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Internal Pressure And Speed Of Sound In Neutron Stars

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***Abstract*—We employ theoretical technics using the Equation of State for neutron stars to obtain the internal pressure and velocity of sound in neutron stars. We confirm the gaseous state of a neutron star at low density and crystalline state at high density. The lowest density that can sustain gravitational stability is found to be 1.598×10^8 g/cm^3 at a pressure of 5.0×10^8 Pa with parameters a=13, α=0.49, b=3.21, β=2.47. A neutron star is considered to be the collapsed core of large solar masses. Other theories describe neutron stars as the corpses left behind after certain normal stars explode as supernovae. According to standard astronomical models, matter is squeezed down so tightly within their cores that nuclei break apart into their constituents, with protons and electrons crushed together into neutrons. In reality, astronomers know little about what happens to matter at such high densities hence the need for theoretical approach in studying the properties of these stars. Understanding the properties of neutron-rich matter could assist in obtaining heavy element synthesis.**

Keywords—Neutron star, Pressure, Density, Force Parameters

1. Introduction

The study of neutron stars has become very important in recent years. The discovery of the first two solar-mass neutron stars (Demorest at al, 2010) has provided critical constraints on the dense matter equation of state. These observations have eliminated the whole classes of models that predicted that the critical mass of a neutron star is $\approx M_{\oplus}$, where $M_{\oplus}$ is the mass of the sun. Combined with the recent observations of the massive neutron stars, the attractive nature of neutron-neutron interaction at low momenta means that the equation of state must be soft at low density, with a rapid transition to high pressure when the higher momentum of neutron-neutron and many body interactions may become important, particularly three neutron and higher interactions. It is important to know that the inner crust of a neutron star is inhomogeneous neutron matter and may be composed of lattice heavy-rich nuclei. Understanding the properties of neutron-rich matter could assist in obtaining heavy element synthesis. Ultimately, the objective is the observation of gravitational waves that can result from the merger of neutron stars (Bauswein et al, 2012), and hence provide direct evidence of the structure of neutron matter and neutron stars.

Some of the high-lights of the neutron star namely, masses, radii and the equation of state (EOS) are now known (Gandolfi et al 2015). Neutron stars are now considered as reservoirs of high density fermions, and these systems are now the largest of their type in the universe. A neutron star is considered to be the collapsed core of large (10-29) solar masses. Neutron stars are the smallest and most dense systems known to exist so far. The typical size of a neutron star is of the order of 10 Km in radius but the mass could be twice or more than that of the sun (Gezerlis, et al 2015). Since the mass is very large and the radius is mall, the density of such stars is very large of the order of ten times ($\rho_n \cong \rho_s$) the saturation density of a heavy nucleus. Hereρ_n is the density of a neutron star and ρ_s is the saturation density of heavy nucleus. Consequently, there could exist strong nucleon-nucleon interactions between the neutrons, and hence a number of different faces can occur.

After the birth of the star, temperatures in the interior of the neutron star falls below a billion degrees Kelvin in less than a year (Wikipedia, 2017). Such temperatures may look high but they are low compared to the characteristic energies such as the Fermi energy, which, for involved nuclear density are of the order of 10-100MeV. This energy corresponds to temperatures of the order of 10^{11}-10^{12}K (kT= energy, k is Boltzmann constant). It is now understood that in the inner crust of a neutron star, the neutrons paired in $^1S_0$ (singlet) state co-exist with a lattice of neutron-rich nuclei; in fact, super fluid neutrons co-exist with a crystal lattice of neutron-rich nuclei and an electron gas.

Another property of matter that determines the state of matter from gaseous to crystalline state is the density of the system. In the case of a neutron star and its crust, it is well known that it is in a crystal state for a wide range of mass densities and temperatures (Rogers, 1964). The density of a neutron star can vary from $1.67 \times 10^{11} g/cm^3$(which is considered as low density) to $3.7 \times 10^{15} g/cm^3$(high density) which is supposed to be the density of the neutron matter at which there may be the onset of the solid phase (Murunga et al, 2017).

2. Theoretical calculations

2.1 Pressure inside the neutron star

Pressure in the neutron star can be expressed in terms of the derivative of energy as;

$$P = \rho_n^2 (\frac{dE}{d\rho_n}) = \rho_n^2 \left[\frac{dE(\rho_n)}{d\rho_n}\right] \quad 2$$

Here $E(\rho_n)$ is the energy per neutron as a function of the neutron density and can be obtained from the equation of state (EoS) for the neutrons.

The EoS calculated using the QMS is given as (Stefano et al, 2015);

$$E(\rho_n) = a\left(\frac{\rho_n}{\rho_0}\right)^{\alpha} + b\left(\frac{\rho_n}{\rho_0}\right)^{\beta} \quad 3$$

Here, ρ_0 is the nuclear saturation density. Generally, we can write as a particular example;

$\rho_n = 10\rho_0$. The constants, a, b, α and β are the fitting parameters. Their values are given in table 1 below;

a (Mev)	α	b (MeV)	β
12.7	0.49	1.78	2.26
12.7	0.48	3.45	2.12
12.8	0.488	3.19	2.20
13.0	0.49	3.21	2.47
12.6	0.475	5.16	2.12
13.0	0.50	4.71	2.49
13.4	0.514	5.62	2.436

Table 1: The fitting parameters values corresponding to different 3N forces.

The derivative of equation 3 is obtained as;

$$\frac{dE}{d\rho_n} = \frac{\alpha a}{\rho_0^{\alpha}}\rho_n^{(\alpha-1)} + \frac{\beta b}{\rho_0^{\beta}}\rho_n^{(\beta-1)} \quad 4$$

On substituting eqn. 4 into eqn. 2, we obtain pressure in the neutron star in terms of neutron density as;

$$P = \rho_n^2\left(\frac{\alpha a}{\rho_0^{\alpha}}\rho_n^{(\alpha-1)} + \frac{\beta b}{\rho_0^{\beta}}\rho_n^{(\beta-1)}\right) \quad 5$$

For EoS of the form of eq. 3, energy density of the neutron star can be given as (Stefano et al, 2015);

$$\in = \rho_0\left[a\left(\frac{\rho}{\rho_0}\right)^{1+\alpha} + b\left(\frac{\rho}{\rho_0}\right)^{1+\beta} + m_n\left(\frac{\rho}{\rho_0}\right)\right] \quad 6$$

The derivative of eq. 6 is obtained as;

$$\frac{\partial \in}{\partial \rho} = \rho_0\left[\frac{a(1+\alpha)}{\rho_0^{(1+\alpha)}}\rho^{\alpha} + \frac{(1+\beta)b}{\rho_0^{(1+\beta)}}\rho^{\beta} + \frac{m_n}{\rho_0}\right] \quad 7$$

Likewise, the Pressure inside the neutron star can be obtained as;

$$P = \rho_0\left[a\alpha\left(\frac{\rho}{\rho_0}\right)^{1+\alpha} + b\beta\left(\frac{\rho}{\rho_0}\right)^{1+\beta}\right] \quad 8$$

2.2 Speed of sound in neutron stars

The derivative of eq. 8 is;

$$\frac{\partial P}{\partial \rho} = \rho_0\left[\frac{a\alpha(1+\alpha)}{\rho_0^{(1+\alpha)}}\rho^{\alpha} + \frac{b\beta(1+\beta)}{\rho_0^{(1+\beta)}}\rho^{\beta}\right] \quad 9$$

The velocity of sound in the neutron star can be calculated using eq. 7 and eq. 9 by applying chain rule as;

$$c_s^2 = \frac{dP}{d\epsilon} = \frac{dP}{d\rho}\cdot\frac{d\rho}{d\epsilon} = \frac{\rho_0\left[\frac{a\alpha(1+\alpha)}{\rho_0^{(1+\alpha)}}\rho^\alpha + \frac{b\beta(1+\beta)}{\rho_0^{(1+\beta)}}\rho^\beta\right]}{\rho_0\left[\frac{a(1+\alpha)}{\rho_0^{(1+\alpha)}}\rho^\alpha + \frac{(1+\beta)b}{\rho_0^{(1+\beta)}}\rho^\beta + \frac{m_n}{\rho_0}\right]} \quad 10$$

Equation 10 can quite easily be evaluated to obtain;

$$c_s^2 = \frac{\rho_0\left[a\alpha(1+\alpha)\rho^\alpha\rho_0^{(1+\beta)} + b\beta(1+\beta)\rho^\beta\rho_0^{(1+\alpha)}\right]}{a(1+\alpha)\rho^\alpha\rho_0^{(1+\beta)}\rho_0 + b(1+\beta)\rho^\beta\rho_0^{(1+\alpha)}\rho_0 + m_n\rho_0^{(1+\alpha)}\rho_0^{(1+\beta)}} \quad 11$$

Or;

$$c_s = \sqrt{\frac{\rho_0[K_1\rho^\alpha + K_2\rho^\beta]}{K_3\rho^\alpha + K_4\rho^\beta + K_5}} \quad 12$$

In eq. 12,

$$K_1 = a\alpha(1+\alpha)\rho_0^{(1+\beta)},$$

$$K_2 = b\beta(1+\beta)\rho_0^{(1+\alpha)},$$

$$K_3 = a(1+\alpha)\rho_0^{(1+\beta)}\rho_0,$$

$$K_4 = b(1+\beta)\rho_0^{(1+\alpha)}\rho_0,$$

$$K_5 = m_n\rho_0^{(1+\alpha)}\rho_0^{(1+\beta)} \quad 13$$

3. Results and discussion

3.1 Pressure variation with density of the neutron star

Using the fitting parameters of table 1 in equation 5, we obtain the variation of pressure and density of the neutron star for each set of parameters as shown in the equations and graphs that follow;

$$P_1 = \rho_n^2[(1.97183\times10^{-5})\rho^{-0.51} + (1.74257\times10^{-25})\rho_n^{1.26}]$$

$$P_2 = \rho_n^2[(2.50116\times10^{-5})\rho^{-0.52} + (1.18031\times10^{-23})\rho_n^{1.12}]$$

$$P_3 = \rho_n^2[(2.08422\times10^{-5})\rho_n^{-0.512} + (1.43297\times10^{-24})\rho_n^{1.2}]$$

$$P_4 = \rho_n^2[(2.01841\times10^{-5})\rho_n^{-0.51} + (1.5104\times10^{-27})\rho_n^{1.47}]$$

$$P_5 = \rho_n^2[(2.79431\times10^{-5})\rho_n^{-0.525} + (1.76533\times10^{-23})\rho_n^{1.12}]$$

$$P_6 = \rho_n^2[(1.59058\times10^{-5})\rho_n^{-0.5} + (1.33247\times10^{-27})\rho_n^{1.49}]$$

$$P_7 = \rho_n^2[(1.17379\times10^{-5})\rho_n^{-0.486} + (6.27879\times10^{-27})\rho_n^{1.436}]$$

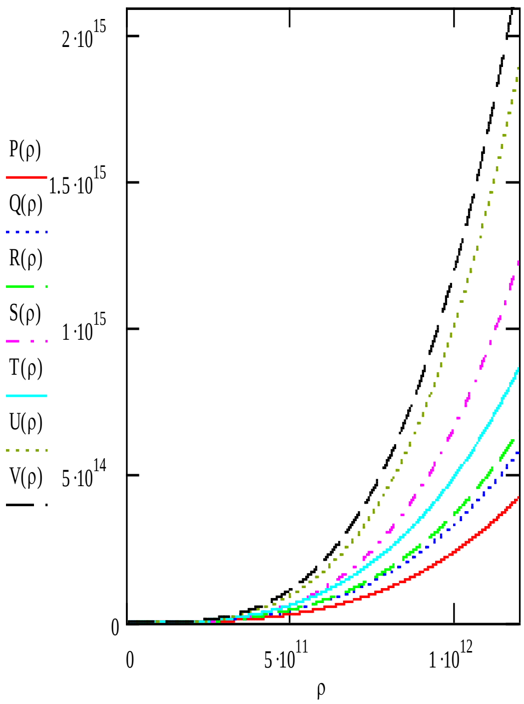

Figure 1: Graphs showing variation of internal pressure with the density of the neutron star. $P(\rho)\rightarrow P_1, Q(\rho)\rightarrow P_2, R(\rho)\rightarrow P_3, S(\rho)\rightarrow P_4, T(\rho)\rightarrow P_5, U(\rho)\rightarrow P_6, V(\rho)\rightarrow P_7$. *Density is in g/cm³*

For each set of fitting parameters, pressure rises exponentially with density. In the soft limit(low density; 0-5×10¹¹g/cm³), the lowest density that can sustain gravitational stability is found to be 1.598×10⁸ g/cm³ at a pressure of 5.0×10⁸ Pa (P_4) with parameters a=13, α=0.49, b=3.21, β=2.47. The highest pressure that can maintain this density is 2.09×10⁸ Pa (P_6). With rising pressure, density of each state approaches saturation where a very large change in pressure produces minimal change in density. At the onset of saturation we expect that the neutron star is in crystalline state. A neutron star with fitting parameters a=13.4, α=0.514, b=5.62, β=2.436 (P_7) shows early crystallization while the one with a=12.7, α=0.49, b=1.78, β=2.26 (P_1) reaches this state much later. In the soft regime, there is an attractive two-body neutron-neutron interaction while the hard regime or crystalline state is reached when very strong nucleon-nucleon many-body interactions set in. Our results show that the crystalline state is reached much earlier ($\rho = 1.0 \times 10^{12} g/cm^3$) than predicted by Murunga-3.7 $\times 10^{15} g/cm^3$, (Murunga, et al., 2017). Our findings are supported by Bauswein (Bauswein et al., 2013) who suggests that during the collision of two neutron stars only a small fraction of the system mass ≈0.1% - 1% remain unbound and is expelled in dynamical time scale of milliseconds.

3.2 Speed of sound as a function of density in the neutron star

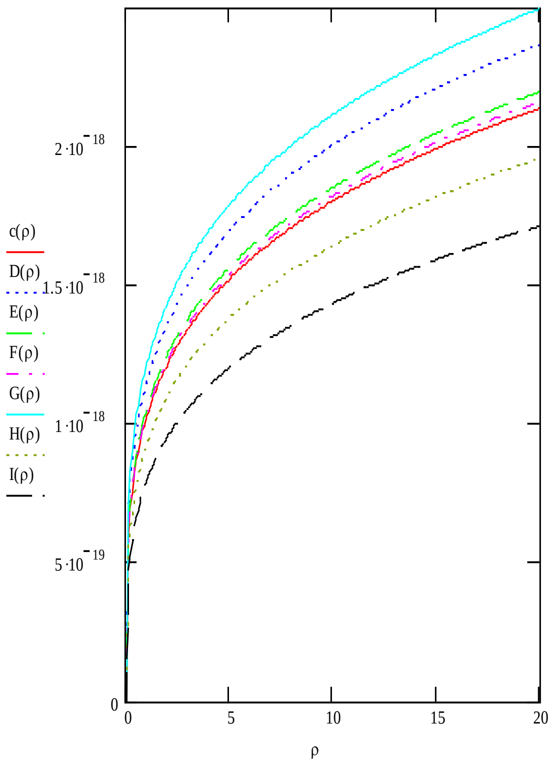

Figure 3: Variation of the speed of sound c_s in the neutron star with density ρ at various fitting parameters.($c(\rho) \rightarrow c_{s1}, D(\rho) \rightarrow c_{s2}, E(\rho) \rightarrow c_{s3}, F(\rho) \rightarrow c_{s4}, G(\rho) \rightarrow c_{s5}, H(\rho) \rightarrow c_{s6}, I(\rho) \rightarrow c_{s7}$)

Figure 3 shows variation of speed of sound in the neutron star at projected very low densities (0-20 g/cm^3). An increase in speed of sound with density of the inner core of the star is declared. Parameters a=12.6, α=0.475, b=5.16, β=2.12 (c_{s5}) give the highest speed while a=13.4, α=0.514, b=5.62, β=2.436 give the lowest speed (c_{s7}) . Highest speeds are experienced at high densities while at low density the speed is low. This predicts well the structure of a neutron star and suggests a gaseous state at low density and a solid or crystalline state at high density since the speed of sound is low in gases and high in solids. At low density, the star is expected to be in a dilute gaseous state with a two-nucleon interaction characterized by s-wave scattering taking pre-eminence (Murunga et al., 2017). The system takes the shape of a superfluid with superconducting properties well explained by high-Tc superconducting theories (Rapando et al.,2016). In such a system, sound transport is poor.

4. Conclusion

In this research, we have confirmed the gaseous nature of a neutron star at low density and the crystalline state at high density. In particular, the change of state is predicted to occur at a density of 5×10^{11}g/cm^3 when pressure per unit density starts to rise sharply and the speed of sound in the star is high with a small rate of change per unit density. The lowest density that can sustain gravitational stability is found to be 1.598×10^8 g/cm^3 at a pressure of 5.0×10^8 Pa with parameters a=13, α=0.49, b=3.21, β=2.47.

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