

Performance of Linear Multiuser Detection in Gaussian and non-Gaussian Channels

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Abstract— Direct-sequence code-division multiple access (DS-CDMA) is a popular wireless technology. In DS-CDMA systems, all of the users' signals overlap in frequency and time and cause multiple access interference (MAI). Multiuser detection schemes are used to detect the users' data in the presence of MAI. In this paper, a comparison study of linear multiuser detectors in the presence of MAI, under the condition of Gaussian noise channels in DS-CDMA communication systems has been presented, the impact of Near-Far problem and non-Gaussian (uncertain) noise on the performance of linear multiuser detectors also have been investigated. Simulation results show that the linear multiuser detectors have more performance gain over the conventional matched filter and the performance of all investigated detectors suffers severely under the condition of a certain range of impulsive noise channels.

Keywords— DS-CDMA, multiuser detection, matched filter, decorrelating, MMSE, non-Gaussian noise.

I. INTRODUCTION

Code-division multiple access (CDMA) is one of several methods of multiplexing wireless users. In CDMA, users are multiplexed by signature waveforms rather than orthogonal time slots as in time-division multiple access (TDMA) or by orthogonal frequency bands as in frequency-division multiple access (FDMA). Each user in CDMA system is allocated the entire available frequency band for transmission, and all users can transmit at the same time.

Direct-sequence code division multiple-access (DS-CDMA) is the most popular CDMA techniques. At the DS-CDMA transmitter, the user's signal is multiplied by a distinct code. At the receiver, the received signal is a composed of the sum of all users' signals that overlap in frequency and time. In a conventional DS-CDMA system, a particular user's signal is detected by correlating the entire received signal with that user's code waveform.

DS-CDMA systems operate just fine under the ideal conditions of orthogonal and synchronized codes of all users. It however suffer under real time non-ideal conditions which are experienced during the practical operation of the systems. Due to non-ideal orthogonality and difficulty in maintaining

synchronization at receiver, each user gets interference from many other users attempting multiple access, the interference due to multiple access is called Multiple Access Interference (MAI). While the MAI caused by any user is generally small, as the number of interferers or their power increases, MAI becomes substantial. The conventional detector does not take into account the existence of MAI. It follows a single-user detection strategy in which each user is detected separately without regard for other users.

In order to mitigate the problem of MAI, Verdu [1] proposed and analyzed the optimum multiuser detection for asynchronous Gaussian multiple access channels. This optimum detector significantly outperforms the conventional detector and it is near-far resistance, but unfortunately its complexity grows exponentially with the number of users, in order to reduce the complexity of the receiver and yet provide acceptable BER performances, significant research efforts have been invested in the field of sub-optimal CDMA multiuser receivers. A sub-optimum linear detector for symbol-synchronous transmission suggested in [2] which was termed the decorrelating detector. A further developed it for asynchronous transmission in Gaussian channel [3]. It was shown that this decorrelator exhibited the same degree of near-far resistance as the optimum multiuser detector. A further sub-optimum multiuser detector investigated was the minimum mean square error (MMSE) detector. A multiuser detector for a synchronous CDMA system designed for a frequency-selective Rayleigh fading channel proposed in [4]. The decorrelating detector of [3] was developed for differentially-encoded coherent multiuser detection in flat fading channel by Zvonar [5]. Zvonar also combined the decorrelating detector with diversity combining, in order to achieve performance improvements in frequency selective fading channels [6].

Mostly, the previous studies focused on the situation in which the ambient noise is additive white Gaussian noise (AWGN). Unfortunately in many physical channels the ambient noise is non-Gaussian. In this paper, we will investigate and compare the performances of conventional, decorrelator, and MMSE detectors, under the condition of Gaussian and impulsive noise channels, and in the presence of the MAI in a synchronous single path DS-CDMA system. The impulsive noise is modeled by a two-term Gaussian mixture distribution model that is known as ϵ -contaminated Gaussian mixture model.

This paper is organized as follows: in Section II, the system model is explained and Section III introduces an impulsive noise model. In Sections IV and V the single user matched filter and its performance in AWGN channel are presented, respectively. The effects of MAI and near far problem are explained in Section VI. The MAI suppression and Linear Multiuser detection techniques are presented in Section VII. In Sections VIII and IX the decorrelating detector and its performance in AWGN channel are presented, respectively. In Sections X and XI the MMSE detector and its performance in AWGN channel are presented, respectively. Simulation results in AWGN channel are presented in Section XII. The impact of impulsive noise on the performance of each detector is simulated in Section XIII, Conclusion presented in Section XIV.

II. SYSTEM MODEL

We assume a K-user BPSK modulated DS-CDMA communication system, where each user transmits its signal as a single DS-CDMA transmitter.

At the receiver, and by assuming a single-path synchronous DS-CDMA channel where all bits of all users are aligned in time and for simplicity we assume that all carrier phases are equal to zero, this enables us to use baseband notation while working only with real signals. So the base-band received signal for one bit interval can be represented as

$$r(t) = \sum_{k=1}^K A_k(t)s_k(t)b_k(t) + n(t), \quad t \in [0, T_b] \quad (1)$$

where $A_k(t)$ is the received amplitude of the k th user's signal, $b_k \in [-1, +1]$ is the bit transmitted by the k th user, T_b is the inverse of the data rate, $n(t)$ is the additive noise, and $s_k(t)$ is the deterministic signature waveform assigned to the k th user normalized so as to have unit energy

$$\|s_k\|^2 = \int_0^{T_b} s_k(t)dt = 1. \quad (2)$$

The signature waveforms are assumed to be zero outside the interval $[0, T_b]$, and therefore there is no inter-symbol interference.

The signature waveform $s_k(t)$ consists of Q chips, each with a duration T_c . The duration of the code sequence is equal to the duration of one data bit T_b . The chip rate $R_c = 1/T_c$ and, therefore, the bandwidth of the signal after modulation with the code is much higher than the data rate $R_b = 1/T_b$ which approximately equal to the bandwidth of the baseband signal. The k th user signature waveform can be expressed as

$$s_k^{(n)} = \sum_{m=0}^{N_c-1} s_{k,m}^{(n)}\psi(t - mT_c) \quad (3)$$

where, $s_{k,m}^{(n)}$ is the m th chip of user k on the symbol interval n , T_c is the length of the chip period, $N_c = T_b/T_c$ is the spreading factor, $\psi(t)$ is the chip waveform which will be assumed binary, i.e., $s_{k,m}^{(n)} \in \{-1, 1\}$. In the short code $s_k^{(n)} = s_k^{(i)} = s_k$.

III. NOISE MODEL

The additive noise $n(t)$ in (1), models the parts of the received signal that not due to the transmitters in the multiuser communication system [7]. The Gaussian noise assumption incorporates the receiver thermal noise and background electromagnetic noise. The additive white Gaussian noise model has been widely used in communication theory due to its mathematical tractability for analysis and optimum solutions and design simplicity. The Gaussian noise assumption is justified by Central Limit Theorem (CLT).

In many situations, the Gaussian noise assumption may not be adequate and justified any more. For example, in many physical channels, such as urban and indoor radio channels [8-10] and under water acoustic channels [11] the ambient noise is known through experimental measurements to be decidedly non-Gaussian. The non-Gaussian noise is characterized as being of impulsive nature because it occurs with noticeable probabilities of large amplitudes for short duration. The non-Gaussian impulsive noise come from man-made or natural. The man-made interference such as car ignition systems, switching transients, neon lights and other electronic devices. The natural noise such as atmospheric noise in radio links due to lightning discharges, ambient acoustic noise in underwater sonar and submarine communications due to ice cracking in the arctic region.

Many models of non-Gaussian have been developed; these models can be divided into two categories: empirical and physical models [12-14]. Middleton class A, B, and C models are widely used physical models [15, 16]. The Symmetric Alpha Stable ($S \alpha S$) probability density functions can accurately model large classes of impulsive noise [17]. An ϵ -mixture (or ϵ -contaminated) model is one of the commonly used empirical models [18]. In this research, we will model additive non-Gaussian noise as ϵ -mixture model.

The first-order probability density function (pdf) of the ϵ -mixture model has the form

$$f_\epsilon(x) = (1 - \epsilon) \cdot f_{bg}(x) + \epsilon \cdot f_{im}(x) \quad (4)$$

where, $\epsilon \in [0, 1]$ represent the mixture weighting coefficient and $f_{bg}(x)$ and $f_{im}(x)$ are the pdf's corresponding to background noise and impulsive noise, respectively. The pdf $f_{bg}(x)$ is usually taken to be Gaussian. The pdf $f_{im}(x)$ is chosen as one of

various heavy-tailed pdf's such as Laplacian or double exponential and the Gaussian with large variance. In case of Gaussian pdf $f_{im}(x)$, the ratio of the variance of impulsive component to the variance of the background one, defined as $\gamma^2 = \sigma_{im}^2 / \sigma_{bg}^2$, is usually taken to be between 1 and 100 [12]. In this paper, we adopt the commonly used two-term Gaussian mixture model. The probability density function (pdf) of this noise model has the form

$$f_\varepsilon(x) = (1 - \varepsilon) \cdot N(0, \sigma_G^2) + N(0, \gamma^2 \sigma_G^2) \quad (5)$$

Where σ_G^2 represents the Gaussian noise variance. The total noise variance

$$\sigma^2 \triangleq (1 - \varepsilon) \cdot \sigma_G^2 + \varepsilon \gamma^2 \sigma_G^2 \quad (6)$$

This model has been used extensively to model physical noise arising in radio acoustic channels [10].

IV. SINGLE USER MATCHED FILTER

The basic receiver that used to recover each user data is called Matched Filter (MF) or (Conventional Detector). The MF is a bank of K correlators, as shown in Fig. 1, where each code waveform is regenerated and correlated with the received signal in a separate detector branch, then the outputs of the MF are sampled at the bit times, which yields "soft" estimates of the transmitted data. The final ± 1 "hard" data decisions are made according to the signs of the soft estimates [19]

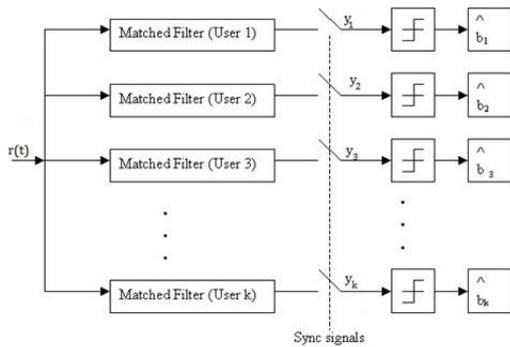


Fig. 1 Matched Filter receiver

The MF is considered as a single-user detector where each branch detects one user and treats the other users as a noise. The success of this detector depends on the properties of the correlations between codes. We require the correlations between the same code waveforms (i.e., the autocorrelations) to be much larger than the correlations between different codes (i.e., the cross-correlations). the correlation value is defined as

$$R_{i,k} = \int_0^{T_b} s_i(t) s_k(t) dt. \quad (7)$$

Here, if $i = k$, $R_{i,k} = 1$ and if $i \neq k$, $0 \leq R_{i,k} < 1$. The output of the k th user's correlator for a particular bit interval is

$$y_k = \int_0^{T_b} r(t) s_k(t) dt. \quad (8)$$

$$= b_k A_k + \sum_{\substack{i=1 \\ i \neq k}}^K R_{i,k} b_i A_i + \int_0^{T_b} n(t) s_k(t) dt. \\ = b_k A_k + MAI_k + n_k \quad (9)$$

where the first term in (9) represents the received data, the second term represents the Multiple Access Interference (MAI), and the third one represents the additive noise. The output of the MF for all users can be represented in a discrete-time matrix-vector model as

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_k \end{bmatrix}_{K \times 1} = \begin{bmatrix} 1 & R_{2,1} & \dots & R_{k,1} \\ R_{1,2} & 1 & \dots & R_{k,2} \\ \vdots & \vdots & \ddots & \vdots \\ R_{1,k} & R_{2,k} & \dots & 1 \end{bmatrix}_{K \times K} \begin{bmatrix} A_1 & 0 & \dots & 0 \\ 0 & A_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & A_k \end{bmatrix}_{K \times K} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_k \end{bmatrix}_{K \times 1} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_k \end{bmatrix}_{K \times 1}$$

Or

$$\mathbf{y} = \mathbf{R} \mathbf{A} \mathbf{b} + \mathbf{n} \quad (10)$$

where for a K user system, the vectors \mathbf{b} and \mathbf{y} are K -vectors that hold the data and matched filter outputs of all K users, respectively; \mathbf{n} is additive random noise vector. In the condition of Gaussian noise it represent a zero-mean Gaussian random vector with covariance matrix equal to $\sigma^2 \mathbf{R}$, the matrix \mathbf{A} is a diagonal matrix containing the corresponding received amplitudes; the matrix \mathbf{R} is a $K \times K$ correlation matrix, where entries contain the values of the correlations between every pair of codes. Note that since $R_{i,k} = R_{k,i}$, the matrix \mathbf{R} is clearly symmetric.

In (9), if the used codes are orthogonal (i.e., $R_{i,k} = 0$ and $i \neq k$), so the value of MAI will be zero and the system will act as a single-user DS-CDMA [20], but the required bandwidth is approximately equal to $B = R_b K / 2$ which achievable by Time Division Multiple Access (TDMA) and Frequency Division Multiple access (FDMA) [7]. By removing the restriction of orthogonal signature waveforms and acceptance of tolerable MAI provides some benefits that make CDMA an attractive multiple access technique for practical communication systems [7].

V. MATCHED FILTER PROBABILITY OF ERROR IN AWGN CHANNEL

As derived before in the synchronous case, the k th user matched filter output is

$$y_k = b_k A_k + MAI_k + n_k.$$

If the signature waveforms of the k th user is orthogonal to all others, resulting in $R_{j,k} = 0$ for $j \neq k$, then the MF output for user k reduced to single user condition:

$$y_k = A_k b_k + n_k \quad (11)$$

In the case of AWGN where $\varepsilon=0$ in (5), the probability of error of a threshold comparison of y_k is $P_k(\sigma) = Q(A_k / \sigma)$.

In the condition of non-orthogonal signature waveforms, we can write the bit-error-rate (BER) of the k th user as:

$$P_k^c = P[b_k = +1]P[y_k < 0 | b_k = +1] + P[b_k = -1]P[y_k > 0 | b_k = -1]$$

$$= \frac{1}{2}P \left[n_k > A_k - \sum_{j \neq k} A_j b_j R_{jk} \right] + \frac{1}{2}P \left[n_k < -A_k - \sum_{j \neq k} A_j b_j R_{jk} \right] \quad (12)$$

And by symmetry

$$P_k^c = P \left[n_k > A_k - \sum_{j \neq k} A_j b_j R_{jk} \right]$$

$$= \frac{1}{2^{K-1}} \sum_{e_1 \in \{-1,1\}} \dots \sum_{e_j \in \{-1,1\}} \dots \sum_{e_k \in \{-1,1\}} Q \left(\frac{A_k}{\sigma} + \sum_{j \neq k} e_j \frac{A_j}{\sigma} \rho_{jk} \right) \quad (13)$$

Where $Q(t) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$, is the complementary Gaussian cumulative distribution function. The average of Q -function in (13) is upper bounded by

$$P_k^c \leq Q \left(\frac{A_k}{\sigma} - \sum_{j \neq k} \frac{A_j}{\sigma} |R_{jk}| \right) \quad (14)$$

The number of operations required for the computation of (13) grows exponentially in the number of users. For this reason, a number of authors have approximated (13) by replacing the binomial random variable

$$\sum_{j \neq k} A_j b_j R_{jk} \quad (15)$$

By a Gaussian random variable with identical variance. The approximated bit-error rate becomes

$$\tilde{P}_k^c(\sigma) = Q \left(\frac{A_k}{\sqrt{\sigma^2 + \sum_{j \neq k} A_j^2 R_{jk}^2}} \right) \quad (16)$$

Whereas at low signal-to-noise ratios the approximation of (13) by (16) is generally good, for high signal-to noise ratios it may be unreliable [7].

VI. MULTIPLE ACCESS INTERFERENCE AND NEAR FAR PROBLEM

At the output of MF, The amount of MAI increases as the number of interfering users increases, and/or

the received signal powers of the interfering users increase. Especially, when there exist interfering users with high powers, the strong MAI dominates over a weak received signal, which results in a near-far problem. The conventional MF detector is highly sensitive to the near-far problem. The near-far problem is thus a limiting factor to the capacity and performance of the conventional DS/CDMA systems in spite of the fact that spread spectrum, by its very nature, is an interference-tolerant modulation [21]. The performance of the conventional MF detector is acceptable if the received signal powers are not too dissimilar and the cross-correlations of the spreading codes are low enough.

To mitigate the effect of MAI, some of the research efforts have focused on several areas [19].

- Signature waveform design: this approach is aimed at the design of spreading sequences with good cross-correlation [22, 23].
- Power control: the use of power control ensures that all users arrive at about the same power (amplitude), open-loop power control and closed loop power control are used for successful DS-CDMA system.
- FEC Codes: The design of more powerful forward error correction (FEC) codes allows acceptable error rate performance at lower signal-to-interference ratio levels.
- Sectorized/adaptive Antennas and Multiple Input Multiple Output (MIMO) antennas systems.
- Multiuser Detectors (MUD): where information about multiple users is used to improve detection of each user [7, 20].

VII. MAI SUPPRESSION AND LINEAR MULTIUSER DETECTION

The most important approach to solve the near-far problem is MAI suppression which is also known as wideband interference suppression. The MAI suppression can be classified into multiuser detection and single-user detection depending on its detection structure as in [21]. Multiuser detection is fully centralized, while single-user detection is fully decentralized. In general, the single-user detection requires knowledge of only one user's (or desired user's) signal parameters such as spreading code delay, and power, but not that of the interfering users' parameters. The multiuser detection requires knowledge of all users' signals parameters [12].

An optimum multiuser detector with minimum Probability of error and near-far resistance was proposed by Verdu [1]. The detector consists of a bank of Matched filters followed by a maximum likelihood sequence detector. The optimum multiuser detector requires knowledge of the spreading codes, delays, and powers of all active users. The computational

complexity increases exponentially with the number of users. Since the detector is too complex to be used in practical DS/CDMA systems, most research efforts have focused on the development of suboptimum multiuser detectors which have good near-far resistance, lower computational complexity, and low probability of error. Most suboptimum multiuser detectors can be classified into one of two categories: linear and nonlinear [19].

A class of linear suboptimum multiuser detectors includes Decorrelating and Minimum Mean Squared Error (MMSE) detectors. This class of detectors applies a linear mapping to the soft output of the conventional MF's to reduce the MAI seen by each user [19] as shown in Fig. 2.

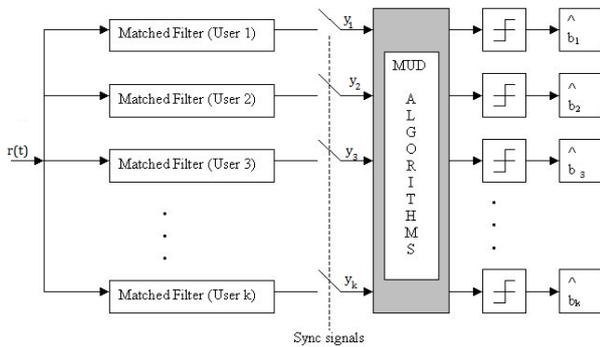


Fig. 2 Multiuser detector for DS-CDMA system

A class of nonlinear suboptimum multiuser detectors is divided into three classes such as successive interference cancellation (SIC) [24-26], multistage detection [27] (or parallel interference cancellation (PIC) [28]), and decision-feedback detection [29, 30]. The basic principle is to subtract out some or all of the MAI by estimates of the MAI at the receiver.

VIII. DECORRELATING DETECTOR

The output vector of the bank of matched filter outputs can be written as in (10)

$$\mathbf{y} = \mathbf{R}\mathbf{A}\mathbf{b} + \mathbf{n}$$

The decorrelating (DEC) detector applies the inverse of the correlation matrix, $L_{dec} = \mathbf{R}^{-1}$ to the matched filter bank outputs. The soft estimate of this detector

$$\tilde{\mathbf{b}}_{dec} = \mathbf{R}^{-1}\mathbf{y} = \mathbf{A}\mathbf{b} + \mathbf{R}^{-1}\mathbf{n} \quad (17)$$

So the k th component of (17) is free from interference caused by any of the other users, in other words there is no MAI. The only source of interference is the background noise. Therefore, the decorrelating detector is seen to give the best joint estimate of the transmitted bits in the absence of any prior knowledge about the received amplitudes [7].

The DEC detector has some attractive properties as:

- It does not require knowledge of the received amplitudes.

- Provide substantial performance over than Matched Filter.
- Has computational complexity significantly lower than that of the maximum likelihood sequence detector.
- Has a probability of error independent of the signal energies.
- It can readily decentralized, where each user can be implemented completely independently.

Disadvantages of the DEC detector as:

- The signature waveforms of all users must be known.
- The timing of all users must be acquired.
- The matrix inversion \mathbf{R}^{-1} must be computed.
- It causes noise enhancement.

IX. DECORRELATING DETECTOR PERFORMANCE ANALYSIS IN AWGN CHANNEL

From (17), the output of decorrelating detector only has two components: one due to the signal of user k , which is equal to $A_k b_k$, and the other due to the additive noise.

In the Gaussian noise assumption where $\epsilon=0$, the noise has zero mean and variance equal to the kk component of the covariance matrix $\sigma^2 \mathbf{R}^{-1}$. consequently, the k th user bit-error-rate (BER) is simply

$$P_k^d(\sigma) = Q\left(\frac{A_k}{\sigma\sqrt{R_{kk}^+}}\right), \quad (18)$$

where, R_{kk}^+ is a shorthand for $(\mathbf{R}^{-1})_{kk}$. The BER of the DEC detector is independent of the interference amplitudes. The multiuser efficiency is the ratio between the effective and actual energies $e_k(\sigma)/A_k^2$, where, the effective energy of user k , $e_k(\sigma)$ is the energy that user k would require to achieve BER equal to $P_k(\sigma)$ in a single-user Gaussian channel with the same additive noise level. The DEC detector's multiuser efficiency is equal to

$$\eta_k^d = \frac{1}{R_{kk}^+}, \quad (19)$$

Which does not depend on either the noise level or the interference amplitudes, and thus, it is equal to the asymptotic multiuser efficiency, that is can be defined as $\eta_k = \lim_{\sigma \rightarrow 0} (e_k(\sigma)/A_k^2)$, and the near-far resistance, that is can be expressed as,

$$\bar{\eta}_k^d = \frac{1}{R_{kk}^+}. \quad (20)$$

So, the decorrelating detector achieves the maximum near-far resistance. Accordingly, knowledge of the received amplitudes is not required to combat the

near-far problem optimally, and the same degree of robustness against imbalances in the received amplitudes as that of the MF detector [7].

From (17), (18) the price paid for the complete elimination of MAI is noise enhancement and by regarding the behavior of decorrelating detector when the background Gaussian noise is dominant, we can say that unless the k th user is orthogonal to all the interferers, the single-user matched filter has lower BER than the decorrelating detector for sufficiently low SNR.

X. MINIMUM MEAN SQUARE DETECTOR

The minimum mean-squared error (MMSE) detector is a linear detector that takes into account the background noise and utilizes knowledge of the received signal powers. This detector implements the linear mapping that minimizes $E[|\mathbf{b} - \mathbf{L}\mathbf{y}|^2]$, the mean-squared error between the actual data and the soft output of the conventional detector.

$$\mathbf{L}_{\text{MMSE}} = [\mathbf{R} + \sigma^2 \mathbf{A}^{-2}]^{-1} \quad (21)$$

The soft estimate of this detector

$$\tilde{\mathbf{b}}_{\text{MMSE}} = \mathbf{L}_{\text{MMSE}} \mathbf{y} \quad (22)$$

The MMSE detector implements a modified inverse of the correlation matrix. The amount of modification is directly proportional to the background noise; the higher the noise level, the less complete an inversion of \mathbf{R} can be done without noise enhancement causing performance degradation. Thus, the MMSE detector balances the desire to completely eliminate MAI with the desire to not enhance the background noise.

Because it takes the background noise into account, the MMSE detector generally provides better probability of error performance than the DEC detector. As the background noise goes to zero, the MMSE detector converges in performance to the DEC detector.

One of the important disadvantage of this detector is that, unlike the DEC detector, it requires estimation of the received amplitudes. Another disadvantage is that its performance depends on the powers of the interfering users. Therefore, there is some loss of the resistance of near-far problem as compared to the DEC detector. Finally, As DEC detector, the MMSE faces the task of implementing matrix inversion [7, 19].

XI. MMSE DETECTOR PERFORMANCE ANALYSIS IN AWGN CHANNEL

Because the linear MMSE detector does not vanish the MAI, the analysis of its BER is not straight forward as that of the DEC detector. For example, the output of linear MMSE for user $k=1$ can be written as

$$(\mathbf{L}_{\text{MMSE}} \mathbf{y})_1 = ([\mathbf{R} + \sigma^2 \mathbf{A}^{-2}]^{-1} \mathbf{y})_1 \\ B_1 \left(b_1 + \frac{1}{B_1} \sum_{k=2}^K B_k b_k \right) + \tilde{n}_1 \quad (23)$$

Where

$$B_k = A_k (\mathbf{L}_{\text{MMSE}} \mathbf{y})_{1k}$$

and in AWGN channel

$$\tilde{n}_1 \sim N(0, \sigma^2 (\mathbf{L}_{\text{MMSE}} \mathbf{R} \mathbf{L}_{\text{MMSE}})_{11}).$$

then, the probability of error follows:

$$P_1^m(\sigma) \\ = 2^{1-K} \sum_{b_2, \dots, b_k \in \{-1, 1\}^{K-1}} Q \left(\frac{A_1}{\sigma} \frac{(\mathbf{L}_{\text{MMSE}} \mathbf{R})_{11}}{\sqrt{(\mathbf{L}_{\text{MMSE}} \mathbf{R} \mathbf{L}_{\text{MMSE}})_{11}}} \left(1 + \frac{1}{B_1} \sum_{k=2}^K B_k b_k \right) \right) \quad (24)$$

To evaluate (24) we face an exponential (in K) number of terms. However, $P_1^m(\sigma)$ can be generally accurately approximated by replacing the MAI by Gaussian random variable with identical variance: $Q(\sqrt{\text{SIR}_1})$, then the probability of error can be written as:

$$P_1^m(\sigma) = Q \left(\frac{\mu}{\sqrt{1 + \lambda^2}} \right), \quad (25)$$

Where

$$\mu = \frac{A_1}{\sigma} \frac{(\mathbf{L}_{\text{MMSE}} \mathbf{R})_{11}}{\sqrt{(\mathbf{L}_{\text{MMSE}} \mathbf{R} \mathbf{L}_{\text{MMSE}})_{11}}}$$

And

$$\lambda^2 = \frac{\mu^2}{B_1^2} \sum_{k=2}^K B_k^2$$

The accuracy of this approximation has been supported by several analytical results in [7, 31].

If we hold all the amplitudes fixed and let $\sigma \rightarrow 0$, then $[\mathbf{R} + \sigma_n^2 \mathbf{A}^{-2}]^{-1} \rightarrow \mathbf{R}^{-1}$. Therefore, as signal-to-noise ratios go to infinity, the linear MMSE detector converges to DEC detector. And that implies that the MMSE linear detector has the same asymptotic efficiency and near-far resistance as the DEC detector.

XII. SIMULATION RESULTS IN AWGN CHANNEL

In this section, we provide some simulation results to demonstrate the performance of the matched filter, decorrelating detector and the Minimum mean square error detector. The simulation is performed for synchronous DS-CDMA system in AWGN channel, the signature waveform for each user is Gold code of length $N_c=31$, and the number of bits for each user equal to 10^5 bits.

First, we compare the performance of the examined detectors against the MAI, which is represented by large number of active users equals 25.

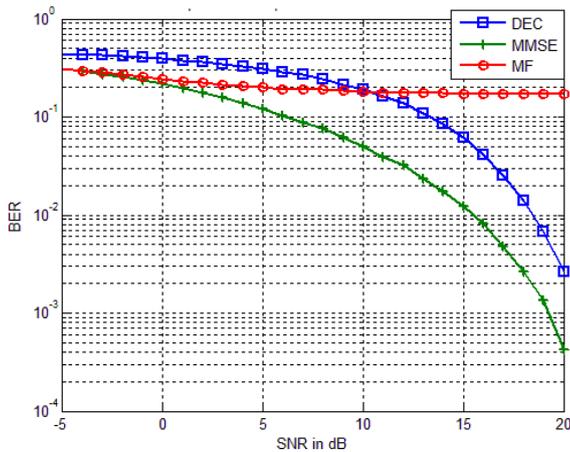


Fig. 3 performance comparison of MF, DEC and MMSE detectors in AWGN, K=25, all users have the same received amplitudes.

The BER comparison versus SNR in AWGN is presented in Fig. 3. It is observed that at low SNR, the performance of MF is better than the DEC detector because it does not enhance the noise; but with sufficient SNR, the DEC performance is better than the MF performance due to the MAI elimination. The MMSE detector has the best performance over the decorrelating and matched filter detector.

Fig. 4 – Fig. 6, show the results of the examination of the resistance of each one of the examined detectors to the Near-Far problem.

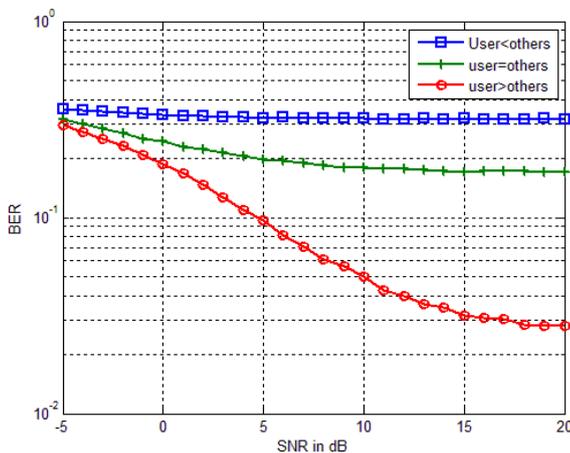


Fig. 4 BER performance of MF detector versus SNR where the observed user amplitude is less than, equal and greater than the other active users

From Fig. 4, it is observed that the performance of MF is affected strongly by the Near Far problem (power of observed user < power of other users), so a perfect power control may be needed for increasing the performance efficiency, which means more complexity.

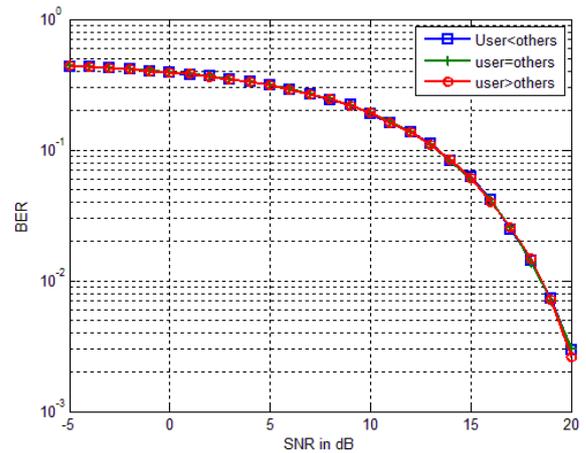


Fig. 5 BER performance of DEC detector versus SNR where the observed user amplitude is less than, equal and greater than the other active users.

It is clear from Fig. 5, that the DEC detector has approximately the same BER performance for all scenarios (power of observed user < / > = power of other users), and this is because the DEC detector completely suppresses the effect of MAI (17).

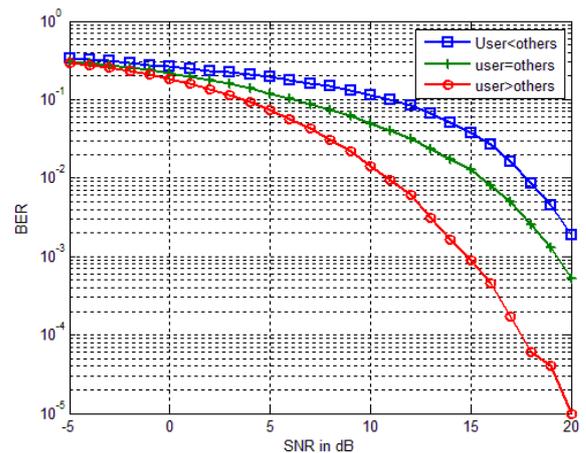


Fig. 6 BER performance of MMSE detector versus SNR where the observed user amplitude is less than, equal and greater than the other active users.

Fig. 6, shows that the MMSE detector performance is affected by the Near Far problem because the MMSE detector balances the desire to decouple the users (and eliminate MAI) with the desire to not enhance the background noise.

The effect of increasing the number of active users in the communication system is presented in Fig. 7. It is clear that as the number of active users in the communication system increases, the performances of all examined detectors degrade (BER increase). Based on the simulation results and analytical evidence, the BER of the MMSE detector is better than that of the DEC detector for all levels of Gaussian background noise, number of users, and cross-correlation matrices.

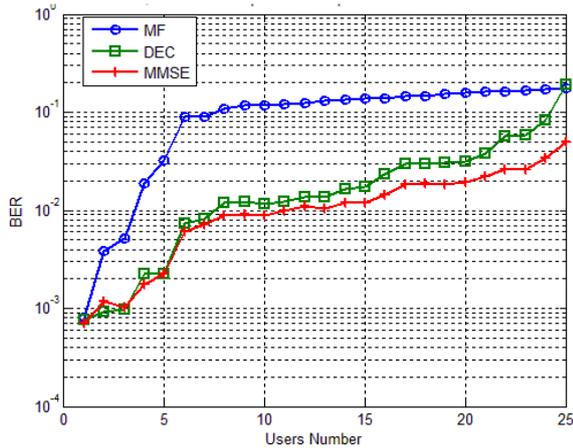


Fig. 7 BER performance versus the no of users in a constant SNR=10dB and equal received amplitudes for all users

XIII. THE IMPACT OF IMPULSIVE NOISE ON THE PERFORMANCE OF EACH DETECTOR

In order to demonstrate the impact of the presence of the impulsive noise on the detectors' performance, the simulation parameters are defined as follows: a synchronous DS-CDMA system, the signature waveform for each user is Gold code of length $N_c=31$, and the number of bits for each user equal to 10^5 bits. We are interested in the effects of variations in the shape of noise distribution on the performance of examined detectors. These variations in the shape of the impulsive noise distribution, acts as uncertain noise environments for the examined detectors.

First, we will vary the parameters ϵ and γ^2 with the total noise variance held constant at each SNR value. In Fig. 8, we demonstrate the performance of MF for several different ϵ -mixture channels where the values of ϵ and γ^2 are corresponding to some practical examples [32].

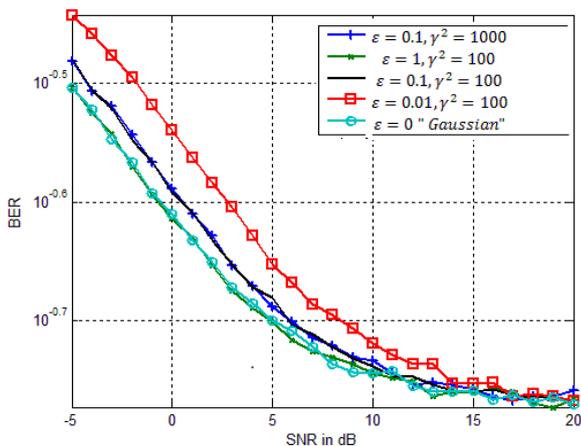


Fig. 8 BER of MF in Gaussian and ϵ -mixture non-Gaussian channels, $K=25$.

Fig. 8, compares impulsive non-Gaussian channels to the Gaussian one, these curves indicates a degradation in the performance over the entire range of interest of SNR's, with fairly significant degradation

in some cases. One interesting observation comes from comparing the two curves in Fig. 8 corresponding to $\epsilon = 0.01, \gamma^2 = 100$ and $\epsilon = 0.1, \gamma^2 = 100$. In these cases with a fixed SNR, increasing the amount of contaminating noise from $\epsilon = 0.01$ to $\epsilon = 0.1$, improving the performance of the MF. This occurs because the total noise variance is held constant and thus, with fixed γ^2 , variations in the performance is not monotonic with changes in ϵ . In fact, the two channels, one with $\epsilon = 0$ "Gaussian" and the other with $\epsilon = 1$, result in identical BER.

In Fig. 9, the BER performance of the MF, DEC and MMSE detectors are examined in various ϵ values with a fixed SNR=5dB. It is clear that, a breakpoint of error probability versus ϵ for this example is approximately between 0^+ and 0.2.

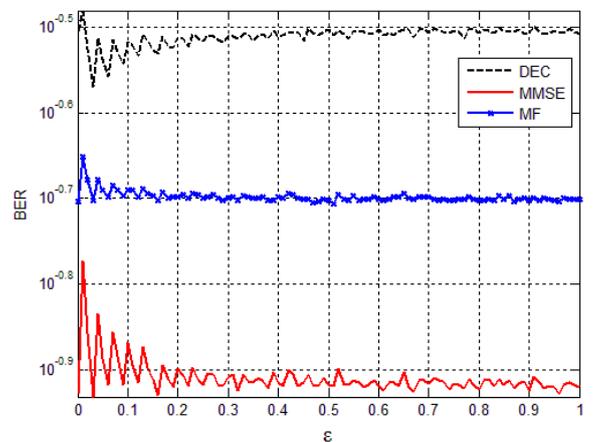


Fig. 9 BER performance of MF, DEC and MMSE detectors versus ϵ with fixed SNR=5dB, number of active users $K=25$.

Fig. 10, compares the BER of the MF, DEC and MMSE detectors in Gaussian and non-Gaussian ϵ -mixture noise with $\epsilon = 0.01$, in different values of signal-to noise ratios for number of users $K=25$. The results indicate a degradation in performance over the entire range of interest of SNR.

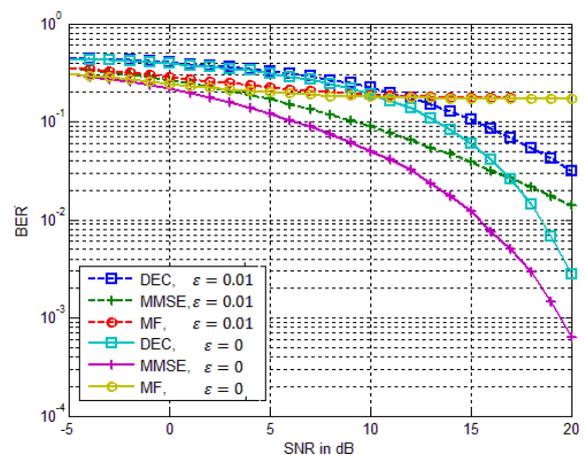


Fig. 10 BER versus SNR of the MF, DEC and MMSE detectors in a synchronous DS-CDMA channel with Gaussian and ϵ -mixture noise at $\epsilon=0.01, K=25$.

XIV. CONCLUSION

Multiple access interference significantly limits the performance and capacity of conventional DS-CDMA systems. In multiuser detection, code and timing information of multiple users is jointly used to better detect each individual user. Linear multiuser detectors have a significant performance gain over the conventional matched filter. MMSE detector generally performs better than the decorrelator detector because it takes the background noise into account. By increasing the number of users, the performance of all examined detectors will degrade; this is because as the number of interfering users increases, the amount (effect) of MAI becomes greater. Also the performance of all detectors become poor in the presence of impulsive noise.

REFERENCES

- [1] S. Verdu, "Minimum probability of error for asynchronous Gaussian multiple-access channels," *IEEE transactions on information theory*, vol. 32, pp. 85-96, 1986.
- [2] R. Lupas and S. Verdu, "Linear multiuser detectors for synchronous code-division multiple-access channels," *IEEE transactions on Information Theory*, vol. 35, pp. 123-136, 1989.
- [3] R. Lupas and S. Verdu, "Near-far resistance of multiuser detectors in asynchronous channels," *IEEE transactions on Communications*, vol. 38, pp. 496-508, 1990.
- [4] Z. Zvonar and D. Brady, "Suboptimal multiuser detector for frequency-selective Rayleigh fading synchronous CDMA channels," *IEEE Transactions on Communications*, vol. 43, pp. 154-157, 1995.
- [5] Z. Zvonar and D. Brady, "Differentially coherent multiuser detection in asynchronous CDMA flat Rayleigh fading channels," *IEEE Transactions on Communications*, vol. 43, pp. 1252-1255, 1995.
- [6] Z. Zvonar, "Combined multiuser detection and diversity reception for wireless CDMA systems," *IEEE Transactions on Vehicular Technology*, vol. 45, pp. 205-211, 1996.
- [7] S. Verdu, *Multiuser detection*: Cambridge university press, 1998.
- [8] K. L. Blackard, T. S. Rappaport, and C. W. Bostian, "Measurements and models of radio frequency impulsive noise for indoor wireless communications," *IEEE Journal on selected areas in communications*, vol. 11, pp. 991-1001, 1993.
- [9] T. K. Blankenship, D. Kriztman, and T. S. Rappaport, "Measurements and simulation of radio frequency impulsive noise in hospitals and clinics," in *Vehicular Technology Conference, 1997, IEEE 47th*, 1997, pp. 1942-1946.
- [10] X. Wang and H. V. Poor, "Robust multiuser detection in non-Gaussian channels," *IEEE Transactions on Signal Processing*, vol. 47, pp. 289-305, 1999.
- [11] P. L. Brockett, M. Hinich, and G. R. Wilson, "Nonlinear and non-Gaussian ocean noise," *The Journal of the Acoustical Society of America*, vol. 80, pp. S64-S65, 1986.
- [12] S. Lee, "Interference suppression for DS/CDMA systems in non-Gaussian impulsive channels," 1998.
- [13] D. Middleton, "Statistical-physical models of urban radio-noise environments-Part I: Foundations," *IEEE Transactions on Electromagnetic Compatibility*, pp. 38-56-, 1972.
- [14] D. Middleton, "Man-made noise in urban environments and transportation systems: Models and measurements," *IEEE Transactions on communications*, vol. 21, pp. 1232-1241, 1973.
- [15] D. Middleton, "Non-Gaussian noise models in signal processing for telecommunications: new methods and results for class A and class B noise models," *IEEE Transactions on Information Theory*, vol. 45, pp. 1129-1149, 1999.
- [16] A. Spaulding and D. Middleton, "Optimum reception in an impulsive interference environment-Part I: Coherent detection," *IEEE Transactions on Communications*, vol. 25, pp. 910-923, 1977.
- [17] M. Shao and C. Nikias, "On symmetric stable models for impulsive noise," *Univ. Southern California, Los Angeles, Tech. Rep. USC-SIPI-231*, 1993.
- [18] K. Vastola, "Threshold detection in narrow-band non-Gaussian noise," *IEEE Transactions on Communications*, vol. 32, pp. 134-139, 1984.
- [19] S. Moshavi, "Multi-user detection for DS-CDMA communications," *IEEE communications magazine*, vol. 34, pp. 124-136, 1996.
- [20] L. Hanzo, L.-L. Yang, E. Kuan, and K. Yen, *Single-and multi-carrier DS-CDMA: multi-user detection, space-time spreading, synchronisation, standards and networking*: John Wiley & Sons, 2003.
- [21] J. Laster and J. Reed, "Interference rejection in digital wireless communications," *IEEE signal processing magazine*, vol. 14, pp. 37-62, 1997.
- [22] J. G. Proakis and D. G. Manolakis, *Digital Signal Processing: Principles, Algorithms, and Applications*: Prentice Hall, 1996.
- [23] M. Sarraf and M. Karim, "W-CDMA and cdma2000 for 3G Mobile Networks," 2002.
- [24] A. J. Viterbi, "Very low rate convolution codes for maximum theoretical performance of spread-spectrum multiple-access channels," *IEEE Journal on Selected Areas in Communications*, vol. 8, pp. 641-649, 1990.
- [25] P. Patel and J. Holtzman, "Analysis of a simple successive interference cancellation scheme in

a DS/CDMA system," IEEE journal on selected areas in communications, vol. 12, pp. 796-807, 1994.

[26] K. I. Pedersen, T. Kolding, I. Seskar, and J. Holtzman, "Practical implementation of successive interference cancellation in DS/CDMA systems," in the Proceedings of the Int. conf. On Universal Personal Communications, 1996, pp. 321-325.

[27] M. K. Varanasi and B. Aazhang, "Multistage detection in asynchronous code-division multiple-access communications," IEEE Transactions on communications, vol. 38, pp. 509-519, 1990.

[28] R. Kohno, H. Imai, M. Hatori, and S. Pasupathy, "Combinations of an adaptive array antenna and a canceller of interference for direct-sequence spread-spectrum multiple-access system," IEEE journal on Selected Areas in Communications, vol. 8, pp. 675-682, 1990.

[29] A. Duel-Hallen, "Decorrelating decision-feedback multiuser detector for synchronous code-division multiple-access channel," IEEE Transactions on Communications, vol. 41, pp. 285-290, 1993.

[30] A. Duel-Hallen, "A family of multiuser decision-feedback detectors for asynchronous code-division multiple-access channels," IEEE Transactions on Communications, vol. 43, pp. 421-434, 1995.

[31] H. V. Poor and S. Verdú, "Probability of error in MMSE multiuser detection," IEEE transactions on Information theory, vol. 43, pp. 858-871, 1997.

[32] B. Aazhang and H. Poor, "Performance of DS/SSMA communications in impulsive channels-Part I: Linear correlation receivers," IEEE transactions on communications, vol. 35, pp. 1179-1188, 1987.