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Numerical Investigation Of A Stirling – Ringbom Engine With An Elastic Element

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***Abstract*— this article deals with numerical simulation of the behavior of a Stirling-Ringbom engine with an elastic element, which supports downward motion of the displacer piston. Engine behavior is investigated under 16 different combinations of working conditions with respect to the ambient pressure, the working fluid mass and the spring constant. Results are presented about the time variation of displacer piston positions as well as the p-V diagram of the engine cycle.**

Keywords*— *Stirling-Ringbom engine, Stirling cycle, spring constant

I. INTRODUCTION

The Stirling engine has had a lot of modifications until now since its invention in 1816. With the development of the science and the technology, the engine has been developing too. There are a lot of Stirling engines types both in terms of the positioning of the two pistons and in terms of ensuring their movement.

In 1907 Ossian Ringbom patented new design of the Stirling engine [1]. Now it is called the Stirling-Ringbom engine. This engine is sometimes called also “hybrid” because the power piston is kinematically bound to flywheel while the displacer is free. In [2] it is shown that the net work of the hybrid Stirling-Ringbom engine cycle at same mass of the working fluid depends strongly on the ambient pressure. Hence, for achieving maximum net work by this engines at given geometrical parameters and ambient pressure the mass of the working fluid must be adjusted correspondingly. This requires each engine to be equipped with a system for automatic adjusting of working fluid mass to compensate the change of the ambient pressure.

It is of practical interest to study the Stirling-Ringbom engine behavior in the case when the displacer piston movement is assisted by an elastic element, spring, and to test if by adjusting the spring constant it is possible to compensate the ambient pressure change at constant mass of the working fluid. This article presents numerical investigation, done by the use of TUS-SRSim simulator, [2], of a Stirling-Ringbom engine with elastic element that assists displacer downward movement. The principle scheme of the engine considered is shown on the Fig.1. The spring is in a ceaseless contact with the displacer piston.

II. MATHEMATICAL MODEL OF THE HYBRID STIRLING-RINGBOM ENGINE WITH AN ELASTIC ELEMENT

Mathematical model of this engine is based on the following assumptions:

A1. The mass of the working fluid is constant;

A2. The working fluid behaves as an ideal gas;

A3. The working fluid temperature in each of the three spaces is constant;

A4. The working fluid temperature in the expansion space is equal to the heat source temperature T_e;

A5. The working fluid temperature in the compression space is equal to the heat sink temperature T_c;

A6. The dead space temperature T_{dead} is the average between T_e and T_c;

A7. The ambient pressure is constant;

A8. The working fluid pressure in each of the three spaces is equal and varies only in time;

A9. The mean pressure of the working fluid across the cycle is equal to the ambient pressure;

A10. The dead volume is constant across the cycle;

A11. The displacer motion is physically limited by rigid stops;

A12. The power piston motion is periodical and is governed by a sinusoidal law;

A13. The gravity and viscous forces are ignored;

A14. The forces acting on the displacer, when not held against one of its stops are: 1) the pressure difference between the ambient and the working fluid pressure acting across the displacer rod; 2) the spring force.

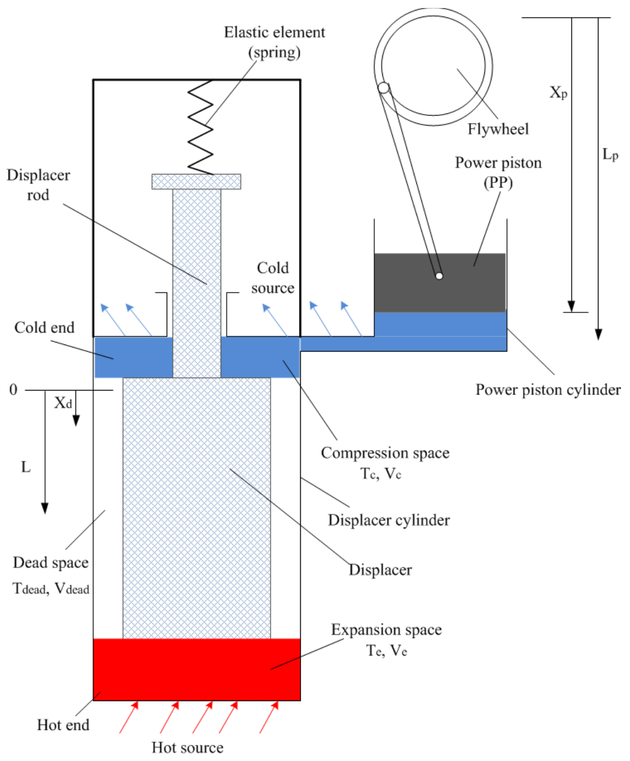

Fig. 1. *Principal scheme of a Stirling-Ringbom engine with an elastic element*

The mathematical model that describes the Stirling-Ringbom engine behavior is presented in full details elsewhere, [2]. For simulation of the cases concerned here some changes are introduced, presented below, to this mathematical model that make possible to account for the spring force acting on the displacer. The other force which acts on the displacer is expressed as a product of the pressure difference (P-B) and the displacer piston rod area A_r. Displacer's speed is calculated as the time rate of change of displacer piston position:

$$U_d = \frac{dX_d}{dt} \tag{1}$$

Where:

U_d- displacer speed, m/s^2;

X_d - displacer position, m;

t – time, s.

$$U_d = \int_t^{t+\Delta t} a_d \, \mathrm{dt} \tag{2}$$

Where:

a_d - displacer acceleration, m/s^2;

Δt – time step of integration.

Displacer acceleration is defined as:

$$a_d = \frac{(\mathrm{B-P})A_r + \mathrm{D}(\mathrm{L}-X_d)}{M_d} \tag{3}$$

Where:

B – ambient pressure, Pa;

P – engine pressure, Pa;

L – displacer half-stroke, m;

M_d – mass of the displacer assembly, kg;

D – the spring constant, N/m.

III. Results and Discussion

The mathematical model of the engine operation is solved in Matlab/Simulink environment by the TUS-SRSim simulator. Design parameters of the investigated Stirling-Ringbom engine with an elastic element are presented in Table I. The working conditions for the reference case (C1) are presented in Table II. The working conditions for all cases concerned in this document are presented in Table III.

TABLE I. Design Parameters

Parameter	Units	Value
Mass of the displacer piston assembly M_d	kg	0.53
Displacer piston cross-section A	m^2	0.2124
Displacer piston rod cross-section A_r	m^2	0.00785
Displacer half-stroke L	m	0.08
Power piston cross section A_p	m^2	0.0314
Power piston half-stroke L_p	m	0.13
Expansion space temperature T_e	K	373
Compression space temperature T_c	K	289

TABLE II. Operating Conditions for Case C1

Working fluid (air) mass	kg	0.0457875
Spring constant D	N/m	1000
Ambient pressure B	Pa	101325
Ambient temperature T_a	K	293.15

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TABLE III. OPERATING CONDITIONS FOR ALL CASES

Case	B, Pa	M, kg	D, N/m
C1	101325	0.0457875	1000
C2	92000	0.0457875	1000
C3	95000	0.0457875	1000
C4	98000	0.0457875	1000
C5	103000	0.0457875	1000
C6	101325	0.0457875	0
C7	101325	0.0457875	800
C8	101325	0.0457875	1200
C9	101325	0.0415740	1000
C10	101325	0.0429290	1000
C11	101325	0.0442850	1000
C12	101325	0.0465440	1000
C13	92000	0.0415740	1000
C14	95000	0.0429290	1000
C15	98000	0.0442850	1000
C16	103000	0.0465440	1000

Along a cycle the pressure of the working fluid in the engine concerned varies around the ambient pressure. In order to ensure stable operation of the engine before the beginning of its operation the mass of the working fluid must be adjusted by equalization of its internal pressure with the ambient pressure at the ambient temperature. Thus, the mass of the working fluid for cases C1, C13 – C16 is calculated by the ideal gas equation of state for the maximum engine volume.

The p-V diagram of the investigated hybrid Stirling-Ringbom engine cycle for case C1 is shown on Fig.2. Actually, when the power piston moves around its upper dead center (UDC) and bottom dead center (BDC), the total volume is changed very little. This small volume changes correspond to the isochoric processes from the idealized Stirling cycle [3] - isochoric cooling and isochoric heating of the working gas.

Time variation of the positions of the two pistons for case C1 is shown on Fig.3. The positive direction of movement of both pistons is given on Fig. 1. The plateaus in X_d time variation appears when displacer holds at its dead centers. The hold-on time of both pistons at its dead centers is duly marked on Fig. 3.

Time variation of the expansion volume V_e, the compression volume V_c and the total volume V for case C1 is shown on Fig. 4. The variation of these volumes is preconditioned by the movement of the displacer piston, which moves the working media from the heated volume to the cooled volume and vice versa.

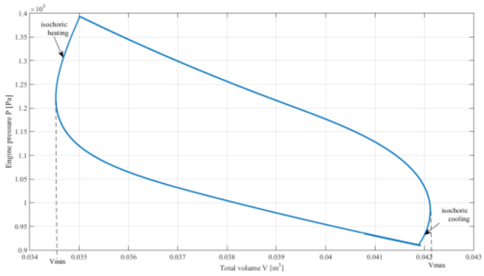

Fig. 2. *Case C1 - actual p-V diagram of the investigated hybrid Stirling-Ringbom engine with an elastic element*

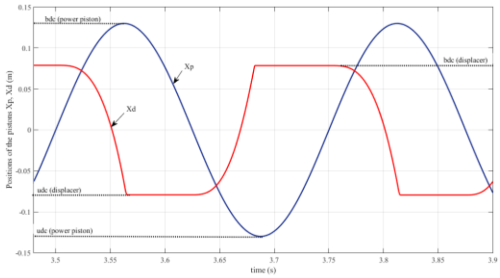

Fig. 3. *Case C1 - time variation of the power piston position X_p and the displacer piston position X_d*

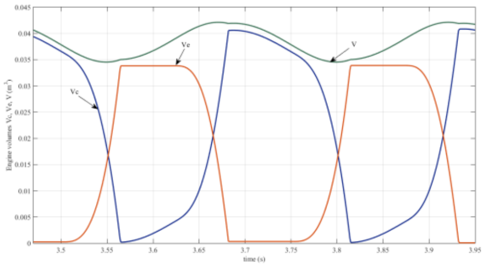

Fig. 4. *Case C1 – time variation of the expansion volume V_e, compression volume V_c and total volume V*

The impact of the ambient pressure changes on the time variation of displacer piston position, X_d, at same mass and same spring constant, i.e. for cases C1-C5, is demonstrated on Fig. 5. It can be easily seen that the hold-on time of the displacer at its dead centers depends strongly on the ambient pressure, i.e. on the pressure difference (P-B) across the displacer piston rod. When ambient pressure decreases and is lower than the one in the reference case C1 the hold-on time of displacer at its upper dead center increases and at its bottom dead center decreases. So displacer motion becomes irregular. The negative impact of this on the p-V diagram of the cycle and hence on cycle net work is visualized on Fig. 6. In the cases of irregular displacer motion (C3 and C4), the net work of the cycle is lower than in the reference case, C1. In case C2 it is even equal to zero – the engine doesn't work. In case C5 ambient pressure is slightly higher than in the reference case (with 1.65%) and the hold-on time of displacer piston at both dead centers is lower than in the reference case C1. Nevertheless, cycle net work is lower than in the reference case C1.

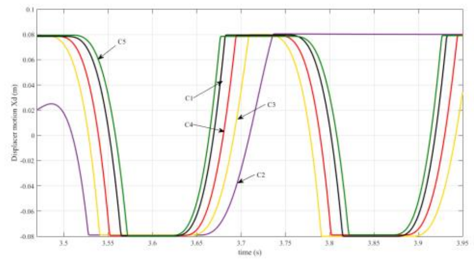

Fig. 5. *Displacer position time variation X_d for cases C1 – C5*

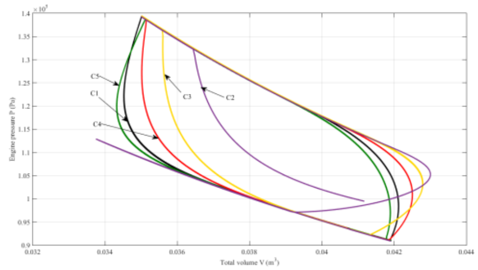

Fig. 6. *Cycle p-V diagram of the Stirling-Ringbom engine with an elastic element for cases C1 – C5*

The purpose of the spring is to support displacer movement from the upper dead center to the bottom dead center. By comparing cases C6, C7, and C8 with the reference case C1 one can investigate the impact of spring constant on engine behavior at same ambient pressure and same working fluid mass. Time variation of displacer piston position for cases C1, C6, C7 and C8 is presented on Fig. 7.

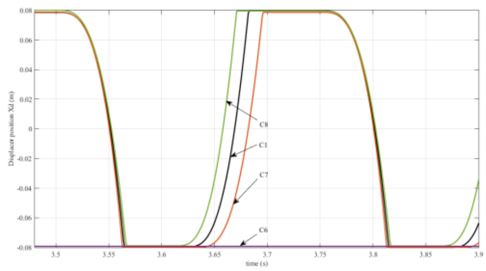

Fig. 7. *Displacer position time variation X_d for cases C1, C6 – C8*

In case C6, there is no spring (D=0 N/m) and the displacer visually stays continuously at the upper dead center. Cycle net work of the engine concerned under these conditions is very small - it is with 99.71% lower than in the reference case. Actually, the engine doesn't work under these conditions. In cases C7 and C8 the spring acts ceaselessly on the displacer. In case C7 (spring constant is 20% lower than the one at the reference case) displacer hold-on time at the upper dead center is longer than in case C1 and its hold-on time at the bottom dead center is shorter. In case C8 (spring constant is 20% higher than the one at the reference case) the situation reverses. The impact of spring constant under these conditions on engine cycle p-V diagram and hence on cycle net work is visualized of Fig. 8.

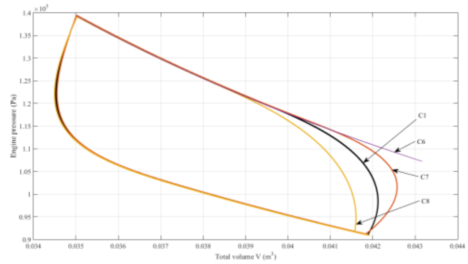

Fig. 8. *Cycle p-V diagram of the Stirling-Ringbom engine with an elastic element for cases C1, C6 – C8*

The impact of fluid mass (M) change, and hence of B to M ratio, at same ambient pressure (B) and same spring constant D on the behavior of engine concerned is investigated by analyzing cases C1, C9 – C12. Time variation of displacer piston position for these cases is presented on Fig. 9 and engine cycle p-V diagram on Fig. 10.

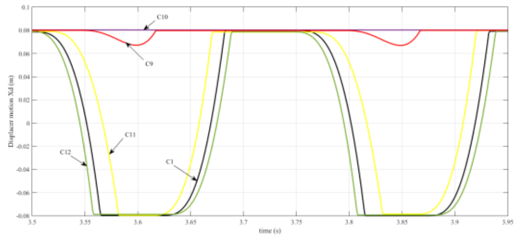

Fig. 9. *Displacer position time variation X_d for cases C1, C9 - C12*

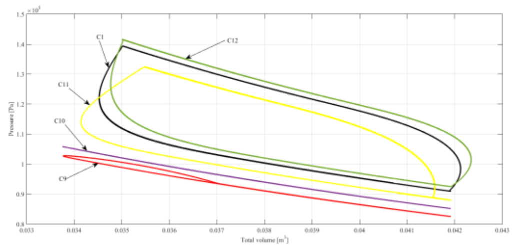

Fig. 10. *Cycle p-V diagram of the Stirling-Ringbom engine with an elastic element for cases C1, C9 – C12*

In cases C9 and C10 the engine almost doesn't work. In case C9 displacer piston holds at its bottom dead center since there is no enough working fluid in the engine, so the pressure generated in the expansion volume is not big enough to overcome spring force. In this case working fluid mass is adjusted for ambient pressure of 92 kPa. In case C10 working fluid mass is higher- it is adjusted for ambient pressure of 95 kPa, and for short periods of time displacer is capable to overcome spring force. In case C11, fluid mass adjusted for ambient pressure of 98 kPa, engine works almost normally. Displacer piston hold-on time at its bottom dead center is greater than for the reference case C1 and at its upper dead center is shorter. For case C12, fluid mass adjusted for ambient pressure of 103 kPa, situation reverses. Cycle net work in case C11 is lower than in the reference case C1 with 11.38%, and in case C12 with 0.9%.

Cases C13 – C16 are meant for studying engine behavior under different ambient pressure while spring constant is the same and B to M ratio is the same. Time variation of displacer piston position for these cases is presented on Fig. 11.

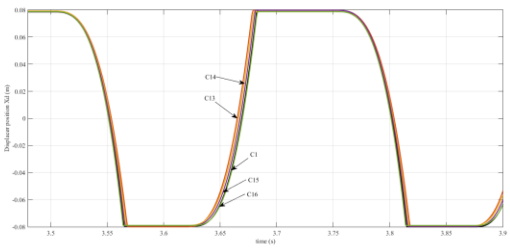

Fig. 11. *Displacer position time variation X_d for cases C1, C13 – C16*

For all these cases (C1, C13 - C16) time variation of displacer piston position is almost the same. There are small differences in the hold-on times of displacer piston at both dead centers.

Cycle p-V diagrams for these cases, Fig. 12, have similar shape but are shifted along the p axis, since engine pressure along the cycle varies around the ambient pressure. Engine pressure variation along the cycle for these cases is presented on Fig. 13.

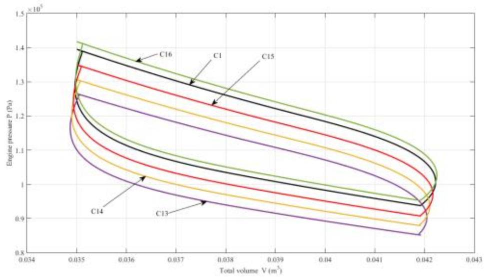

Fig. 12. *Cycle p-V diagram of the Stirling-Ringbom engine with an elastic element for cases C1, C13 – C16*

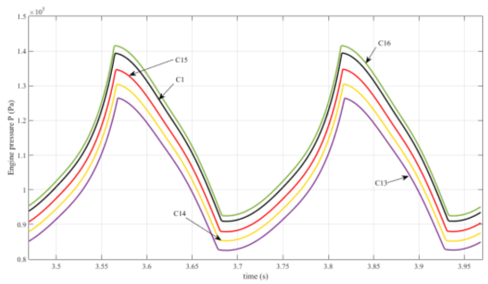

Fig. 13. *Pressure variation along the cycle for cases C1, C9 – C12*

For all these cases presure variation along the cycle has similar behavior. With respect to the reference case C1 the locus of pressure maximum for cases C13, C14 and, C15 is shifted to the right, i.e. it happens later and the locus of pressure minimum is shifted to the left, i.e it hepens earlier. As a result cycle net work for case C13 is lower than in the reference case with 11.12%, in case C14 with 9.17% and in case C15 with 4.05%. Cycle net work for case C16 is slightly higher than in the reference case – with 0.39%.

IV. Conclusions

The behavior of a hybrid Stirling-Ringbom engine with an elastic element supporting displacer piston downward motion is numerically investigated under 16 combinations of working conditions.

The efficiency of the engine concerned depends strongly on the working conditions.

Since the working fluid pressure in the investigated engine varies around the ambient pressure a stable operation of the engine, with maximum possible cycle net work, at a given ambient pressure can be achieved by adjusting both the working fluid mass and spring constant. Small variations of the ambient pressure may be compensated by adjusting spring constant. Large and continuous change of ambient pressure must be compensated by changing the mass of engine working fluid.

The near future tasks can be summarized as follows:

- Development of an algorithm for calculation of the combination of working fluid mass and spring constant that ensures maximum possible cycle net work of the engine concerned under given design parameters;
- Improving the mathematical model in the direction of taking into account all forces acting on both pistons.

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