

# High Resolution Techniques for Direction of Arrival Estimation of GPR Signals

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**Abstract—** In this paper, the Direction of Arrival (DOA) estimation is implemented using a stepped-frequency ground penetrating radar data for anti-personal landmines detection. Different experiments for different anti-personal landmines for different depths have been achieved. The techniques used in this paper are high-resolution techniques such as Multiple Signal Classification (MUSIC), and EigenVector (EV) methods. These techniques are compared with the classical method, Fast Fourier Transform (FFT). The practical results show the better performance of high resolution techniques when compared with the classical method.

**Keywords—** GPR; Landmines; IFFT; DOA; MUSIC; EV

## I. INTRODUCTION

The problem of estimating the wave number or angle of arrival of a plane wave is referred to as direction finding or DOA estimation problem.

In signal processing literature, DOA denotes the direction from which usually a propagating wave arrives at a point, where usually a set of sensors are located. This set of sensors forms what is called a sensor array. In many signal processing applications a set of unknown parameters should be estimated from measurements collected by array of sensors.

It has a large application in radar, sonar, seismic systems, electronic surveillance, medical diagnosis and treatment, radio astrology and other areas.

Because of its widespread applications and difficulty of obtaining an optimum estimator, the topic has a received a significant amount of attention over the last several decades.

Several methods exist to address the problem of estimating the direction of - arrivals (DOAs) of multiple sources using the signals received at the sensors. The application of the array processing requires either the knowledge of a reference signal or the direction of the desired signal source to achieve its desired objectives.

The algorithms of DOA estimation can be classified into two algorithms classical such as Fast Fourier Transform (FFT) and subspace method such as Multiple Signal Classification (MUSIC)[1,2], and EigenVector (EV) [3]. The main advantage of

subspace methods over the conventional method is that the aperture of the entire array does not limit the resolution, therefore these also known as super resolution techniques [4]. The aims of this research are to make an investigation about modern DOA estimation methods such as Fast Fourier Transform (FFT) [5], Multiple Signal Classification (MUSIC), and Eigenvector (EV) methods. These methods based on eignanalysis of an autocorrelation matrix of the received signal. Then, these methods are compared with the conventional method Fast Fourier Transform (FFT).

There are more than 119 million mines were buried in 71 countries in the world. The number of mine victims is greater than the number of the victims of nuclear and chemical weapons together.

The detection and identification techniques for buried objects have been a great interest to researchers for many years. There are many techniques have been studied. One of them is the metal detector approach (MD) which is still used, but it can't be used for the detection of plastic and low metal landmines. The most important technique to be used is the ground penetrating radar (GPR) which is being used widely in many science fields [6].

The Ground Penetrating Radar (GPR) is used the difference in permittivity of both mine and the surrounding medium to detect the target [7]. However it is difficult for the GPR to detect the target if it has very small dimensions or has a permittivity near to that of the ground. In these cases, the reflected signal of the target is very weak compared to that of the ground and noise, making it difficult to distinguish between both without proper signal processing. Thus, in order to extract useful information about the target, it is necessary to apply proper signal processing by using a stepped frequency continues wave radar (SFCW) to overcome the problem of high instantaneous bandwidth and high sampling rate of the pulsed systems[8].

Generally, ground penetration radar (GPR) is a narrow bandwidth device and its radar range is normally high, a wide bandwidth is greatly desired to enclose all target images, which is difficult to make because it is limited by antenna size in the low-frequency range and underground propagation characteristics in the high-frequency range [9, 10]. In order to overcome these problems, improvement of

frequency resolution is greatly desired. Moreover, improvement of resolution is very important for GPR to trace out closely buried targets, like gas pipes, water pipes, cables, and so forth, in an urban area and also to detect the buried land mines that cause thousands of human life every year throughout the world [11, 12].

Several signal processing approaches have been suggested to improve the performance of GPR systems. These include: simple mean scan subtraction [13], two-dimensional digital filtering [14], wavelet packet decomposition [13], likelihood ratio test [13], [16], parametric system identification [17], and Kalman filter [15], [16]. Most of these methods depend on the background signal estimation by taking the mean value of the unprocessed ensemble collected GPR data, followed by employing the simple mean scan subtraction. Although these methods have been used widely in GPR applications. Various researchers have shown interest in subspace techniques including Singular Value Decomposition (SVD) [13], [18], Linear Discriminant Analysis (LDA) [19], Principal Component Analysis (PCA) [20], [21], and Independent Component Analysis (ICA) [10], [22]–[24].

This work set out to explore various spectral estimation techniques that can be applied to GPR imaging systems. The goal of this work is to advance the use of the most popular DOA algorithm to estimation GPR Signals. In this paper, the application possibility of super resolution method MUSIC algorithm, Eigenvector (EV) methods and conventional FFT (Fast Fourier Transform) are examined for signal processing of GPR.

## II. DATA MODEL

Let us consider the received GPR data being represented by the A-scans  $x_i(k)$  ( $i=1,2,..,m$ ;  $k=1,2,..,K$ ), where  $i$  denotes the antenna position index and  $k$  denotes the time index.  $X$  is the matrix holding the  $M$  A-Scans in each row with  $K$  time samples,  $X = [X_1, \dots, x_M]^T$  where  $x_i(t) \in \mathbb{C}^{1 \times K}$  And  $X(t) \in \mathbb{C}^{M \times K}$

## III. FOURIER TRANSFORM

Fourier analysis is an important tool in the area of signal analysis and processing. With its help, it can be determined which harmonic signals with different amplitudes, frequencies, and phases. Traditional FFT-based methods to process signal and phase history data into images are widely used, even though they suffer from poor resolution and high side lobe artifacts. However, modern spectral estimation methods provide an attractive alternative that can improve resolution, help eliminate image speckle effects, and increase the accuracy of interferometric height estimates. These methods promise to improve the clarity and applicability of GPR imaging for many applications.

Discrete Fourier Transform

It is a kind of Discrete Transform which is used in Fourier analysis. It transforms one function into another, which is called the frequency domain representation, or simply the DFT, of the original function.

In most SF GPR systems, the windowed Inverse Discrete Fourier Transform (IDFT) processing is used to transform the raw radar data into the spatial domain, providing the impulse response of the radar [25, 26, 27]. (Typically the Inverse Fast Fourier Transform is used, which is simply a computationally efficient implementation of the IDFT). Often the amplitudes of the resulting impulse response are related to the reflectivity of targets without consideration for the properties of the medium [25, 26, 27].

Fourier transforms of the temporal records  $x(t)$  defined as:

$$X(f) = \int_0^{\infty} x(t) e^{-2j\pi ft} dt \quad (1)$$

It is noted that the lower integration limit is 0 since  $x(t) = 0$  when  $t < 0$

Since the collected measurements are discrete data point within finite time duration, assume that  $N$  points of data are evenly sampled at time interval  $\Delta t$ . The signal  $x(t)$  at  $t = n \Delta t$  ( $n = 0, 1, 2, \dots, N-1$ ), are replaced by discrete form.

$$x_n = x(n\Delta t) \quad n = 0, 1, 2, \dots, N-1 \quad (2)$$

In addition, the Fourier transform at  $N$  discrete frequency values  $f_k$  ( $k = 0, 1, 2, \dots, N-1$ ) is computed, where

$$f_k = k f = \frac{k}{N\Delta t} \quad (3)$$

Note that, for computational purposes, the number of discrete frequency values is the same as the number of sampled data. Then the discrete Fourier transform is obtained via

$$X_k = \frac{X(f_k)}{\Delta t}, \quad k = 0, 1, 2, \dots, N-1 \quad (4)$$

Thus, the discrete version of (Equ.1) are readily shown as

$$X_k = \sum_{n=0}^{N-1} x_n \exp\left[\frac{-2j\pi kn}{N}\right] \quad (5)$$

## IV. MUSIC

Multiple Signal Classification (MUSIC) is the most popular technique used in DOA estimation. We can summarize DOA estimation as the work of estimating the direction of an unknown incoming signal to a receiver antenna by some processing techniques.

The MUSIC method is a relatively simple and efficient eigen structure method of DOA estimation. It has many variations and is perhaps the most studied method in its class. In its standard form, also known as spectral MUSIC, the method estimates the noise subspace from available samples. This can be done by either eigenvalue decomposition of the estimated array correlation matrix or singular value decomposition of the data matrix, with its columns being the snapshots of the array signal vectors. The latter is preferred for numerical reasons. Once the noise subspace has been estimated, a search for angle pairs in the range is made by looking for steering vectors that are as orthogonal to the noise subspace as possible. This is normally accomplished by searching for peaks in the MUSIC spectrum.

The measured value of M-A-Scans reflected signal from the target with a vector network analyzer can be expressed using vector notation as follows:

$X = [X_1, \dots, x_M]^T$  where  $X$  is the matrix holding the  $M$  A-Scans in each row with  $K$  time samples,  $x_j(t) \in C^{1 \times K}$  And  $X(t) \in C^{M \times K}$ .

In general, MUSIC assumes that every  $X_{ik}$  is a linear combination of each  $S_j$  as follows:

$$X_M(t) = \sum_j^N a_m(\theta_j) s_j(t) + w(t) \quad (6)$$

$j = 1, 2, \dots, N$

or in the matrix notation

$$X = As + W \quad (7)$$

where  $X = [X_1, \dots, x_M]^T$  is the matrix holding the  $M$  A-Scans in each row with  $K$  time samples,  $T$  represents transpose.  $X$  is the  $(M \times K)$  matrix,  $A$  is an  $(M \times N)$  matrix and  $S$  is the  $(N \times K)$  matrix

Again  $a(\theta_i)$  is the array steering vector corresponding to the DOA of the  $i$ -th signal., so it is called a mode vector. The symbol  $A$  is a delay parameter matrix which has  $M$  numbers of arrays and the  $i$ th element of row. So,  $M$  can be regarded as the number of signals while the symbol  $s$  is the  $(N \times K)$  matrix.  $s_j(k)$  ( $j = 1, 2, \dots, N$ ) is the reflection coefficient of the  $N$ th reflection point .and  $W$  is the  $(M \times K)$  noise matrix. We assume that  $N(1 \leq N \leq M - 1)$ .

The  $M \times M$  signal covariance matrix  $R_{xx}$  of  $x$  vector is represented by

$$R_{xx} \in C^{M \times M}$$

$$R_{xx} = AE[SS^H]A^H + E[WW^H]$$

$$R_{xx} = AR_{ss}A^H + R_{nn}$$

Where

$R_{ss} \in C^{N \times N}$  is the signal correlation matrix  $E[SS^H]$ ,  $H$  is the hermitian (transpose of complex conjugate) ,  $R_{nn} \in C^{M \times M}$  is the signal correlation matrix  $E[WW^H]$  and  $E[\bullet]$  denotes the statistical expectation.

Assuming uncorrelated AWGN

$$R_{xx} = AR_{ss}A^H + \sigma_k^2 I \quad (9)$$

$\sigma_k^2 I$  is the noise correlation matrix,  $\sigma_k^2$  is the power of noise,  $I$  is The unit matrix of  $M \times M$ .

The eigen decomposition of matrix  $R$  yields

$$R_{xx} = \sum_{i=1}^M \lambda_i q_i q_i^H$$

where  $\lambda_i$ , and  $q_i$  are the  $i$ -th eigenvalue and  $i$ -th corresponding eigenvector, respectively. In the ideal environment, we have

$$\lambda_1 \geq \lambda_2 \geq \lambda_N > \lambda_{N+1} = \dots = \lambda_M = \sigma^2$$

The eigenvalues of  $R_{xx}$  are the values  $\lambda_1, \lambda_2, \dots, \lambda_M$  such that:

$$|R_{xx} - \lambda_i I| = 0 \quad (10)$$

Using Equation (9):

$$|AR_{ss}A^H + \sigma_k^2 I - \lambda_i I| = |AR_{ss}A^H - (\lambda_i - \sigma_k^2) I|$$

Therefore, the eigenvalues of,  $v_i$  of  $AR_{ss}A^H$  are:

$$v_i = \lambda_i - \sigma_k^2 \quad (11)$$

Since  $A$  is comprised of steering vectors which are linearly independent, it has full column rank, and the signal correlation matrix  $R_{ss}$  is non-singular as the incident signals are not highly correlated. A full column rank  $A$  matrix and non-singular  $R_{ss}$  guarantees that, when the number of incident signals  $N$  is less than the number of array elements  $M$ , the  $M \times M$  matrix  $AR_{ss}A^H$  is positive semi-definite with rank  $N$ . From elementary linear algebra, this implies that  $M - N$  of the eigenvalues  $v_i$ , of  $AR_{ss}A^H$  are zero. From Equation (11), this means that  $M - N$  of the eigenvalues of  $R_{xx}$  are equal to the noise Variance  $\sigma_k^2$ . We then sort the eigenvalues of  $R_{xx}$  such that  $\lambda_1$  is the largest eigenvalue, and  $\lambda_M$  is the smallest eigenvalue. Therefore,

$$\lambda_N, \dots, \lambda_M = \sigma_k^2 \quad (12)$$

In practice, however, when the autocorrelation matrix  $R_{xx}$  is estimated from a finite data sample, all the eigenvalues corresponding to the noise power will not be identical. Instead they will appear as a closely spaced cluster, with the variance of their spread decreasing as the number of samples used to obtain an estimate of  $R_{xx}$  is increased. Once the multiplicity,  $z$ , of the smallest eigenvalue is determined, an estimate of the number of signals,  $\hat{N}$  can be obtained by Equation (13).

$$\hat{N} = M - z \quad (13)$$

In this research, the number of incident signals  $N$  is known. The eigenvector  $q_i$  associated with a particular eigenvalue  $\lambda_i$  the vector such that

$$R_{xx} - \lambda_i I = 0 \quad (14)$$

For eigenvectors associated with the  $(M - N)$  smallest eigenvalues, we have

$$(R_{xx} - \sigma_k^2)q_i = AR_{ss}A^Hq_i + \sigma_k^2I - \sigma_k^2I = 0$$

$$AR_{ss}A^Hq_i = 0$$

Since A has full rank and  $R_{ss}$  is non-singular, this implies that

$$A^Hq_i = 0 \quad (15)$$

This means that the eigenvectors associated with the  $(M - N)$  smallest eigenvalues are orthogonal to the  $N$  steering vectors that make up A.

$$\{a(\theta_1), \dots, a(\theta_N)\} \perp \{q_{N+1}, \dots, q_M\}$$

This is the essential observation of the MUSIC approach. It means that one can estimate the steering vectors associated with the received signals by finding the steering vectors which are most nearly orthogonal to the eigenvectors associated with the eigenvalues of  $R_{xx}$  that are approximately equal to  $\sigma_k^2$ .

This analysis shows that the eigenvectors of the covariance matrix  $R_{xx}$  belong to either of the two orthogonal subspaces, called the principle eigensubspace (signal subspace) and the non-principle eigensubspace (noise subspace). The steering vectors corresponding to the DOA lie in the signal subspace and are hence orthogonal to the noise subspace. By searching through all possible array steering vectors to find those which are perpendicular to the space spanned by the non-principle eigenvectors, the DOA can be estimated.

To search the noise subspace, we form a matrix  $U_k \in \mathbb{C}^{M \times M-N}$  containing the noise eigenvectors:

$$U_k = [q_{N+1} \ q_{N+2} \ \dots \ q_M] \quad (16)$$

Since the steering vectors corresponding to signal components are orthogonal to the noise subspace eigenvectors,  $a^H(\hat{\theta})U_kU_k^Ha(\hat{\theta})$  for  $(\hat{\theta})$  corresponding to the DOA of a multipath component. Then the DOAs of the multiple incident signals can be estimated by locating the peaks of a MUSIC spatial spectrum given by:

$$P_{\text{MUSIC}}(\hat{\theta}) = \frac{1}{a^H(\hat{\theta})U_kU_k^Ha(\hat{\theta})} \quad (17)$$

$(\theta)$ : Array steering vector, and  $Q_n$  noise subspace  $= [q_{N+1}, q_{N+2}, \dots, q_M]$ .

$U_k$  denotes an  $M$  by  $M - N$  dimensional matrix with its  $M - N$  columns being the eigenvectors corresponding to the  $M - N$  smallest eigenvalues of the array correlation matrix.

$a^H$  is the hermitian (transpose of complex conjugate) of the steering vector that is used for scanning the range of meaningful angles for the user.

The orthogonality between the noise subspace and the steering vectors will minimize the denominator and hence will give rise to peaks in the MUSIC spectrum defined in Equation (17). The largest peaks in the MUSIC spectrum correspond to the signals impinging on the array. From Equ. (17), we can estimate the DOA by searching the peak value [28].

## V. EIGEN VECTOR METHOD (EV)

In addition to the MUSIC algorithm, a number of other eigenvector methods have been proposed for estimation the DOA. One of these, the EigenVector (EV) method. The EigenVector is closely relate to the MUSIC algorithm. Specifically, the EV method estimates the exponential frequencies from the peaks of the eigenspectrum:

$$P_{\text{EV}}(\theta) = \frac{1}{\sum_{i=N+1}^M \frac{1}{\lambda_i} |U_k^H a(\theta)|^2} \quad (18)$$

Where  $\lambda_i$  is the eigenvalue associated with eigenvector  $Q_n$ .  $(\theta)$ : Array steering vector

The only difference between the EV method and MUSIC is the use of inverse eigen value (the  $\lambda_i$  are the noise subspace eigen values of R) weighting in EV and unity weighting in MUSIC, which causes EV to yield fewer spurious peaks than MUSIC [28] The EV Method is also claimed to shape the noise spectrum better than MUSIC.

## VI. GPR MEASUREMENTS

The data have been acquired with a bistatic-stepped frequency GPR system at IESK, Magdeburg University, Germany. The system consists of a network analyzer (Rohde & Schwarz) and two ultra-wideband (UWB) transmitting and receiving antennas [29]. A wooden box with dimensions  $1.1 \times 1.1 \times 1.1$  m whose internal sides are covered by absorption material and is filled by sand of 0.5 m height has been used. The transmitting and receiving antennas are mounted on a 2D scanning system and were placed above the ground surface at height 30 cm.

The measurement grid covers the area bounded by  $x = 27 \rightarrow 76$  cm and  $y = 39 \rightarrow 89$  cm with a distance between the measurements of 1 cm in both x and y directions. The measurements then form a two dimensional matrix, referred to as a B-scan. Column vector of the B-scan matrix (image) is called an A-scan and it represents the data, at each individual point on the surface of the soil. Using this experimental setup, two different measurements were made. In the first measurement the radar system was operated in the frequency range from 1 GHz to 4 GHz and the number of samples was 1024 for each A-scan.

In the second measurement the radar system was operated in the frequency range from 1.5 GHz to 20 GHz and the number of samples was 1601 for each A-scan. Examples of A-scans in the presence and absence of a landmine for both measurements in the frequency domain and time domain are displayed in Fig. 1 and Fig. 2 respectively. PMN anti-personal landmine was used at different depths.

Sample B-Scans showing PMN targets at different depths are displayed in Fig.3.

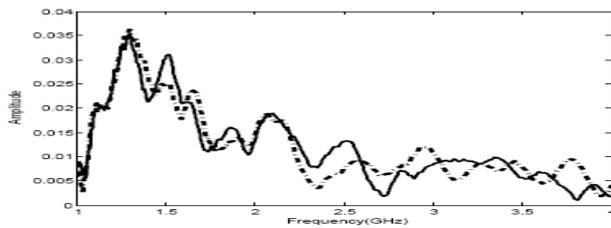


Fig.1 a) A-scans in the presence (dashed) and absence (solid) of a mine for data with 3 GHz bandwidth in the frequency domain.

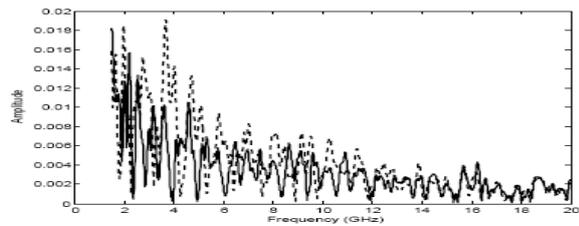


Fig.1 b) A-scans in the presence (dashed) and absence (solid) of a mine for data with 18.5 GHz bandwidth in the frequency domain.

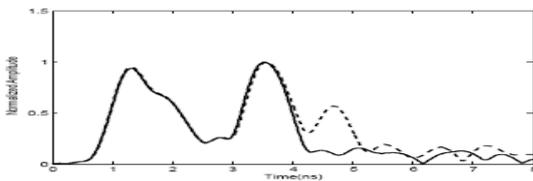


Fig.2 a) A-scans in the presence (dashed) and absence (solid) of a mine for data with 3 GHz bandwidth in the time domain.

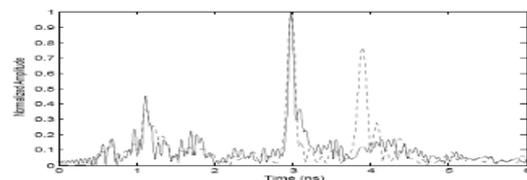


Fig.2 b) A-scans in the presence (dashed) and absence (solid) of a mine for data with 18.5 GHz bandwidth in the time domain.

It is observed in Figure 4 that the IDFT response gives correct estimation of target signal. However, resolution is very poor and increasing by using window with IDFT. In MUSIC response, the output signal is very sharp and the resolution is very high as it is estimated from the peak of the MUSIC function. From the results from different experiments, the IFFT could not resolve two closely located targets well, that is, the delay-time difference between the successive signal is very small. On the other hand, MUSIC could resolve the same two closely located targets. Also eigenvector method (EV) gives the same result as Music method.

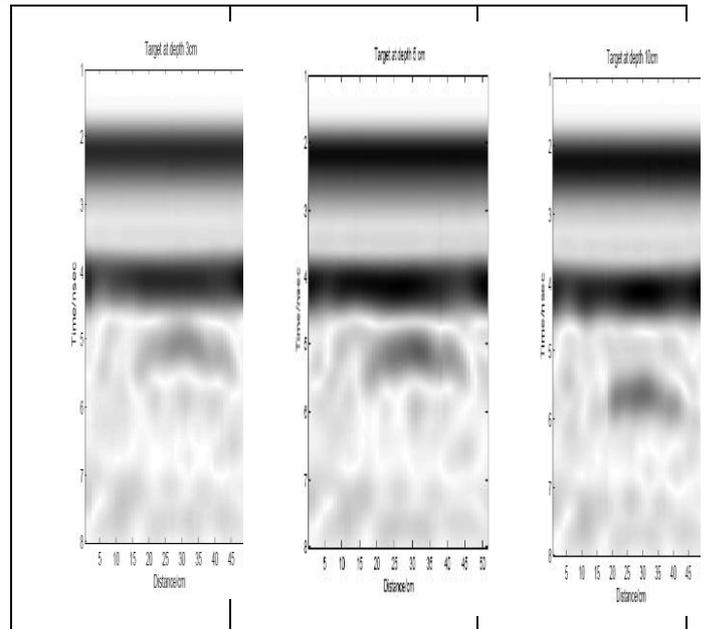


Fig.3 B-Scans of PMN anti-personal landmine at depth 3, 5, and 10cm, respectively.

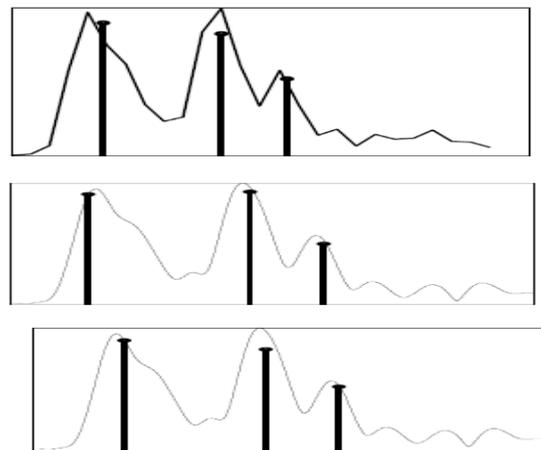


Fig.4 a) the result of using IDFT and music algorithm. B) the result of using IDFT with window and music algorithm. C) the result of using IDFT with window and eigenvector method (EV).

## VII. CONCLUSION

Three different signal processing techniques for detection anti-personal Landmine in GPR data have been presented, namely, inverse discrete fourier transform with and without window, music and eigenvector (EV) method. The time-domain response of IFFT, music and eigenvector (EV) method have been compared.

The efficiency of techniques has been experimentally verified using two different sets of raw SFGPR. From experimental, IDFT response gives correct estimation of target signal but resolution is very poor. on the other hand, the output signal of music is very sharp and the resolution is very high. Also eigenvector method (EV) gives the same result as Music method.

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