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Modified Integrated Multi-Point Approximation And GA Used In Truss Topology Optimization

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***Abstract*— Genetic algorithm (GA) is a kind of optimization method which does not require the differentiability and continuity of the function or variables, and thus can be used for problems if only the adaptation values of species can be obtained. It is appropriate to use GA solving complicated problem as structural topology optimization. However, GA requires the calculation all the adaptation values of the given species, which means an extremely large number of structural analyses are needed to conduct if GA is directly applied to topology optimization, especially for large-scale problems. A modified integrated multi-point approximate concepts and GA for truss topology optimization including continuously cross-sectional size variables and discrete topology variables is proposed. The primal truss optimization problem is at first transformed into a series of approximately mixed-variable problems using the proposed approximate formula; after that the topology variables and cross-sectional areas of the truss are determined by GA and dual method respectively through a layered strategy. The final optimization solution can be reached after few iterations. The numerical examples show the truss optimization with topology variables can be obtained after less structural analysis compared with others, which presents that the proposed modified integrated multipoint approximation has higher quality than the former integrated multi-point approximation.**

Keywords— Topology Structural Optimization; Multi-point approximation; GA.

I. INTRODUCTION

Topology optimization is recognized as one of the most challenging problem in structural synthesis. It is usually a mathematical programming problem with discrete variables, and may exist singular optima. Although approximation concepts are very important meaning and will remarkably effect on the efficiency of the logarithms of structural optimization, it is hardly to establish a high quality approximate problem for topology structural optimization because of its discrete properties. Generally, on discrete structures such as trusses the topology optimization is concerned with finding an optimal distribution of material within a specified domain. One of the approaches of the topology optimization problems is so-called ground structure method [1,2]. The ground structure approach was first proposed by Dorn et al. [3]. In the ground structure problem with all possible pair-wise interconnection with fixed nodal locations, the optimal topology solution is to be selected. The most difficulties in such approach are that there may exist many local optimal solutions. The original multi-point approximation (MA) [4] is one of the most effective approximation functions for structural sizing optimization, but it is no longer effective when design variable x_k approaches to zero, which is always happened in truss topology optimization. With two-level approximation concepts and GA [5], and the integrated multi point approximation (IMA) [6] formula was put forward to solve truss topology optimization problems, the original problem is transformed into a series of first-level explicit approximate problems using IMA, and then a layered optimization strategy is introduced. The topology variables of the trusses are optimized through GA in the external layer, and the cross-sectional areas of bars are optimized in the internal layer through a series of second level approximate problems that can be solved by the duel method. In this work, the integrated multi-point approximate function is further studied, thus the modified integrated multipoint approximation (MIMA) is proposed, in which MA and IMA are combined as one function with two branches for conditions when a bar exists or removed respectively. The new approximation function not only can avoid the singularity of the original MA for topology optimization, but also keeps the satisfied accuracy features for sizing problems. The numerical examples show the truss optimization with topology variables can be obtained after notable few structural analysis, which presents that the proposed MIMA has higher quality than the former IMA.

II. MODIFIED INTEGRATED MULTIPOINT APPROXIMATION FUNCTION

The Integrated multipoint approximation model, [6] which its accuracy has been tested through a series of explicit and implicit functions, [7]. The model can be represented here as follows: -

$$\begin{cases} Find & X=\{x_1,x_2,\ldots,x_n\}^T \\ & \alpha=\{\alpha_1,\alpha_2,\ldots,\alpha_n\}^T \\ Min & W=\sum_{i=1}^{n}\alpha_i f_i(X) \\ St & g_j(X)\le 0 \qquad j=1,\cdots,m \\ & \alpha_i x_i^L+(1-\alpha_i)x_i^b\le x_i \quad i=1,\cdots,n \\ & x_i\le \alpha_i x_i^U+(1-\alpha_i)x_i^b \\ & \alpha_i=0 \quad or \quad \alpha_i=1 \end{cases} \qquad (1)$$

where X is the vector of cross-sectional size variables, α is the vector of topology variables, n is the group number of linked bar, m is the number of response constraints, x_i^U and x_i^L are the upper and lower bounds of the size variables, and x_i^b is a very small value usually takes (0.01 x_i^L) used to substitute the cross-sectional size of a removed bar, $f_i(X)$ is weight of bars in group i, and $g_j(X)$ represents a response constraint function.

A. The layered strategy to solve the approximate problems

To solve the problem (1), a series of approximate problem called as the first-level approximate problems are created through introducing a Modified Integrated Multi-Point Approximate (MIMA) function. The MIMA function is established by improving the former Integrated Multipoint Approximate (IMA) function. At p-th stage the approximate problem can be stated as: -

$$\begin{cases} Find & X=\{x_1,x_2,\ldots,x_n\}^T \\ & \alpha=\{\alpha_1,\alpha_2,\ldots,\alpha_n\}^T \\ Min & W=\sum_{i=1}^{n} f_i(X) \\ St & g_j^{(p)}(X)\le 0 \qquad j=1,\cdots,J_1 \\ & \alpha_i x_{i(p)}^L+(1-\alpha_i)x_i^b\le x_i \quad i=1,\cdots,n \\ & x_i\le \alpha_i x_{i(p)}^U+(1-\alpha_i)x_i^b \\ & \alpha_i=0 \quad or \quad \alpha_i=1 \\ & x_{i(p)}^U=\min\{x_i^U,\ \tilde{x}_{i(p)}^U\} \quad , \quad x_{i(p)}^L=\max\{x_i^L,\ \tilde{x}_{i(p)}^L\} \end{cases} \qquad (2)$$

Where J_1 is the number of active response constraint of the original problem (1); $x_{i(p)}^U$ and $x_{i(p)}^L$ are the upper and lower bounds of the size variables in problem (2) after considering the move limits $\tilde{x}_{i(p)}^L$ and $\tilde{x}_{i(p)}^U$ at the p-th stage; and $g_j^{(p)}(X)$ represents the approximate constraint function constructed with the proposed Modified Integrated Multi-point Approximate function, which is based on the information of primal function $g_j(X)$ at multiple known points, and effective even if some bars are removed (e.g. $x_k=x_k^b$, that means the bars in k-th group are removed). But the explicit approximate function $g_j^{(p)}(X)$ was formed based on MA in structural optimization problem which is higher quality as it is used for cross-sectional size optimization then it becomes singular and no longer effective if the design variable x_k approaches to zero or is substituted by very small value x_k^b, which can be happened in truss topology optimization for this reason IMA was proposed [6], which is a weighted representation of the Taylor expansions with respect to intermediate variables. IMA is still effective even if some bars are removed so it can be used for Topology optimization. In order to constructed an approximate function that has satisfied accuracy in both size and topology optimization a Modified Integrated Multipoint Approximation (MIMA) is proposed in this paper for $g_j^{(p)}(X)$ in (2), which integrates MA and IMA as one function with two branches for conditions when the corresponding bar exists or is removed respectively, which can be written as: -

$$g_j^{(p)}(X)=\sum_{t=1}^{H}\left\{g(X_t)+\sum_{i=1}^{n}\tilde{g}_{com,i,t}(X)\right\}h_t(X)$$

Where, (3)

$$\tilde{g}_{com,i,t}(X)=\begin{cases} \dfrac{1}{r_{tO}}\dfrac{\partial g(X_t)}{\partial x_i}.x_{it}^{1-r_{tO}}\left(x_i^{r_{tO}}-x_{it}^{r_{tO}}\right) & \ldots\ldots\ x_i\ge x_i^L \\ \dfrac{1}{r_{tM}}\dfrac{\partial g(X_t)}{\partial x_i}\left(1-e^{-r_{tM}(x_i-x_{it})}\right) & \ldots\ldots\ x_i<x_i^b \end{cases}$$

Where, X_{it} (t=1,…,H; i =1,…,n) are the known points; H is the number of points to be counted; n is the number of design variables in a point; $g(X_t)$ is the function values and $\tilde{g}(X)$ is the approximated values. And, $h_t(X)$ is the weighting function, which can be determined as: -

$$h_t(X)=\frac{\bar{h}_t(X)}{\sum_{l=1}^{H}\bar{h}_l(X)} \qquad t=1,\ldots,H,$$

$$\bar{h}_l(X)=\prod_{\substack{S=1\\S\ne l}}^{H}(X-X_S)^T(X-X_S) \qquad l=1,\ldots,H$$

The exponent r_{tO} and r_{tM} are the adaptive parameters to control the non-linearity of the approximation, they to be found from the following equations respectively: -

$$g(X_H)=\left[g(X_t)+\frac{1}{r_{tO}}\sum_{i=1}^{n}\frac{\partial g(X_t)}{\partial x_i}.x_{it}^{1-r_{tO}}(X_{iH}^{r_{tO}}-X_{it}^{r_{tO}})\right]$$

$$g(X_H)=\left[g(X_t)+\frac{1}{r_{tM}}\sum_{i=1}^{n}\frac{\partial g(X_t)}{\partial x_i}.(1-e^{-r_{tM}(x_{iH}-x_{it})})\right]$$

However, the proposed MIMA is expected to be of the advantages of both MA and IMA. The layered strategy to solve the first-level approximate problems (2) passes through a series of a layered optimization, where the topology variables of the trusses are optimized using GA technique in the external layer, then the problem transferred to the second-level approximation where the cross-sectional areas of the bars are optimized in the internal layer and solved by the duel method. The required structural analysis for truss topology optimization can be dramatically decreased as GA is only used to solve the approximate problems in the external layer where no structural analysis is needed. On the other hand, a relatively small number of species is taken in GA as the design variables of cross-sectional areas are determined in internal layer.

III. NMERICAL EXAMPLES

Different examples from literature are chosen to demonstrate the validity and compare the efficiency of the modified integrated multipoint approximation function with the most well-known examples 10-bar and the 72-bar trusses, and other example is a ground truss structure:

A. *Example 1. The Ten-Bar Truss*

For the structure response, the 10-bar truss, Fig.1, is loaded by $1*10^5$ lbs. at node 2 and 4 in –Y direction, where the displacement constraints ±2in, and the stress limits $\pm25*10^3$psi, The Young's modulus is $E=1*10^7$ lb./ in^2 and the specific weight 0.1 lb./in^3. Initial cross-sectional area of each bar is 30in^2 and the area lower limit is equal to 0.1in^2. The Optimized Ten-bar, truss is shown in Fig. 2. Comparison is made for both sizing and topology optimization. The iteration history data is shown in table (1). Table (2) shows the final optimum design variable results and number of analysis. Results of the proposed MIMA method reaches the optimum solutions after a relatively less number of iterations compared with others. The proposed method shows very efficient results.

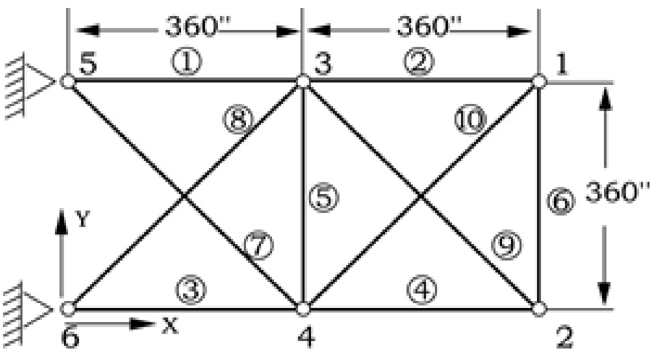

Fig. 1. *The Ten Bar Truss*

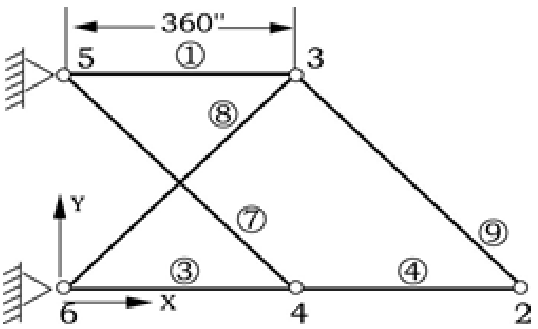

Fig. 2. *The Optimized Ten-Bar Truss*

TABLE I. ITERATION HISTORY DATA FOR 10-BAR TRUSS

No. Of Analysis	Weights (lbs)				
	MIMA	IMA Ref. [6]		Ref. [4]	Ref. [8]
	Topology	Sizing	Topology	Sizing	Topology
1	12589.4	12589.4	12589.4	8266.1	12589.4
2	6041.31	3963.83	3858.32	6061.3	6082.56
3	5028.62	6682.63	4813.14	5816.3	5687.49
4	4928.81	5959.35	6041.31	5482.0	5215.43
5	4898.90	5928.46	5970.80	5540.1	4851.24
6	4898.97	5798.99	5891.55	5106.2	4816.31
7		5579.61	5721.89	5262.0	4882.06
8		5252.91	5407.93	5076.9	4867.04
9		5130.36	5229.04	5065.1	4884.60
10		5100.00	5028.62	5075.1	4873.68
11		5074.42	4910.50	5062.7	4881.59
12		5068.69	4928.81	5067.4	4896.10
13			4898.90		4895.04
14			4899.68		4897.34
…					…
19					4899.39

TABLE II. FINAL OPTIMUM DESIGN VARIABLE RESULTS, FOR10-BAR TRUSS.

No. Of Variables	Weights (lbs)				
	MIMA	IMA Ref. [6]		Ref. [4]	Ref. [8]
	Topology	Sizing	Topology	Sizing	Topolog y
1	30.62	30.79	30.1107	30.62	30.2897
2	0	0.0935	0	0.1	0
3	22.1332	23.154	22.1317	23.28	21.4207
4	15.0568	15.086	15.0522	15.13	15.1451
5	0	0.161	0	0.1	0
6	0	0.670	0	0.529	0
7	6.0659	7.3	6.0724	7.503	6
8	21.3065	21.327	21.2948	21.1	21.4184
9	21.2935	21.334	21.2871	21.4	21.4184
10	0	0.130	0	0.1	0
No. Of Analysis	6	12	14	12	19
Weights [lbs.]	4898.97	5068.69	4899.68	5067.4	4899.39

B. *Example 2. The 72-Bar Truss*

For the structure response, the 72-bar truss, Fig. 3. Where the displacement constraints ±0.25, for other details see ref. [9]. The Optimized 72-bar, truss is shown in Fig. 4. The iteration history results are shown in table. (3). The final optimum design variable results are in table (4) Clearly, the present study has remarkable number of analysis for the topology comparing with the former one and the final optimal design variable results have considerable precise agreement.

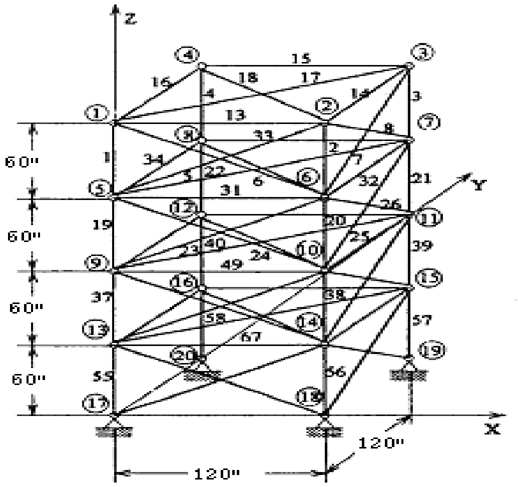

Fig. 3. *The 72-Bar Truss*

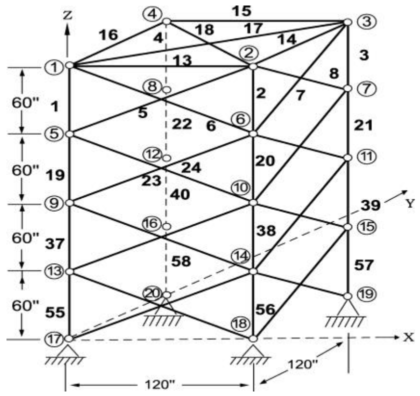

Fig. 4. *The optimized 72-Bar Truss*

TABLE III. ITERATION HISTORY DATA FOR 72-BAR TRUSS

No. Of Analysis	Weights (lbs)				
	MIMA	IMA Ref. [6]		Ref. [4]	Ref. [8]
	Topology	Sizing	Topology	Sizing	Topology
1	853.09	853.09	853.09	656.77	853.09
2	386.88	345.19	154.68	386.4	650.83
3	352.56	558.57	267.28	368.17	518.23
4	356.75	428.51	386.88	364.82	452.33
5	365.11	409.39	322.48	364.69	438.97
6	366.12	393.32	339.74		420.77
7	362.93	384.64	352.56		414.75
8	365.95	388.69	356.75		403.64
9	362.41	375.11	365.11		360.90
10	362.92	370.01	366.12		327.15
11		367.42	362.93		375.01
12		365.85	365.95		368.34
13		364.69	362.41		362.54
14			364.12		360.64
…			…		362.64
20			362.36		362.58
21					362.35
…					…
28					362.30

TABLE IV. FINAL OPTIMUM DESIGN VARIABLE RESULTS FOR 72-BAR TRUSS

No. Of Variables	Weights (lbs)				
	MIMA	IMA Ref. [6]		Ref. [4]	Ref. [8]
	Topology	Sizing	Topology	Sizing	Topology
1	0.169	0.168	0.167	0.158	0.167
2	0.534	0.535	0.535	0.537	0.535
3	0.454	0.434	0.452	0.412	0.452
4	0.583	0.593	0.571	0.562	0.572
5	0.521	0523	0.519	0.508	0.519
6	0.519	0.519	0.517	0.520	0.517
7	0	0.0219	0	0.1	0
8	0.112	0.0747	0.129	0.1	0.128
9	1.296	1.285	1.29	1.280	1.293
10	0.519	0.516	0.517	0.515	0.517
11	0	0.0213	0	0.1	0
12	0	0.0155	0	0.1	0
13	1.890	1.892	1.885	1.899	1.8846
14	0.519	0.516	0.517	0.516	0.517
15	0	0.0213	0	0.1	0
16	0	0.0155	0	0.1	0
No. Of Analysis	10	13	20	5	28
Weights [lbs.]	362.92	364.69	362.364	364.69	362.302

C. *Example 3. The Ten-Node, 2D Truss ground structure*

The proposed method is applied to the ten-node truss Fig. 5, with ground structure with all possible interconnection a total of 34 members; this example is taken from reference [10]. The parameters used for this example are as follows: The structure is loaded by $10*10^3$ lbs at node 3, 5, & 7 in –Y direction, where the displacement constraints ±2in, and the stress limits $\pm 25*10^3$psi, The Young's modulus is $E=1*10^7$ lb/ in^2 and the specific weight 0.1lb/in^3.

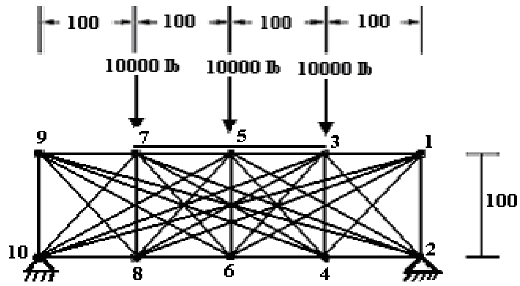

Fig. 5. *Ten-Node truss ground structure*

Optimized Ten-node, truss ground structure is shown in Fig. 6, as well as the optimized solution from ref. [6] & [10] for comparison. The optimized solution table 5 shows present study has same topology as ref. [10], and overlapping members are shown in the optimum solution. The cross-sectional areas are different for those overlapping members, but are almost identical for others; the objective function is comparable.

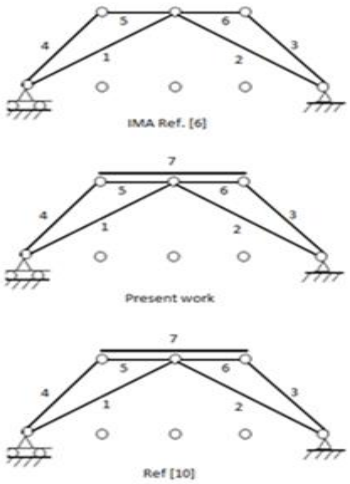

Fig. 6. *The Optimized Ten-Node Truss ground structure*

TABLE V. CROSS-SECTIONAL AREA OF THE OPTIMIZED TEN-NODE TRUSS GROUND STRUCTURE

Member No.	Cross-sectional area (in^2)		
	MIMA	IMA, Ref. [6]	Ref. [10]
1	0.566	0.447	0.477
2	0.447	0.447	0.477
3	0.447	0.566	0.566
4	0.566	0.566	0.566
5	0.2	0.4	0.082
6	0.2	0.4	0.082
7	0.2	0	0.321
No. of Analysis	10	-	-
Weights [lb]	43.995	44.2708	44.033

IV. CONCLUSION

One can conclude from the results based on the conducted computational trusses examples, that the efficiency of the proposed MIMA method for truss topology and size optimization further improved. It also presents that the MIMA has a higher quality of approximation compared to IMA, as GA is just used to solve the approximate problems. The proposed method can reach the optimum solution of a problem with both topology and size design variables after few structural analyses, which is even comparable to solve the optimization problem with only the cross-sectional size as variable. Moreover, the ground trusses structure results also assure that the proposed model results are satisfying and comparable for topology optimization.

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