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Dynamics of Poverty and Drug Addiction in Sylhet, Bangladesh

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***Abstract*— In this article, we introduce a compartmental mathematical model, which is attempted to understand the dynamics of poverty and drug addiction. In the model, we have an addicted compartment, which allows for an approach of government and non-government interventions. The stability analysis in this model holds for an addiction free equilibrium. We instated that, the equilibrium is locally asymptotically stable when the reproduction number is less than 1 and unstable when it is greater than 1. Numerical simulations of the systems have been presented to show the variations of the population in different situations. We also find out that, the high rate of interventions will reduce poverty and drug addiction and snatching faster. Our aim is to reduce poverty and addiction to their barest minimum. Data that have been used for simulations are based on the addiction happens in the district of Sylhet in Bangladesh. Some of the data we used are actually based on estimations from the results of our personal survey and secondary source (since primary data are not always available because of government restrictions and strictness in law and order). But we found that, our approximations are actually correct enough to present the model appropriately.**

Keywords*— *ODE; epiedmiology; stability Analysis; compartmental model; reproduction number; poverty; drug addiction; intervention

I. INTRODUCTION

Nowadays epidemiological researches are becoming more and more realistic based on many social problems. Some epidemiologists consider socio-economical problems like:- poverty, crime etc. as epidemics. They started to develop mathematical models based on correlation between poverty, crime, prostitution etc. These works are actually based on works by Kelly, 2000 [4]; Block and Heineke, 1975. These have inspiration from Becker's economic theory of crime (1968), described in [3]. According to [4] [22], the people living in poverty have a eminent chance of committing property crime than the general population. Property crime can be defined as burglary, larceny, or theft (O'Conner, 2005). In Becker's paper [3], he used statistical and economic analysis to obtain the optimal control of crime. On the other hand, in the paper [6], the respective epidemiologists developed a model explaining how crime becomes concentrated in certain neighbourhoods, where the prospective financial return from committing a crime (the possibility of not being arrested times the reward of the crime) exceeds the opportunity costs for crime. Wang and colleagues [21] generalized this phenomena by allowing opportunity costs to be heterogeneous across potential criminals and depend on the level of crime in a particular neighbourhood and derived the equilibrium amount of criminal activities in a neighbourhood. In the paper by McMillon and colleagues [23], they tried to find the general patterns among the parameters that are independent of the basal particulars. They compute analytical expressions for the equilibrium and for the tipping points between high-crime and low-crime equilibriums of the models. They examined the effects of longer prison terms and increased incarnation rates on the prevalence of crime. In modern days, these kinds of mathematical epidemic modelling are becoming very popular. So we choose social problems like:- poverty and addiction happen among the society for this research.

There is a strong relationship between poverty and drug addiction. Some of the reasons of addiction are directly or indirectly related to poverty such as:- ignorance, unhealthy social environment, inability to deal with life and stress, unavailability of social and psychological help, family and social damages, family troubles and strained relationship, inability to complete education [9] etc. Some of the people living in poverty started taking drugs. Large cities with many low income citizens are the frequently oppressed with drug addiction. Drugs are actually pieces of cake to get on the streets, and young people living in poverty are often encompassed by drug dealers and drug addicts. Moreover, the people living in non-poverty/non-impoverished class (rich or middle class) may also get addicted to drugs. In the big cities and districts in Bangladesh like:- Sylhet has a huge rate of drug addiction, which is increasing day by day. It is causing social imbalance. There is also a significant rate 24.1% [19] of poverty in Sylhet. So it is a modern day challenge to control addiction along with poverty in Sylhet.

In previous works [1] and [2] the respective epidemiologists have developed the mathematical models on Poverty and Crime as well as Poverty and Prostitution. Their works were based on the works by Kelly, 2000 [4]; Block and Heineke, 1975. These have inspiration from Becker's economic theory of crime (1968), described in [3]. In Becker's paper [3], he used statistical and economic analysis to obtain the optimal control of crime. In the papers [1] and [2] the respective epidemiologists used a system of ODEs to try and get more realistic, dynamical solution based on the correlation between poverty, crime, prostitution etc. In the paper [2], the respective epidemiologists considered prostitution as a crime and use a mathematical model to understand the dynamics of the system and how both poverty and prostitution can be reduced to the barest minimum, just like in the paper [1], the respective epidemiologists use a mathematical model to understand the dynamics of the system and how both poverty and crime can be reduced to the barest minimum.

In this article, our aim is to develop a compartmental mathematical model which examines the dynamics of poverty and drug addiction (can be shortly named as PD model) just like their approach. We conduct a system of ODEs and obtain a more husky and realistic solution of the system. Our model includes an addicted compartment that allows for an approach of government and non-government interventions. The basic difference between the models described in [1] and [2] with our model is that, we consider the fact, people living in non-poverty/non-impoverished class (rich or middle class) may also get addicted to drugs. So our model is much improved one comparing to the models described in [1] and [2].

We'll try to obtain a useful approach to minimizing the drug addiction and inevitably the poverty in Sylhet. In Sec. II, we formulate the model. In Sec. III, we present the mathematical analysis of the model. In Sec. IV, we represent the numerical results and simulate the system in MATLAB to present the figures. Finally, in sec. V, we finish our work as conclusion.

II. Formulation of the Model

We know that, it is very difficult to stop drug addiction totally from our society. But it is possible to enhance interventions in order to control addiction reasonably while the cost of intervention is minimal. This model deals with the fact. In this model, the population is divided into five compartments: the non-impoverished class N, the poverty/impoverished class P, the drug addicted class D, the rehabilitation class I and the recovery class R (from the poverty class or rehabilitation class). All the variables are functions of time t. α denotes the rate of flow from the non-impoverished class to the poverty class/impoverished class, which is dependent upon unemployment and underemployment rate, normally is directly related to poverty. β denotes the rate at which the individuals in the poverty/impoverished class get into the drug addicted class. Similarly, φ denotes the rate at which the individuals in the non-impoverished class get into to the drug addicted class. γ is the rate at which an addict recruited into the rehabilitation class. A person in the P class will resort to addiction after coming in contact with an addict over a given period of time. On the other hand, a person in the N class will also resort to addiction after coming in contact with an addict over a given period of time. The term $\frac{\varphi ND}{T}$ represents the conversion rate from the N class to the D class, where the transmission rate is φ. Similarly the term $\frac{\beta PD}{T}$ represents the conversion rate from the P class to the D class, where the transmission rate is β. A recovered individual may also involve to addiction again but at reduced rates $\frac{\beta\sigma RD}{T}$ and $\frac{\varphi\sigma RD}{T}$, where $0 \le \sigma \le 1$ is the reduction fraction that account for the phenomena. The addicts, already been rehabilitated, immediately move to the class R at a rate δ and then due to contact with other addicts may return back to addiction at some reduced rate $\beta\sigma$ and $\varphi\sigma$. There are also disease induced deaths due to infection in different classes, but the death rates are very low. Our assumption is that, the total population T remains constant ($T' = 0$) i.e. the per capita death rate is equal in magnitude to the per capita birth rate ($\mu = \lambda$). All parameters are assumed to be non-negative and $'$ has the meaning of differentiation with respect to t.

Now the relationship between poverty and drug addiction can be presented by the following system of ODEs:

$$
\begin{aligned}
N' &= \mu T - (\mu+\alpha)N - \frac{\varphi ND}{T} \\
P' &= \alpha N - (\mu+\rho)P - \frac{\beta PD}{T} \\
D' &= \frac{\beta PD}{T} + \frac{\varphi ND}{T} + \frac{\beta\sigma RD}{T} + \frac{\varphi\sigma RD}{T} - (\mu + k_1 + \gamma)D \\
I' &= \gamma D - (\mu + k_2 + \delta)I \qquad (1) \\
R' &= \delta I + \rho P - \mu R - \frac{\beta\sigma RD}{T} - \frac{\varphi\sigma RD}{T} \\
T &= N + P + D + I + R
\end{aligned}
$$

In the following tables, the variables and parameters used in the model are defined:

TABLE I. MODEL VARIABLES

Variables	Description
$N(t)$	Non-impoverished class
$P(t)$	Poverty/impoverished class
$D(t)$	Drug addicted class
$I(t)$	Rehabilitation class
$R(t)$	Recovery class
$T(t)$	Total Population

TABLE II. MODEL PARAMETERS

Parameters	Description
α	The rate of flow from the non-impoverished class N to the poverty/impoverished class P
β	The rate at which the individuals in the poverty/impoverished class P get into the drug addicted class D
$\mu=\lambda$	Per capita death rate = Per capita birth rate
φ	The rate at which the individuals in the non-impoverished class N get into to the drug addicted class D
γ	The rate at which an addict recruited into the rehabilitation class I.
ρ	The conversion rate from the poverty class P to the recovery class R due to intervention
δ	The conversion rate from the rehabilitation class I to the recovery class R
σ	The rate of R to D transmission $(0\le\sigma\le1)$
k_1	Disease induced death rate due to infection in D
k_2	Disease induced death rate due to infection in I

The flow diagram for the system is given below:

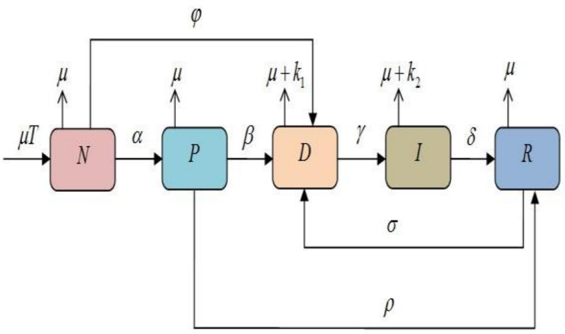


Fig. 1. *Flow diagram for the system (1).*

III. MATHEMATICAL ANALYSIS OF THE MODEL

In this section, we discuss the existence and uniqueness of the Addiction Free Equilibrium (AFE) of the model and its stability analysis. It can be obtained by considering the different compartments as proportions and setting the LHS of the system (1) as zero and then solving the system simultaneously.

But first of all we consider an equilibrium named as Addiction Equilibrium (AE) as,

$$E^*=\left(\frac{N^*}{T},\frac{P^*}{T},\frac{D^*}{T},\frac{I^*}{T},\frac{R^*}{T}\right) \text{ with } D^*>0 \tag{2}$$

Now considering $\frac{D^*}{T}=x\Rightarrow D^*=xT$ and LHS of the system (1) as zero and then solving the system simultaneously, we have obtained,

$$\frac{N^*}{T}=\frac{\mu}{[(\mu+\alpha)+\varphi x]},\ \frac{P^*}{T}=\frac{\alpha\mu}{[(\mu+\alpha)+\varphi x][(\mu+\rho)+\beta x]},$$

$$\frac{I^*}{T}=\frac{\gamma x}{(\mu+k_2+\delta)} \quad \text{and}$$

$$\frac{R^*}{T}=\frac{\frac{\delta\gamma x}{(\mu+k_2+\delta)}+\frac{\alpha\rho\mu}{[(\mu+\alpha)+\varphi x][(\mu+\rho)+\beta x]}}{[\mu+(\beta+\varphi)\sigma x]} \tag{3}$$

Then considering $\frac{D^*}{T}=x=0\Rightarrow D^*=0$ we have $\frac{I^*}{T}=0\Rightarrow I^*=0$ and eventually, we have obtained the Addiction Free Equilibrium (AFE) which is given by,

$$E_0=\left(\frac{N_0}{T},\frac{P_0}{T},\frac{D_0}{T},\frac{I_0}{T},\frac{R_0}{T}\right)$$
$$=\left(\frac{\mu}{(\mu+\alpha)},\frac{\alpha\mu}{(\mu+\alpha)(\mu+\rho)},0,0,\frac{\alpha\rho}{(\mu+\alpha)(\mu+\rho)}\right) \tag{4}$$

When $D^*>0\Rightarrow\frac{D^*}{T}>0\Rightarrow x>0,$ then $\frac{I^*}{T}>0\Rightarrow I^*>0$, since it is directly depending on x.

Then for E^*, considering the LHS as zero, the third equation of the system (1) can be written as:

$$0=\beta\frac{P^*}{T}x+\varphi\frac{N^*}{T}x+\beta\sigma\frac{R^*}{T}x+\varphi\sigma\frac{R^*}{T}x-(\mu+k_1+\gamma)x$$

implies

$$\left[\beta\frac{P^*}{T}+\varphi\frac{N^*}{T}+\beta\sigma\frac{R^*}{T}+\varphi\sigma\frac{R^*}{T}-(\mu+k_1+\gamma)\right]x=0$$

Since $x>0,$ so clearly,

$$\left[\beta\frac{P^*}{T}+\varphi\frac{N^*}{T}+\beta\sigma\frac{R^*}{T}+\varphi\sigma\frac{R^*}{T}-(\mu+k_1+\gamma)\right]=0$$

$$\Rightarrow\beta\frac{P^*}{T}+\varphi\frac{N^*}{T}+\beta\sigma\frac{R^*}{T}+\varphi\sigma\frac{R^*}{T}=(\mu+k_1+\gamma) \tag{5}$$

Again, since $x > 0$ and all the parameters are non-negative, we can conclude from (3) and (5) that,

$$\frac{P^*}{T} > 0 \Rightarrow P^* > 0, \frac{N^*}{T} > 0 \Rightarrow N^* > 0$$

and $\frac{R^*}{T} > 0 \Rightarrow R^* > 0$

So there is existence of a unique AE i.e. E^* when $D^* > 0$ and $I^* > 0$. We assumed that, it is only exists when the AFE i.e. E_0 is unstable.

Now using the approach of *Next Generation Matrix Operator* described in [10], [11], [12], [13], [14] and [15], we obtained the *reproduction number*.

By using the approach we obtained,

$$F = \begin{pmatrix} \frac{\beta\alpha\mu + \varphi\mu(\mu+\rho) + (\beta+\varphi)\sigma\alpha\rho}{(\mu+\alpha)(\mu+\rho)} & 0 \\ 0 & 0 \end{pmatrix} \quad \text{and}$$

$$V = \begin{pmatrix} (\mu + k_1 + \gamma) & 0 \\ -\gamma & (\mu + k_2 + \delta) \end{pmatrix} \tag{6}$$

Then, the next generation matrix will be,

$$FV^{-1} = \begin{pmatrix} \frac{\beta\alpha\mu + \varphi\mu(\mu+\rho) + (\beta+\varphi)\sigma\alpha\rho}{(\mu+\alpha)(\mu+\rho)(\mu+k_1+\gamma)} & 0 \\ 0 & 0 \end{pmatrix} \tag{7}$$

The *reproduction number* then can be termed as,

$$\Re = \Re_P + \Re_N + \Re_R = \frac{\beta\alpha\mu + \varphi\mu(\mu+\rho) + (\beta+\varphi)\sigma\alpha\rho}{(\mu+\alpha)(\mu+\rho)(\mu+k_1+\gamma)} \tag{8}$$

where,

$\Re_P = \frac{\beta\alpha\mu}{(\mu+\alpha)(\mu+\rho)(\mu+k_1+\gamma)}$ = The input from the poverty/impoverished class P (9)

$\Re_N = \frac{\varphi\mu}{(\mu+\alpha)(\mu+k_1+\gamma)}$ = The input from the non-impoverished class N (10)

and

$\Re_R = \frac{(\beta+\varphi)\sigma\alpha\rho}{(\mu+\alpha)(\mu+\rho)(\mu+k_1+\gamma)}$ = The input from the recovery class R (11)

We move forward to show the stability of the equilibria. To study the behaviour of the system (1) around the AFE, we refer the linearlized stability principals. We evaluate the partial derivatives of the system (1) at AFE i.e. E_0 to get the *Jacobian matrix* for AFE which is given by,

$$J_{[AFE_s]} = J_{E_0} = \begin{pmatrix} -(\mu+\alpha) & 0 & \frac{-\varphi N_0}{T} & 0 & 0 \\ \alpha & -(\mu+\rho) & \frac{-\beta P_0}{T} & 0 & 0 \\ 0 & 0 & \frac{\beta P_0}{T} + \frac{\varphi N_0}{T} + \frac{\beta\sigma R_0}{T} + \frac{\varphi\sigma R_0}{T} - (\mu+k_1+\gamma) & 0 & 0 \\ 0 & 0 & \gamma & -(\mu+k_2+\delta) & 0 \\ 0 & \rho & -\frac{\beta\sigma R_0}{T} - \frac{\varphi\sigma R_0}{T} & \delta & -\mu \end{pmatrix} \tag{12}$$

where

$$\frac{N_0}{T} = \frac{\mu}{(\mu+\alpha)}, \quad \frac{P_0}{T} = \frac{\alpha\mu}{(\mu+\alpha)(\mu+\rho)} \quad \text{and}$$

$$\frac{R_0}{T} = \frac{\alpha\rho}{(\mu+\alpha)(\mu+\rho)} \tag{13}$$

Now we try to calculate the eigenvalues of J_{E_0} by finding the characteristic equation.

Let the eigenvalues be defined as θ. By proceeding forward we have the characteristic equation,

$$\left[\theta - \left(\frac{\beta P_0}{T} + \frac{\varphi N_0}{T} + \frac{\beta\sigma R_0}{T} + \frac{\varphi\sigma R_0}{T}\right) + (\mu+k_1+\gamma)\right]. \\ [\theta+\mu][\theta+(\mu+k_2+\delta)][\theta+(\mu+\alpha)][\theta+(\mu+\rho)] = 0 \tag{14}$$

From the characteristic equation we obtain the eigenvalues,

$\left(\frac{\beta P_0}{T} + \frac{\varphi N_0}{T} + \frac{\beta\sigma R_0}{T} + \frac{\varphi\sigma R_0}{T}\right) - (\mu+k_1+\gamma)$, $-\mu$, $-(\mu+k_2+\delta)$, $-(\mu+\alpha)$ and $-(\mu+\rho)$.

Now, E_0 is only exists when all the eigenvalues of J_{E_0} are non-positive [17]. Also E_0 is stable if all the eigenvalues of J_{E_0} has negative real parts [16].

These holds only if, $\left(\frac{\beta P_0}{T} + \frac{\varphi N_0}{T} + \frac{\beta\sigma R_0}{T} + \frac{\varphi\sigma R_0}{T}\right) - (\mu+k_1+\gamma) < 0$

$$\Rightarrow \left(\frac{\beta P_0}{T} + \frac{\varphi N_0}{T} + \frac{\beta\sigma R_0}{T} + \frac{\varphi\sigma R_0}{T}\right) < (\mu+k_1+\gamma)$$

$\Rightarrow \frac{\beta\alpha\mu + \varphi\mu(\mu+\rho) + (\beta+\varphi)\sigma\alpha\rho}{(\mu+\alpha)(\mu+\rho)(\mu+k_1+\gamma)} < 1$ Hence, $\Re < 1$

Again, E_0 is unstable if at least one of the eigenvalues of J_{E_0} has positive real part [16].

These holds only if, $\left(\frac{\beta P_0}{T} + \frac{\varphi N_0}{T} + \frac{\beta\sigma R_0}{T} + \frac{\varphi\sigma R_0}{T}\right) - (\mu+k_1+\gamma) > 0$

$$\Rightarrow\left(\frac{\beta P_0}{T}+\frac{\varphi N_0}{T}+\frac{\beta\sigma R_0}{T}+\frac{\varphi\sigma R_0}{T}\right)>(\mu+k_1+\gamma)$$

$$\Rightarrow\frac{\beta\alpha\mu+\varphi\mu(\mu+\rho)+(\beta+\varphi)\sigma\alpha\rho}{(\mu+\alpha)(\mu+\rho)(\mu+k_1+\gamma)}>1 \quad \text{Hence, } \Re>1$$

On the other hand, a unique E^* is only exists if E_0 is unstable. So a unique E^* is only exists if $\Re>1$. □

We can now conclude our findings in the following theorem:

Theorem 1

For the system (1), the AFE i.e. E_0 is locally asymptotically stable if $\Re<1$ and unstable if $\Re>1$. Also the system has only the AFE i.e. E_0 if $\Re<1$ and an unique AE i.e. E^* if $\Re>1$.

IV. Numerical Simulations from the Model

We are now going to discuss the numerical simulation and findings from the model in this section. We solved the system (1) by using MATLAB. The baseline variables and used parameters estimated for Sylhet district of Bangladesh are shown in the following tables. Although some of the data we used are actually based on estimations from the results of our personal survey for research purpose and secondary source (since primary data are not always available because of government restrictions), but we found that, our approximations are actually correct enough to present the model appropriately. In further research we will try to use primary data and try to find more accurate simulations.

TABLE III. Estimation of the Baseline Variables

Variables	Values	Sources
T	3567138	[18]
$N(0)$	2707458	Estimated on the basis of [18] and [19]
$P(0)$	859680	Estimated on the basis of [18] and [19]
$D(0)$	86890	Estimated from the survey results
$I(0)$	25689	Estimated from the survey results
$R(0)$	12923	Estimated from the survey results

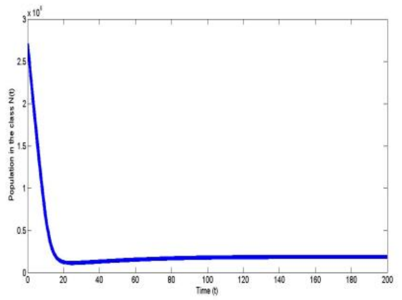
TABLE IV. Estimation of the Parameters used

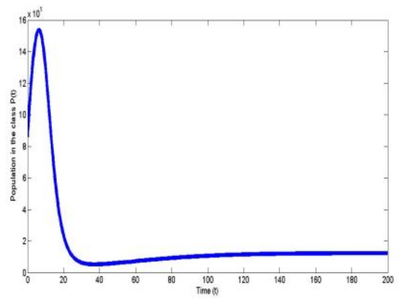
Parameters	Values	Sources
α	0.08	Estimated from the survey results
β	0.28	Estimated from the survey results
$\mu=\lambda$	0.01413	Estimated on the basis of [20]
φ	0.47	Estimated from the survey results
γ	0.37	Estimated from the survey results
ρ	0.12	Estimated from the survey results
δ	0.61	Estimated from the survey results
σ	0.40	Estimated from the survey results
k_1	0.02	Estimated from the survey results
k_2	0.01	Estimated from the survey results

Results and Discussion

For the system (1), the parameters γ and ρ can be treated as the *intervention parameters*. First we will analyze the situation of the different population classes with respect to time without implementing any kind of interventions considering the values of both the intervention parameters as zero. Then we will analyze the situation of the different population classes with respect to time with interventions considering the values of both the intervention parameters as our estimation in *Table III*. Finally we will analyze the situation of the different population classes with respect to time with high and low interventions considering the values intervention parameters as $\gamma=0.40$, $\rho=0.15$ (for high intervention) and $\gamma=0.34$, $\rho=0.09$ (for low intervention).

When there are no interventions i.e. $\gamma=0$ and $\rho=0$, we have plotted the following graphs in MATLAB:





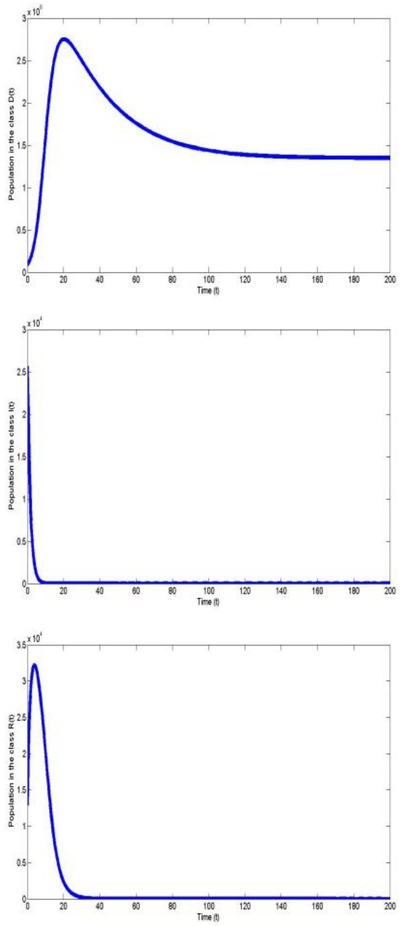


Fig. 2. *Situations of the different population classes with respect to t without interventions.*

The plots in the *Fig. 2* show that, when there are no interventions, The population in the class *N(t)* decreases with respect to time *t* and after a certain period of time it increases again. The population in the class *P(t)* increases with respect to time *t* till a small amount of time. After that it decreases with respect to time *t* and after a certain period of time it increases again. The population in the class *D(t)* increases with respect to time *t* till a certain period of time. After that it started to decrease and then after another certain period of time becomes constant. The population in the class *I(t)* decrease towards zero with respect to time *t*. Also the population in the class *R(t)* increases with respect to time *t* till a small amount of time. After that it decreases towards zero with respect to time *t*.

We see that, when there are no interventions, poverty grows continuously after a small reduction. The population of the addicted class after a time period becomes constant, while the population of rehabilitation and recovery classes decreases towards zero. So the control of poverty and addiction becomes impossible. The existence of government and non-government approach of interventions is a must while controlling poverty and drug addiction.

When there are interventions (taking intervention parameters as our estimation in *Table IV*), we have plotted the following graphs in MATLAB:

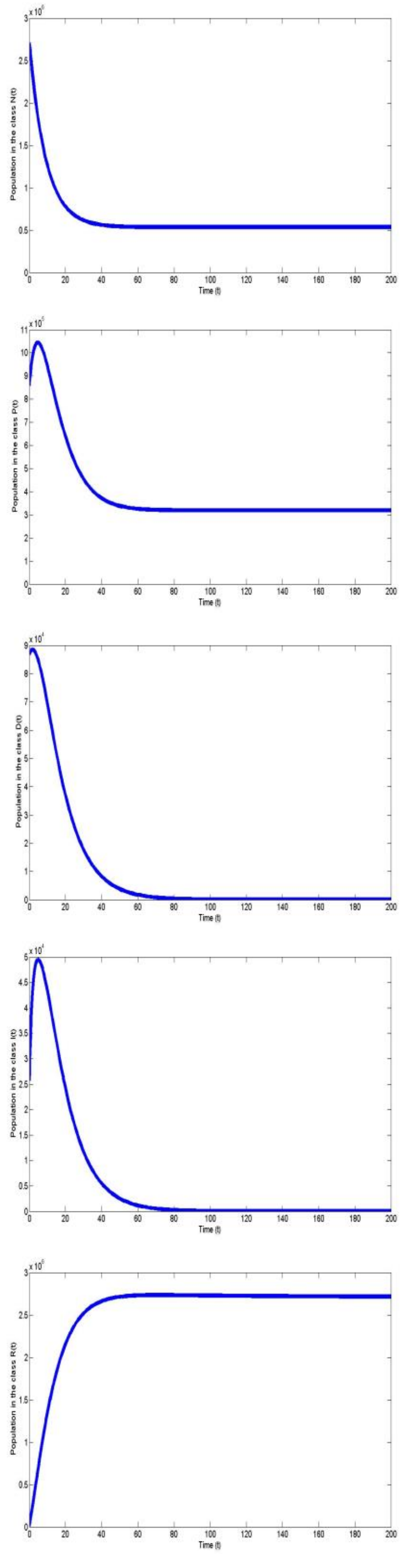


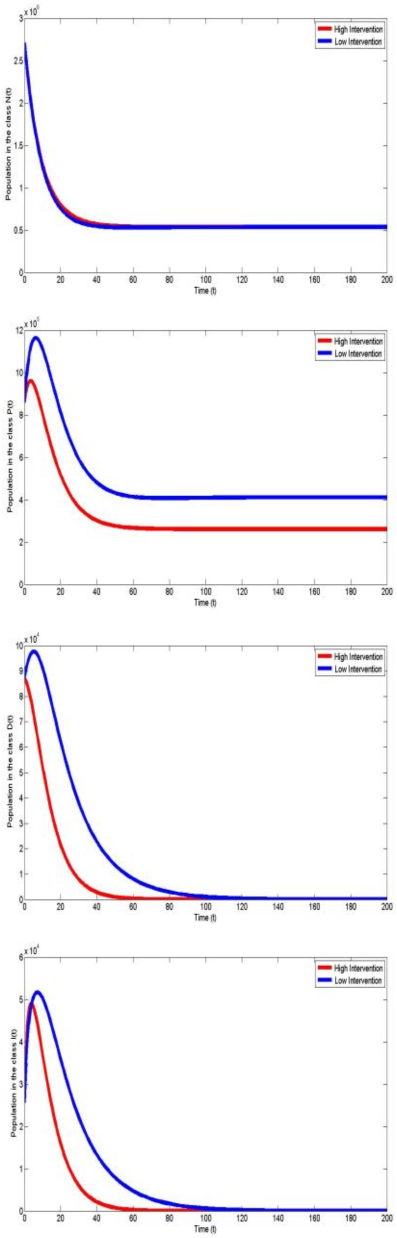
Fig. 3. *Situations of the different population classes with respect to t with interventions.*

The plots in the *Fig. 3* show that, when there are interventions, The population in the class *N(t)* decreases with respect to time *t* and after a certain period of time becomes constant. The population in the class *P(t)* increases with respect to time *t* till a small amount of time. After that it decreases with respect to time *t* and after another certain period of time becomes constant. The population in the class

D(t) increases with respect to time *t* till a very small period of time. After that it started to decrease towards zero. The population in the class *I(t)* increases with respect to time *t* till a very small period of time. After that it started to decrease towards zero. The population in the class *R(t)* increases very high with respect to time *t* till a large amount of time. After that it started to decrease again. That means the controlling has reached its target i.e. population in the poverty class almost becomes constant and population in the addicted class almost becomes zero.

So existence of government and non-government approach of interventions can reduce poverty and drug addiction.

For high and low interventions, taking intervention parameters as $\gamma = 0.40,$ $\rho = 0.15$ (for high intervention) and $\gamma = 0.34,$ $\rho = 0.09$ (for low intervention), we have plotted the following graphs in MATLAB:



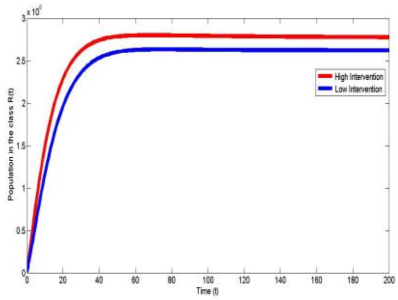


Fig. 4. *Situations of the different population classes with respect to t with high vs low interventions*

The plots in the *Fig. 4* show the effects on the population classes with respect to time *t* for high and low approach of interventions. The population in the class *N(t)* with respect to time *t* becomes constant faster for high interventions. The population in the classes *P(t), D(t) and I(t)* decrease more faster for high interventions rather than for low interventions. Also the population in the class *R(t)* increases more faster for high interventions rather than for low interventions. So we can say that, the high rate approach of government and non-government interventions will reduce poverty and drug addiction faster towards barest minimum.

By government and non-government approach of interventions for controlling the poverty we mean policies, strategies and programmes which are resulting empowerment of the poverty class. These programmes include skill development, soft loan, job opportunities, educational opportunities, entrepreneurship opportunities, self employment, raising awareness, food for work, good governance etc. On the other hand by interventions for controlling the addiction we mean policies, strategies and programmes which are resulting reduction of drug addiction. These programmes include raising awareness, rehabilitate the addicts, increase of rehabilitation centres, counselling, controlling the availability of drugs, financial assistance, raising religious values, strictness of law and order, free treatment etc. Since mainly the youth (in 70-80% cases) are involved to drug addiction, most of the awareness raising has to be made through the social media and educational institutions. It takes about 7-8 days of continuous use of drugs for getting addicted (physically or mentally). So controlling the availability of drugs and educating the people about the bad effects of drugs are very important. The government and NGOs are both important catalysts for these kinds of interventions. Our research deals with all the types of interventions towards the people of Sylhet no matter what their gender is (male, female or third gender). Also we believe that, the high intervention policies, strategies and programmes should be closely monitored for effectiveness if poverty and addiction are to be reduced to their barest minimum.

V. CONCLUSION

We have provided a practical but simple compartmental mathematical model (can be shortly named as PD model) that helps us to study the dynamics of poverty and addiction in Sylhet, Bangladesh. We introduced a compartment that focuses on recovering rehabilitated addicts from all genders. The model introduced in this article, is obviously more developed than other models related to poverty and crime. The effect of poverty in almost every other crime in the society cannot be underestimated. The elimination and control of poverty and addiction should be the focus of good governance in the cities and districts of developing countries like:- Sylhet, Bangladesh as can be seen in the result of the model. We established in this model that the addiction free equilibrium is locally asymptotically stable when the reproduction number $\Re<1$ and unstable when $\Re>1$. Numerical simulations of the system have been presented to show the variations of the population in different situations. Also we find out that, the high rate of interventions will reduce poverty and drug addiction faster towards their barest minimum. Data that have been used for the simulations are based on the drug addiction happens in Sylhet district of Bangladesh. Some of the data we used are actually based on estimations from the results of our personal survey and secondary source (since primary data are not always available because of government restrictions and strictness in law and order). But we believe that, our approximations are actually correct enough to present the model appropriately. The model is useful to study the dynamics of poverty and addiction not only in Sylhet, but also across the country and even all over the world. In further research we will try to use primary data and try to find more accurate simulations. We will try to evaluate the model on the basis of the data collected for the poverty and addiction situation of Sylhet. We will also involve other probable phenomena related to this model like:- drug smuggling, Drug Induced Diseases (DIDs) infections etc. to make the model more useful.

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