory plan of the joints.

Acceleration (rad/sec2) Time (sec) a. Acceleration-time graphic of the first joint	Acceleration (rad/sec2) Time (sec) b. Acceleration-time graphic of the second joint

Figure 4. Acceleration-time diagrams followed by manipulator joints.

Position (rad) Time (sec) a. Position-time response	Speed (rad/sec) Time (sec) b. Speed-time response

Figure 5. Time response of the first joint.

Position (rad) Time (sec) a. Position-time response	Speed (rad/sec) Time (sec) b. Speed-time response

Figure 6. Time response of the second joint

Position (rad) Time (sec) a. Position-time response of the first joint	Speed (rad/sec) Time (sec) b. Position-time response of the second joint

Figure 7. Time Position-time responses of the joints against 100 Nm disturbing ramp inputs

6. RESULTS

Controller structure designed in this study meets the most significant design conditions desired for robot manipulator control. These can be listed as follows.

a. Independent joint control is applied by reducing the interaction between the joints.
b. High resistance is ensured against modeling errors and external disturbing inputs since, the controller that is used has sequential feedback network.
c. As can be seen in the simulations, in high acceleration disturbing inputs (50 rad/sec^2) no disturbance is observed in the system behavior.
d. The controller that is used ensures limitation of the state variables. Therefore, by limiting the currents drawn by direct current motors, smaller, cheaper and lighter motors can be used for manipulator design [7].

SYMBOLS

L_a = Armature inductance (Henry)
i_a = Armature current (amp)
R_a = Armature resistance (ohm)
e_a = Armature voltage (V)
e_b = Feedback voltage (V)
K_b = Feedback voltage coefficient (V.sec/rad)
K_i = Current-torque coefficient (N.m/amp)
T_m = Torque produced by the motor (N.m)
Θ = Angular displacement of the motor shaft (rad)
ω = Angular speed of the motor shaft (rad/sec)

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