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Analysis Of Wave Motion In Rotating Thermo Visco-Elastic Transversely Isotropic Half Space

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Abstract— **Present paper explores the Matlab implementation of the normal mode analysis and predicts the waves propagating in rotating thermo visco-elastic transversely isotropic half space. At the beginning the paper deals with the mathematical development of the physical problem satisfying the boundary conditions. Resulting linear equation has been solved with the help of Matlab software. To demonstrate the behaviors of phase velocities and attenuation quality factor the Grapher has been used.**

Keywords— Elasticity;transversely isotropic; phase velocity;Attenuation quality factor.

I. INTRODUCTION

The study of wave propagation in anisotropic materials has been a subject of extensive significance in the literature. It is of great importance in a variety of applications ranging from seismology to nondestructive testing of composite structures used in aircraft, spacecraft, or other engineering industries. Polymers or polymer-based matrix composites are widely used in these industrial environments. These materials possess isotropic or anisotropic properties that can strongly affect the propagation of waves. The dynamical interaction between the thermal and mechanical fields in solids also has a great number of practical applications in modern aeronautics, astronautics, nuclear reactors and high energy particle accelerators. The generalized theory of thermoelasticity has drawn widespread attention because it removes the physically unacceptable situation of the classical theory of thermoelasticity, that is, that the thermal disturbance propagates with the infinite velocity. The Lord-Shulman theory [1] and Green–Lindsay theory [2] are two important generalized theories of thermoelasticity. Recently, Chandrasekharaiah [3], Hetnarski and Ignazack [4] in their surveys, considered the theory proposed by Green and Naghdi [5-7] as an alternate way of formulating the propagation of heat. This theory is developed in a rational way to produce a fully consistent theory that is capable of incorporating thermal pulse transmission in a very logical manner. The development is quite general and the characterization of material response for the thermal phenomena is based on three types of constitutive functions that are labeled as type I, type II, and type III.

Some researchers in past have investigated different problem of rotating media. Chand et al. [8] presented an investigation on the distribution of deformation, stresses and magnetic field in a uniformly rotating homogeneous isotropic, thermally and electrically conducting elastic half-space. Many authors (Schoenberg and Censor [9]; Clarke and Burdness [10]; Destrade [11]) studied the effect of rotation on elastic waves. (Sharma and Thakur [12], Sharma [13]) discussed effect of rotation on different type of waves propagating in a thermoelastic medium. Othman [14] investigated plane waves in generalized thermoelasticity with two relaxation times under the effect of rotation. Othman and Song [15] presented the effect of rotation in magneto-thermoelastic medium. Mahmoud [16] discussed the effect of Rotation, Gravity Field and Initial Stress on Generalized Magneto-Thermoelastic Rayleigh Waves in a Granular Medium.

The inelastic behavior of the Earth's material plays an important role in changing the characteristics of seismic waves, in defining seismic source functions [17], and in determining the internal structure of the Earth. The general theory of viscoelasticity describes the linear behavior of both elastic and inelastic materials and provides the basis for describing the attenuation of seismic waves due to inelasticity. Gupta [18] discussed the reflection of waves in visco-thermoelastic transversely isotropic medium. In spite of these studies, relatively less attention has been paid to studying the reflection of waves in viscoelastic transversely isotropic half space by considering the equations of generalized thermoelasticity [6,7], which has motivated the authors to carry out the present work.

In this article, effect of viscoelasticity on the propagation of waves in a rotating transversely isotropic medium in the context of thermo-visco-elasticity with GN theory of type-II and III has been investigated. A cubic equation resulting in the three values of phase velocities and attenuation quality factor has been obtained. Furthermore the expressions for the amplitude ratios of the reflected wave corresponding to the three incident waves have been obtained. These expressions are then evaluated numerically and plotted graphically (by using Grapher) to manifest the effect of viscoelasticity.

II. FORMULATION OF THE PROBLEM

In the context of thermoelasticity based on Green-Naghdi theory of type II and type III, the equation of motion for the transversely isotropic medium, taking the rotation term about y-axis as a body force is

$$t_{ij,j} = \rho[\ddot{u}_i + (\vec{\Omega}\times\vec{\Omega}\times\vec{u})_i + (2\vec{\Omega}\times\dot{\vec{u}})_i] \tag{1}$$

where $\vec{\Omega}$ is the uniform angular velocity and ρ is the density of the medium. The generalized energy equation can be expressed as

$$K_{ij}\dot{T}_{,ij}+K_{ij}^{*}T_{,ij}=(T_0\beta_{ij}\ddot{u}_{i,j}+\rho c^{*}\ddot{T}),\quad i,j=1,2,3, \tag{2}$$

The constitutive equations have the form

$$t_{ij}=C_{ijkl}e_{kl}-\beta_{ij}T, \tag{3}$$

where $C_{ijkl}=C'_{ijkl}+C''_{ijkl}\frac{\partial}{\partial t}$, the deformation tensor is defined by $e_{ij}=(u_{i,j}+u_{j,i})/2$, t_{ij} are components of stress tensor, u_i the mechanical displacement, e_{ij} are components of infinitesimal strain, T the temperature change of a material particle, T_0 the reference uniform temperature of the body, K_{ij} is the thermal conductivity, K_{ij}^{*} are the characteristic constants of the theory, $\beta_{ij}=C_{ijkl}\alpha_{kl}$ are the thermal elastic coupling tensor, α_{kl} are the coefficient of linear thermal expansion, c the specific heat at constant strain, C_{ijkl} are characteristic constants of material following the symmetry properties $C_{ijkl}=C_{klij}=C_{jikl}$, $K_{ij}^{*}=K_{ji}^{*}, K_{ij}=K_{ji}, \beta_{ij}=\beta_{ji}$. The comma notation is used for spatial derivatives and superimposed dot represents time differentiation.

Following Slaughter [19], the appropriate transformations have been used on the set of equations (3), to derive equations for transversely isotropic medium. We restrict our analysis for two dimensions, in which we consider the component of the displacement vector in the form

$$\vec{u}=(u_1,0,u_3) \tag{4}$$

Here we consider plane waves propagating in plane such that all particles on a line parallel to x_2-axis are equally displaced. Therefore, all the field quantities will be independent of x_2 coordinate, i.e. $\partial/\partial x_2\equiv 0$. Thus, the field equations and constitutive relations for such a medium reduces to:

$$C_{11}u_{1,11}+\frac{C_{55}}{2}u_{1,33}+(C_{13}+\frac{C_{55}}{2})u_{3,13}-\beta_1 T_{,1}=\rho\left[\ddot{u}_1-\Omega^2u_1+2\Omega\dot{u}_3\right], \tag{5}$$

$$\frac{C_{55}}{2}u_{3,11}+C_{33}u_{3,33}+(C_{13}+\frac{C_{55}}{2})u_{1,13}-\beta_3 T_{,3}=\rho\left[\ddot{u}_3-\Omega^2u_3+2\Omega\dot{u}_1\right], \tag{6}$$

$$K_1\dot{T}_{,11}+K_3\dot{T}_{,33}+K_1^{*}T_{,11}+K_3^{*}T_{,33}=\rho c^{*}\ddot{T}+T_o(\beta_1\ddot{u}_{1,3}+\beta_3\ddot{u}_{3,1}), \tag{7}$$

where $\beta_1=C_{11}\alpha_1+C_{13}\alpha_3, \beta_3=C_{31}\alpha_1+C_{33}\alpha_3$ and we have used the notations $11\to 1, 13\to 5, 33\to 3$, for the material constants.

It is convenient to change the preceding equations into the dimensionless forms. To do this, the non-dimensional parameters are introduced as:

$$x_i'=\frac{x_i}{L}, u_i'=\frac{u_i}{L}, t_{ij}'=\frac{t_{ij}}{C_{11}}, t'=\frac{t}{t_o}, T'=\frac{T}{T_o}, \tag{8}$$

where L, t_o, T_o are parameters having dimension of length, time and temperature respectively.

III. PLANE WAVE PROPAGATION AND REFLECTION OF WAVES

Let $\vec{p}=(p_1,0,p_3)$ denote the unit propagation vector, c and ξ are respectively the phase velocity and the wave number of the plane waves propagating in x_1-x_3 plane. For plane wave solution of the equations of motion of the form

$$(u_1,u_3,T)=(\bar{u}_1,\bar{u}_3,\bar{T})e^{i\xi(p_1x_1+p_3x_3-ct)} \tag{9}$$

With the help of equations (8) and (9) in equations (5)-(7), three homogeneous equations in three unknowns are obtained. Solving the resulting system of equations for non-trivial solution results in

$$Ac^6+Bc^4+Cc^2+D=0, \tag{10}$$

where

$$A=f_{10}, B=-f_5f_{10}-f_1f_{10}+f_6f_8+f_3f_7+f_9,$$

$$C=-f_5(f_9+f_1f_{10}-f_3f_7)-f_1(f_9-f_6f_8)-f_2(f_4f_{10}+f_6f_7)+f_3f_4f_8,$$

$$D=f_1f_5f_9-f_2f_4f_9$$

$$f_1=p_1^2d_1+p_3^2d_3-d_{13}Pp_3^2/2,$$

$$f_2=p_1p_3d_3-d_{13}Pp_1p_3/2, f_3=ip_1d_4,$$

$$f_4=p_1p_3d_2-d_{13}Pp_1p_3/2,$$

$$f_5=p_1^2d_3+p_3^2d_5+d_{13}Pp_1^2/2, f_6=ip_3d_6, f_7=ip_7d_{11},$$

$$f_8=ip_3d_{12}, f_9=i\omega p_1^2-d_8p_1^2+i\omega\bar{k}p_3^2-d_9p_3^2$$

$$f_{10}=\varepsilon_1, d_1=\frac{C_{11}t_o^2}{\rho L^2}, d_2=\frac{(C_{13}+C_{55}/2)t_o^2}{\rho L^2},$$

$$d_3=\frac{C_{55}t_o^2}{2\rho L^2}, d_4=\frac{\beta_1T_ot_o^2}{\rho L^2}, d_5=\frac{C_{33}t_o^2}{\rho L^2}, d_6=\frac{\beta_3T_ot_o^2}{\rho L^2},$$

$$\bar{k}=d_7=\frac{k_3}{k_1}, d_8=\frac{k_1^{*}t_o}{k_1}, d_9=\frac{k_3^{*}t_o}{k_1}, \varepsilon_1=d_{10}=\frac{\rho C^{*}L^2}{k_1t_o},$$

$$d_{11}=\frac{\beta_1L^2}{k_1t_0}, d_{12}=\frac{\beta_3L^2}{k_1t_0}, d_{13}=\frac{t_0^2}{\rho L^2}.$$

The roots of this equation give three values of c^2. Three positive values of c will be the velocities of propagation of three possible waves. The waves with velocities c_1, c_2, c_3 correspond to three types of quasi waves. We name these waves as quasi-longitudinal displacement (qLD) wave, quasi thermal wave (qT) and quasi transverse displacement (qTD) wave.

IV. NUMERICAL RESULTS AND DISCUSSION

In order to illustrate the theoretical results obtained in the preceding sections, we now present some numerical results. The following relevant physical constants for Cobalt material are taken from Dhaliwal et al. [20] for a thermoelastic transversely isotopic material,

$$C'_{11} = 3.071\times10^{11}\,Nm^{-2},\ C'_{12} = 1.650\times10^{11}\,Nm^{-2},$$

$$C'_{13} = 1.027\times10^{11}\,Nm^{-2},\ C'_{33} = 3.581\times10^{11}\,Nm^{-2},$$

$$C'_{55} = 1.51\times10^{11}\,Nm^{-2},\ \beta_1 = 7.04\times10^{6}\,Nm^{-2}K,$$

$$\beta_1 = 6.98\times10^{6}\,Nm^{-2}K,\ \ \rho = 8.836\times10^{3}\,Kgm^{-3},$$

$$T = 298K.$$

$$K_1 = 6.90\times10^{2}\,Wm^{-1}K,\ K_3 = 7.01\times10^{2}\,Wm^{-1}K,$$

$$K_1^* = 1.313\times10^{2}\,W\sec,\ K_3^* = 1.54\times10^{2}\,W\sec,$$

$$c^* = 4.27\times10^{2}\,J\,Kg\,K.$$

For a particular model of a thermo-visco-elastic transversely isotropic solid, the relevant parameters are expressed as:

$$C_{11} = C'_{11}(1+iQ_1^{-1}),\ C_{12} = C'_{12}(1+iQ_2^{-1}),$$

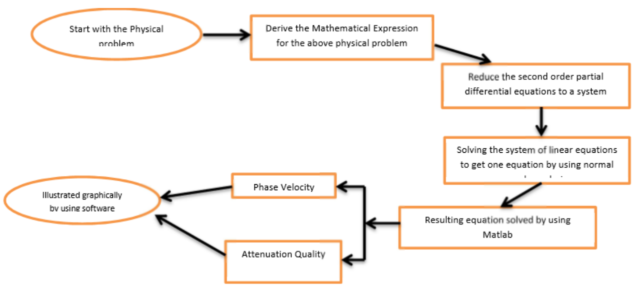
$$C_{13} = C'_{13}(1+iQ_3^{-1}),$$

$$C_{33} = C'_{33}(1+iQ_4^{-1}),\ C_{55} = C'_{55}(1+iQ_5^{-1}),$$

where

$$Q_1^{-1} = 10.05, Q_2^{-1} = 2.05, Q_3^{-1} = 3.02, Q_4^{-1} = 1.05, Q_5^{-1} = 4.0.$$

A. *Flow Chart:*



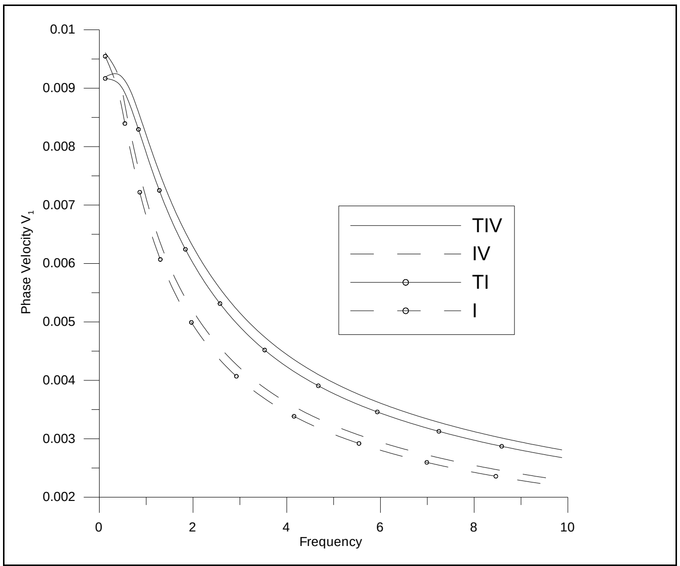
B. *Program*

MATLAB is an integrated technical computing environment that combines numeric computation, advanced graphics and visualization, and a high-level programming language oriented towards matrix computation. There are functions for data analysis and visualization, numeric computation, engineering and scientific graphics, modeling, simulation, and prototyping, programming and application development. Writing the Matlab program to solve the problem is rather straightforward. To simplify the code, we initially assume that the motion lies in a plane (x-z plane). We convert the second order partial differential equations into a system of homogeneous linear equations by introducing a plane wave solution (represented by equation 9). Resulting system of homogeneous equations possess a unique if and only if the determinant of the coefficient matrix is non-zero. This will lead to a cubic equation in c^2. We can readily code such an equation in Matlab by invoking one of the available routines: roots (c). As an input we just need to provide the values of the coefficients of the cubic equation, viz., A, B, C and D. The calling function is:

```
function r = roots(c)
if size(c,1)>1 && size(c,2)>1
    error('MATLAB:roots:NonVectorInput', 'Input must be a vector.')
end
if ~all(isfinite(c))
    error('MATLAB:roots:NonFiniteInput', ...
        'Input to ROOTS must not contain NaN or Inf.');
end
c = c(:).';
n = size(c,2);
r = zeros(0,1,class(c));
inz = find(c);
if isempty(inz),      % All elements are zero
    return
end  % Strip leading zeros and throw away.  % Strip trailing zeros, but remember
them as roots at zero.
nnz = length(inz);
c = c(inz(1):inz(nnz));
r = zeros(n-inz(nnz),1,class(c));    % Prevent relatively small leading coefficients
from introducing Inf  by removing them.
d = c(2:end)./c(1);
while any(isinf(d))
    c = c(2:end);
    d = c(2:end)./c(1);
end  % Polynomial roots via a companion matrix
n = length(c);
if n > 1
    a = diag(ones(1,n-2,class(c)),-1);
    a(1,:) = -d;
    r = [r;eig(a)];
end
```

C. Graphs

Graphical representation (with the help of Grapher) is given for the variations of the phase velocities and attenuation quality factors for the qLD, qTD and qT waves to compare the results in two cases, one for transversely isotropic thermo-visco elastic half-space (TIV) and other from isotropic thermo-viscoelastic half-space (IV). Fig. 1(a, b, c) represents the variations of phase velocities of the waves while Fig. 2(a, b, c) represents the attenuation quality factors for the three waves. In these figures solid line represents the variations for transversely isotropic medium (TI) while the dotted curves represent the variations for isotropic medium (I). The curves with center symbol correspond to the variations with the viscoelastic effect while the curves without center symbol represent the behavior after neglecting the viscoelastic effect.



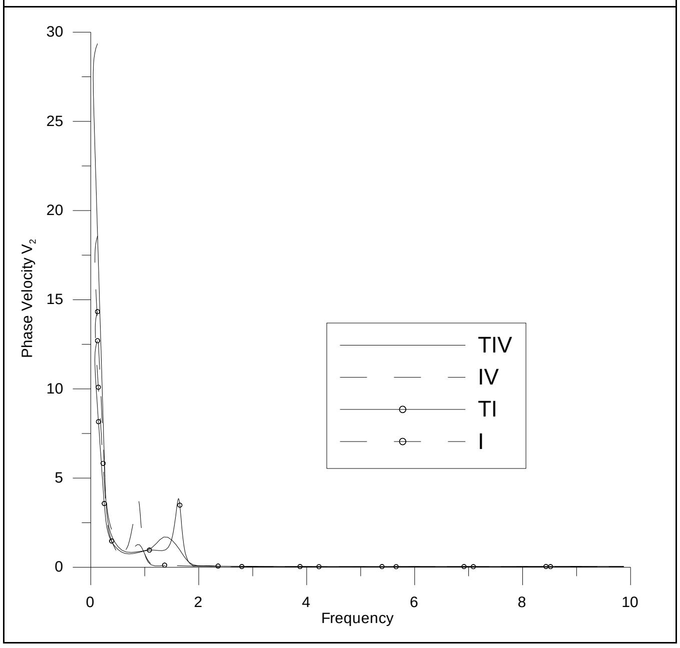
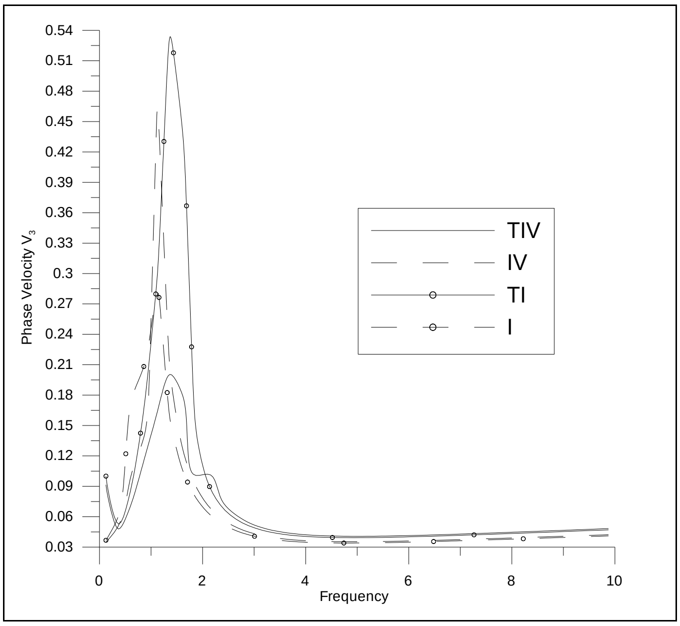
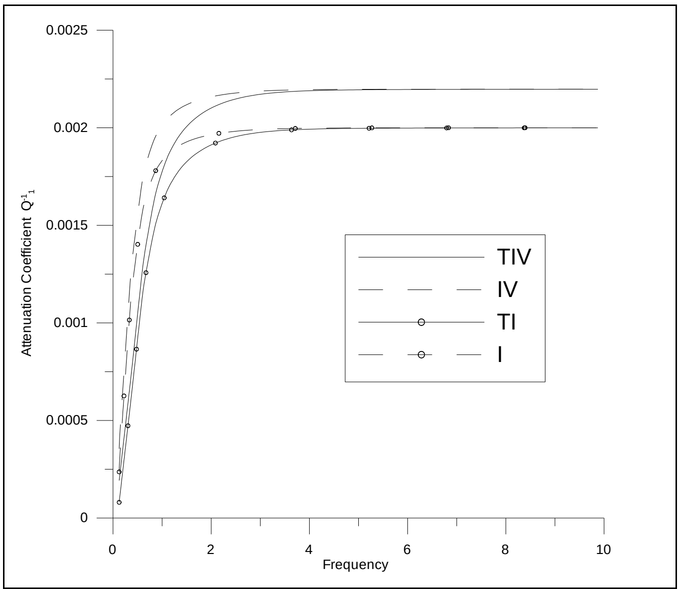
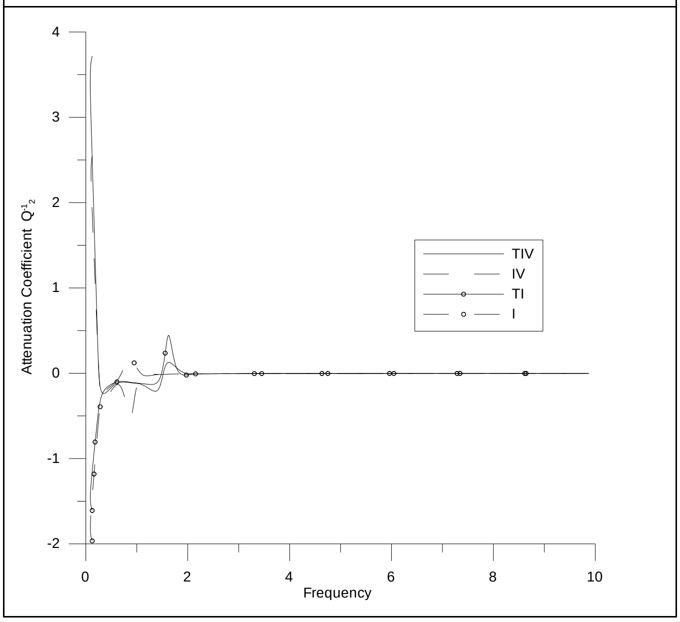


Fig. 1 Phase velocities of (a) qLD wave, (b) qT



wave, and (c) qTD wave





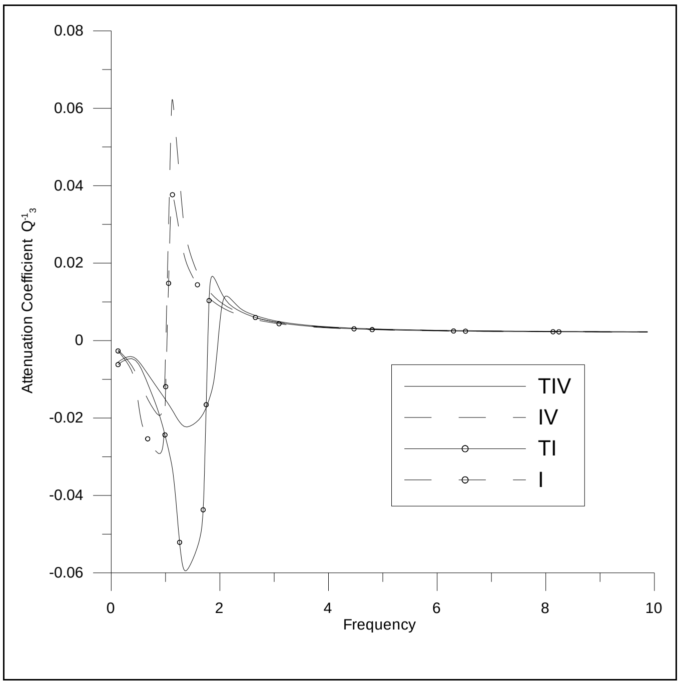


Fig. 2 Attenuation Quality factors of (a) qLD wave, (b) qT wave, and (c) qTD wave

V. Conclusion

The output of the function is used to get the values of the phase velocity and attenuation coefficient. The physical quantities displacement, temperature, amplitude ratios depend not only on time 't' and space coordinates but also on the characteristic parameter of the Green-Naghdi theory of type II and type III. Here, all variables are taken in non-dimensional form. The approach adopted to discuss the propagation of waves is summarized into the flow chart shown in the Appendix. It is depicted that for all the waves the value of phase velocity and attenuation quality factor starts with initial oscillation and attains a constant behavior.

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