

Splitting And Vibrating Phenomenon Of Optical Solitons In The Optical Secure Communication Model

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Abstract—The propagation properties with different initial pulse are considered in the Optical fiber secure communication model. Results show that the optical soliton can propagate stably under small perturbation parameter. Furthermore we consider the parameters' role in the propagation of soliton in the perturbed system. Through varying the dispersive parameter, solitons vibrate sharply. Through varying the nonlinear parameters, the soliton is split into two, three or more solitons in the propagation.

Keywords— *Perturbed NLS equation; Optical soliton; Splitting; Vibrating; Secure communication*

I. INTRODUCTION

The nonlinear Schrödinger (NLS) equation

$$i \frac{\partial q}{\partial x} + \beta \frac{\partial^2 q}{\partial t^2} + \gamma |q|^2 q = 0 \quad (1)$$

is extensively used in many practical applications such as nonlinear fiber optics, optics communications, plasma theory [1-6]. For the cases of communication models, the NLS equation is an excellent option. The NLS equation admits optical solitons which can achieve ultra-long-distance and large-capacity communications. Here x represents the propagation distance, t represents the normalized time, as well as β and γ are real valued constants, which denote the coefficients of dispersion and nonlinearity, respectively. The dependent variable function $q(x, t)$ is the complex envelope of optical soliton pulse. In a general way, Eq. (1) is a nonlinear partial differential equation, which is completely integrable on the infinite line or periodic boundary conditions in one dimension. Many researches have focused on the properties of the nonlinear Schrödinger equation such as soliton interactions [7, 8], collision of optical solitons [9, 10] and some specific behaviors connecting with specific effects higher order dispersion and nonlinearity [11-14]. Effects of third-order dispersion and higher order dispersion on soliton in unperturbed NLS equations have been researched in [2, 15].

What's more interesting, Eq.(1) admits unusual waves which are called rogue wave. Rogue waves are spontaneous nonlinear waves with amplitudes significantly larger than the surrounding average wave crests and appear from nowhere and disappear without a trace[16]. Their relevant attributes have been investigated by many researchers. Multi-rogue waves and rational solutions of the coupled nonlinear Schrödinger equations have been studied in [17]. Schober investigated rogue waves by Melnikov analysis and inverse spectral analysis [18].

The aims of this paper lie in the two following points. The first one is that we want to know the propagation stability for optical solitons under small perturbation. The second one is to analyze the parameters' sensitivity, that is, we will discuss the parameters' roles on the propagation. Through analyzing the parameters' sensitivity of perturbed system, we can get a series of reasonable parameters to guarantee the propagation of soliton stably.

In this paper, we consider the perturbed NLS equation

$$i \frac{\partial q}{\partial x} + \beta \frac{\partial^2 q}{\partial t^2} + \gamma |q|^2 q = kq, \quad (2)$$

where $k(>0)$ is the perturbed parameter. It is noted equation (2) is difficult to be solved analytically, so the numerical method is applied. Based on the simplicity, flexibility and relatively modest computing cost split-step Fourier method [2], NLSE problem can be solved. According to the split-step Fourier method, the propagation of the optical pulses from x to $x+h$ is carried out in two steps, where h is a small distance.

In the first step from x to $x + \frac{h}{2}$. Nonlinearity acts alone, while in the second step from $x + \frac{h}{2}$ to $x+h$, only the linearity terms act alone.

Our paper is organized as follows. In Section 2, we study the propagation stability under small perturbation. In Section 3, we consider parameter' sensitivity of the perturbed system. Conclusions are present in Section 4.

II. ANALYSIS ON PROPAGATION STABILITY WITH DIFFERENT INPUT POWERS

In general, system given by Eq. (2) can be split into a linear and a nonlinear part by split-step Fourier method,

$$\frac{1}{2} \frac{\partial q}{\partial x} = i\gamma |q|^2 q - ikq, \quad (3)$$

$$\frac{1}{2} \frac{\partial q}{\partial x} = -i\beta \frac{\partial^2 q}{\partial t^2}. \quad (4)$$

In the first step, $|q|^2$ is regarded as invariable.

Eq. (3) can be exactly solved and the iterative scheme can be described as

$$q(t, x + \frac{h}{2}) = q(t, x) \exp(i\gamma |q|^2 h - ikh). \quad (5)$$

Taking the Fourier transformation of (5), we have

$$Q(w, x + \frac{h}{2}) = F(q(t, x) \exp(i\gamma |q|^2 h - ikh)), \quad (6)$$

where $Q(w, x + \frac{h}{2})$ is the Fourier transformation of $q(t, x + \frac{h}{2})$. In the second step, we take the

Fourier transformation of Eq.(4) in the same way we obtain

$$Q(w, x + h) = Q(w, x + \frac{h}{2}) \exp(-i\beta w^2 h). \quad (7)$$

After taking (6) into (7) and carrying out the reversal Fourier transformation, the numerical analysis on the pulse propagation can be worked out in the following.

On the one hand, we consider the initial soliton pulse is taken to be of the form,

$$q(t, 0) = 2 \operatorname{sech}(t). \quad (8)$$

When the soliton pulse has been added into the perturbed system given by Eq.(2) with fixed $\beta=0.5, \gamma=1$, we find that the soliton can propagate stably based on the facts that the soliton propagation performs a periodic oscillation along with the increase of propagation distance, which is shown in Fig. 1(a) and in Fig. 1(b).

III. SPLITTING AND VIBRATING OF THE PERTURBED SYSTEM

Equation (2) has two system parameters and every parameter plays an important role in the process for the soliton propagation. Next we will study the propagation properties of two input power with different parameters of dispersion and nonlinearity.

Case I: Firstly, we focus on the effect on the dispersion parameter γ with fixed $\beta=0.5$. Numerical results are shown in Fig. 2(a) and 2(b) with $\gamma=2$, Fig.

3(a) and 3(b) with $\gamma=4$, respectively, From Fig 2.(a) , it is easy to find that the soliton can be split into two solitons when $\gamma=2$. From Fig 3.(a), when, $\gamma=4$ the soliton can be split into three solitons.

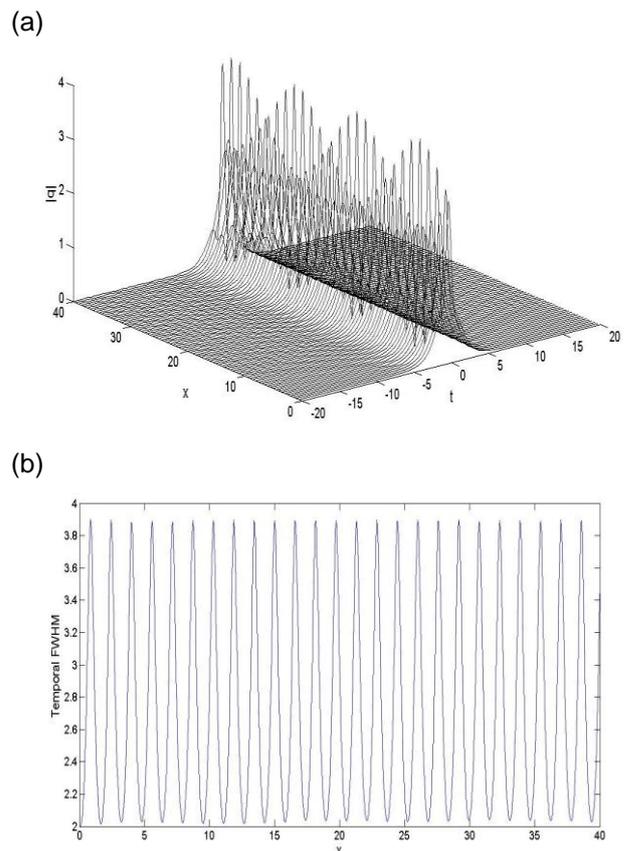
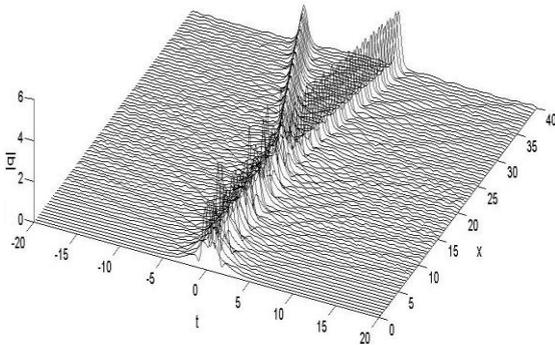


Fig.1. (a) Propagation of solitons with $\beta=0.5, \gamma=1, k=0.05$; (b) Variation of its amplitude with propagation distance for $k=0.05$.

Remark. The initial soliton pulse can be taken other forms just like abundant exact solutions to the NLS equation found by Ref. [19]. All smooth solitons can propagate stably under small perturbation parameter.

Case II: Secondly, we study the effect on the nonlinearity parameter β with fixed $\gamma=1$. Numerical results are shown in Fig. 4(a) and 4(b) with $\beta=1$, Fig. 5(a) and 5(b) with $\beta=3$, respectively. From Fig 4.(a) , it is easy to find that the soliton turns to be in a vibrating state when $\beta=1$. From Fig 5.(a) , when $\beta=3$, the soliton vibrate more sharply.

(a)



(b)

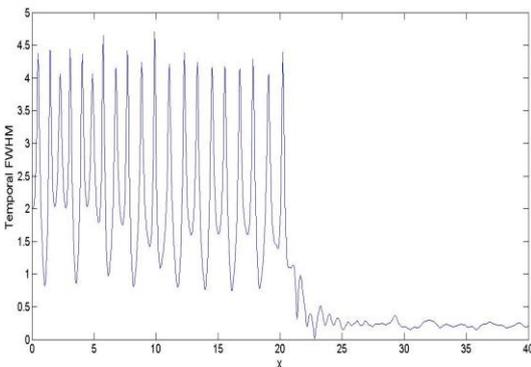
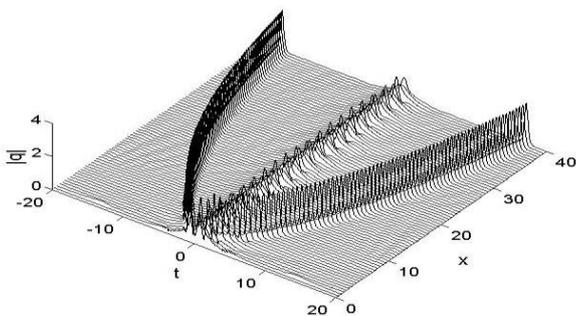


Fig. 2. (a) Propagation of soliton with $\beta=0.5, \gamma=2$;(b) Variation of its amplitude with propagation distance.

(a)



(b)

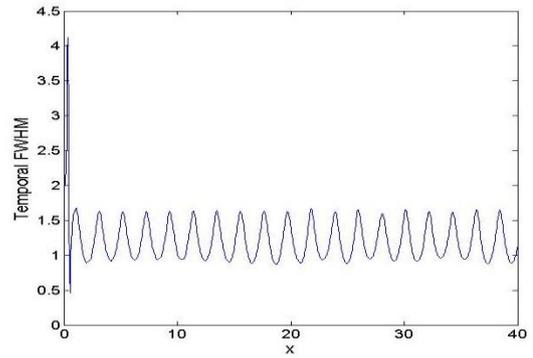
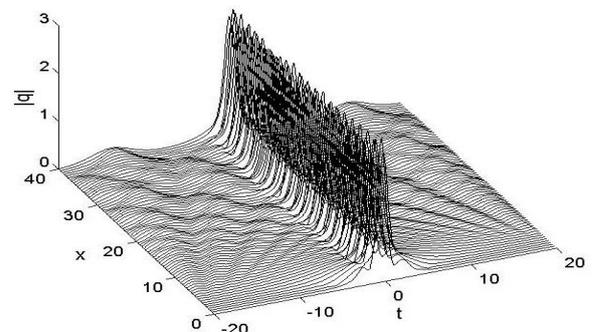


Fig.3.(a) Propagation of soliton with $\beta=0.5, \gamma=4$;(b) Variation of its amplitude with propagation distance.

(a)



(b)

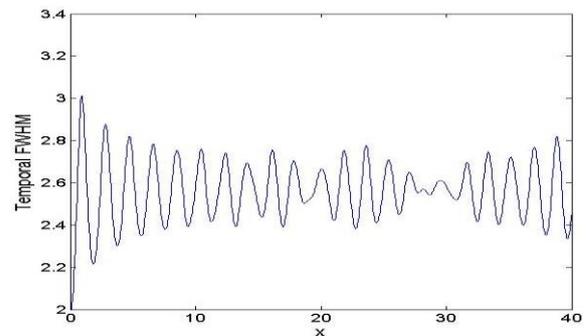


Fig. 4 (a) Propagation of soliton with $\beta=1, \gamma=1$;(b) Variation of its amplitude with propagation distance.

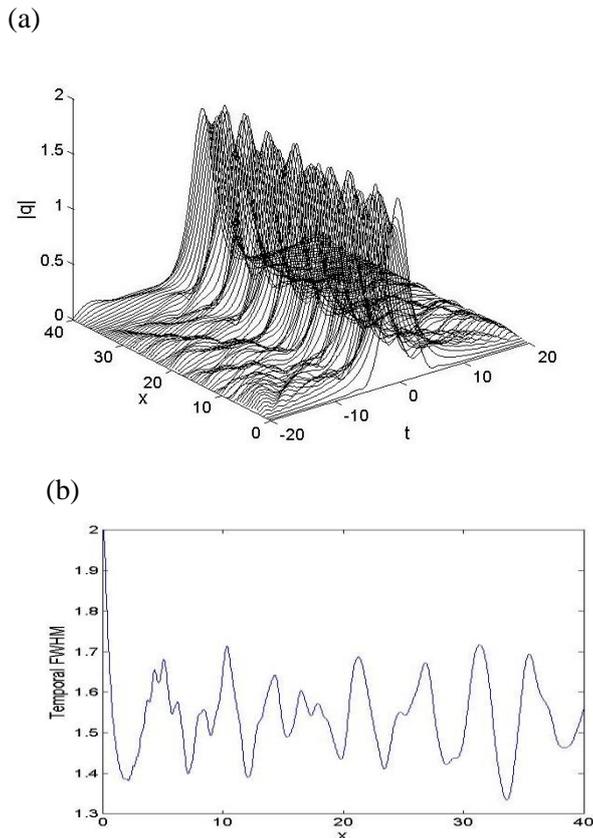


Fig.5. (a) Propagation of soliton with $\beta=3, \gamma=1$; (b) Variation of its amplitude with propagation distance.

IV. CONCLUSIONS

Based on the split-step Fourier method, we studied the time-independent propagation of different input pulse, which lead to different results. We analyze the system with a perturbation and parameters' sensitivity of the perturbed system. It can be extensively applied to other soliton propagation system. Our study may be useful to further understand the effect of the nonlinearity and dispersion term.

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REFERENCES

[1] Y Y Wang, J T Li, C Q Dai, X F Chen and J F Zhang. Solitary waves and rogue waves in a plasma with nonthermal electrons featuring Tsallis distribution. *Physics Letters A*, 2013, 377, 2097-2104

[2] Y T Fang, J Zhou. Effects of third-order dispersion on soliton switching in fiber nonlinear directional coupler. *Optik* 2008, 119, 86-89.

[3] L C Zhao and C Liu and Z Y Yang, The rogue waves with quintic nonlinearity and nonlinear dispersion effects in nonlinear optical fibers, *Communications in Nonlinear Science and Numerical Simulation*, 2015, 20, 9-13.

[4] M Onorato, D Proment. Approximate rogue wave solutions of the forced and damped nonlinear Schrödinger equation for water waves. *Physics Letters A* 2012, 376, 3057-3059.

[5] F. Geran Gharakhili and M. Shahabadi and M. Hakkak, Higher-order soliton-effect pulse compression in a nonlinear left-handed transmission line, *International Journal of Electronics and Communications*, 2011, 65, 848-851.

[6] A. Biswas and S. Konar, Quasi-particle theory of optical soliton interaction, *Communications in Nonlinear Science and Numerical Simulation*, 2007, 12, 1202-1228.

[7] X J Shi, L Li, R Y Hao, Z H Li and G S Zhou. Stability analysis and interaction of chirped femtosecond soliton-like laser pulses. *Optics Communications*, 2004, 241, 185-194.

[8] Z. Xu, L. Li, G. Zhou, Soliton interaction under the influence of higher-order effects, *Opt. Comm.* 2002, 210, 375-384.

[9] Y Xiao, A Biswas, Collision of optical solitons with Kerr law nonlinearity. *Optik* 2007, 118, 243-248.

[10] J. Miguel Hickmann and A.S.L. Gomes, Dynamics of femtosecond soliton-effect optical compression in monomode fibers, *Optics Communications*, 1993, 97, 333-336.

[11] D. Urzagasti and A. Aramayo and D. Laroze, Soliton antisoliton interaction in a parametrically driven easy-plane magnetic wire, *Physics Letters A*, 2014, 378, 2614-2618.

[12] L. Cherbi and N. Lamhene and F. Boukhelkhal and Anjan Biswas, Ultra-short pulse compression at 1065 nm in nonlinear photonic crystal fiber, *Optik -International Journal for Light and Electron Optics*, 2014, 125, 133-136.

[13] W P Hong. Modulational instability of optical waves in the high dispersive cubic

[14] X F Wu and G S Hua and Z Y Ma, Novel rogue waves in an inhomogeneous nonlinear medium with external potentials, *Communications in Nonlinear Science and Numerical Simulation*, 2013, 18, 3325-3336.

[15] H Q Zhang, New exact complex travelling wave solutions to nonlinear Schrödinger (NLS) equation, *Communications in Nonlinear Science and Numerical Simulation* 2009, 14, 668-673.

[16] N. Vishnu Priya and M. Senthilvelan, Generalized Darboux transformation and Nth order rogue wave solution of a general coupled nonlinear Schrödinger equations, *Communications in Nonlinear*

Science and Numerical Simulation, 2015, 20, 401-420.

[17] B G Zhai, W G Zhang, X.L Wang, H.Q Zhang. Multi-rogue waves and rational solutions of the coupled nonlinear Schrödinger equations. Nonlinear Analysis: Real World Applications 2013, 14, 14-27.

[18] C.M. Schober. Melnikov analysis and inverse spectral analysis of rogue waves in deep water, European Journal of Mechanics B/Fluids 2006, 25, 602-620.

[19] W X Ma, M. Chen, Direct search for exact solutions to the nonlinear Schrödinger equation, Applied Mathematics and Computation, 2009, 215, 2835-2842