Debye And Debye – Type Potentials In Physical Acoustic Problems

A. A. Kleshchev
St. Petersburg State Marine Technical University, st. Lotsmanskaya 3, St. Petersburg, 190008 Russia
E-mail: alexalex-2@yandex.ru

***Abstract*—In a review article demonstrated efficiency of an use of Debye and Debye-type potencials for a solution of a three-dimensional problems of a diffraction and a propagation for elasic bodies (isotropic and anisotropic) of analytical and non-analytical forms. Analytical solutions are performed computer calculations. are performed computer calculations.**

Keywords—diffraction, propagation, radiation, sound wave, boundary conditions, elastic body, integral equation, Debye potential.

1. INTRODUCTION

This work shows how to use potentials Debye and Debye-type are solved following three-dimensional dynamic elasticity problems: a sound diffraction at elastic bodies of spheroidal and cylindrical forms (isotropic and anisotropic); a sound diffraction at elastic bodies of a non-analytical form (a Green's functions method); a calculation of phase velocities of three-dimensional elastic waves in cylindrical bars and shells (isotropic and anisotropic). Originally Debye potentials were used in three-dimensional diffraction problems of electromagnetic waves on different bodies. The purpose of this study – to show that Debye and Debye – type potentials are essential in the dynamic theorie of elasticity.

2. A SOUND DIFFRACTION AT ELASTIC BODIES OF SPHEROIDAL AND CYLINDRICAL FORMS (ISOTROPIC AND ANISOTROPIC)

Debye first proposed expanding the vector potential $\vec{A}$ in scalar potentials *U* and *V* in his publication [1] devoted in studying the behavior of light waves near the local point or focal line. Later this approach was used in solving diffraction problems for the cases of electromagnetic wave diffraction by a sphere, a circular disk and a paraboloid of revolution [2 – 7], as well as for the diffraction of longitudinal and transverse waves by spheroidal and cylindrical bodies [8, 9].

As applied to problems based on the dynamic elasticity theory the introduction of the Debye potentials occurs as follows. The displacement vector $\vec{u}$ of an elastic isotropic medium obeys the Lame equation:

$$(\lambda+\mu) grad div \vec{u} - \mu curl curl \vec{u} = -\rho\omega^2\vec{u}, \qquad (1)$$

where λ and μ are Lame constans, ρ is the density of the isotropic medium and ω is the circular frequency of harmonic vibrations. According to the Helmholtz theorem, the displace- ment vector $\vec{u}$ is expressed through scalar Φ and vector $\vec{A}$ potentials:

$$\vec{u} = -grad\Phi + curl\vec{A} \qquad \text{............................(2)}$$

Substituting Eq. (2) in Eq. (1), we obtain two Helmholtz equations, which include one scalar equation for Φ and one vector equation for $\vec{A}$:

$$\Delta\Phi + h^2\Phi = 0, \qquad (3)$$

$$\Delta\vec{A} + k_2^2\vec{A} = 0. \qquad (4)$$

Here $h = \omega / c_1$ is the wavenumber of the longitudinal elastic wave, c_1 is the velocity of this wave, $k_2 = \omega / c_2$ is the wavenumber of the transverse elastic wave and c_2 is the velocity of the transverse wave.

In the three-dimensional case, variables involved in scalar equation (3) can be separated into 11 coordinate systems. As for Eq. (4), in the three-dimensional problem, this equation yields three independent equations for each of components of the vector function $\vec{A}$ in Cartesian coordinate system alone. To overcome this difficulty, one can use Debye's potentials U and V, which obey the Helmholtz scalar equation

$$\Delta V + k_2^2 V = 0; \quad \Delta U + k_2^2 U = 0. \qquad (5)$$

Vector potential $\vec{A}$ (according to Debye) is expanded in potentials V and U as

$$\vec{A} = curl curl(\vec{R}U) + ik_2 curl(\vec{R}V), \qquad (6)$$

where $\vec{R}$ is the radius vector of a point of the elastic body or the elastic medium.

Representation (6) for the vector potential is not the only possible one. Let us consider two other representations that the Debye-type potentials. First , they include the potentials proposed by Buchwald

[10] in his study of the behavior of a Rayleigh wave in a transversaly iso-tropic medium. Displacement vector in a transversaly isotropic medium is represented in the form [10]:

$$\vec{u} = grad\,\Phi + curl\,\overline{\Psi} + grad\,\theta, \qquad (7)$$

Here, it is assumed that vector potential $\overline{\Psi}$ has only one nonzero component, namely the component Ψ_z, in addition, potentials Φ and $\Psi_z = \Psi$ are functions of the *x* and *y* coordinates, while potential θ is a function of the *z* coordinate.

Second, there are the Debye-type potentials proposed in [11]. In this case, vector potential $\vec{A}$ in expressed through the Debye-type potentials χ and ψ as follows

$$\vec{A} = \chi\vec{e}_z + a \cdot curl\left(\psi\vec{e}_z\right), \qquad (8)$$

where $\vec{e}_z$ is the unit vector in the direction of the Z axis and a is the radius of the transversely isotropic circular cylindrical bar placed in an elastic medium.

Using Debye potentials [expression (6)] were solved three-dimensiunal problems of diffraction on spheroidal scatterers (Fig. 1) and on cylindrical shell radiated by a point source [9].

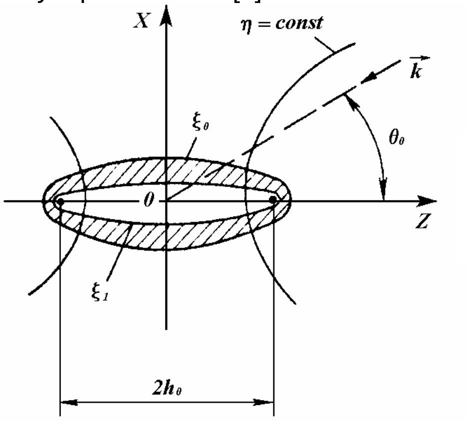


Fig. 1. Elastic spheroidal shell in a plane harmonic wave field.

Fig. 2 shows relative backscattering cross sections of prolate spheroidal scatterers, fig. 3 - of oblate spheroidal spheroids.

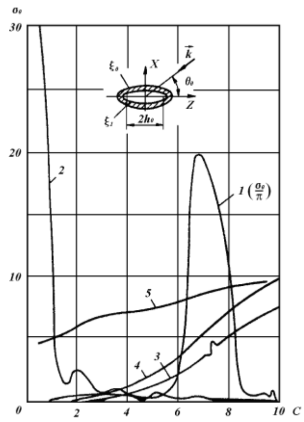


Fig. 2. Relative backscatterings cross sections of prolate spheroidal scatterers.

Fig. 3. Relative backscattering cross sections of oblate spheroids.

With a help Debye-type potentials [expression (8)] were solved thre-dimensional problems of diffraction on a cylindrical shell radiated by a plane sound wave (Fig. 4) [9] and on a transversely isotropic bar (Fig. 5) [11].

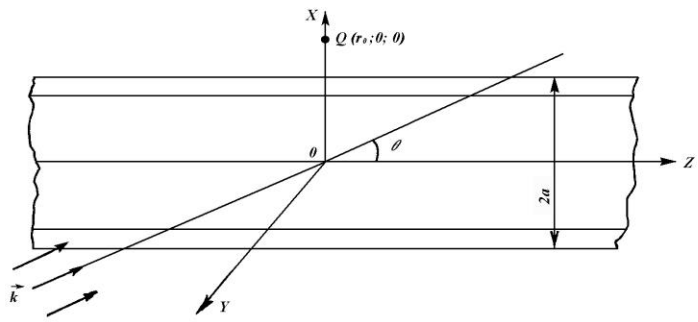


Fig. 4. The cylindrical shell radiated by the plane sound wave.

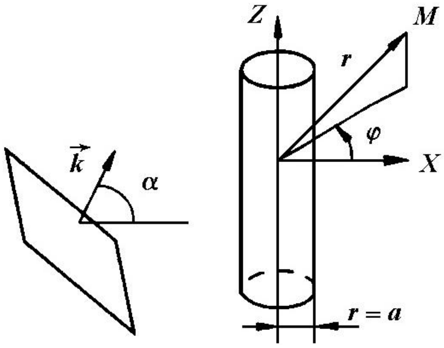


Fig. 5. The transversely isotropic bar radiated by the plane sound wave [11].

On fig. 6 and 7 show normalized amplitudes of a backscatter of a polarized shear wave for a transversely isotropic bar.

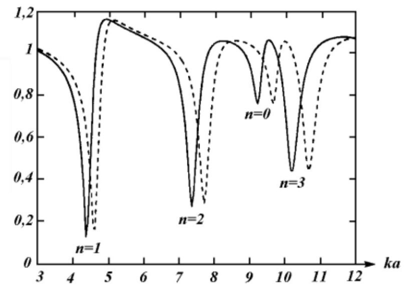


Fig. 6. The normalized amplitude of the axially polarized shear wave backscatter.

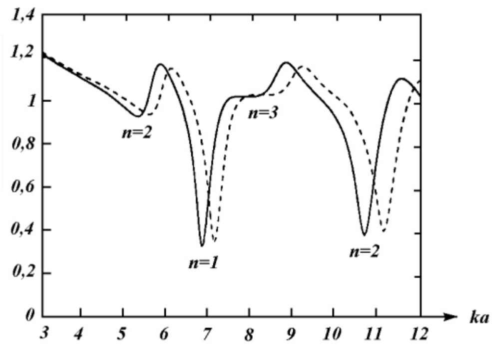


Fig. 7. The normalized amplitude of the polarized in the plane $r-\varphi$ shear wave backscatter.

3. A SOUND DIFFRACTION AT ELASTIC BODIES OF A NON-ANALYTICAL FORM (A GREEN'S FUNCTIONS METHOD)

Refering to the method of Green's functions [9, 12 – 15], which was developed for solving problems of diffraction by bodies with mixed boundary conditions. In this work, this approach is used in study sound scattering by elastic bodies of non-analytical forms [16]. As non-analytical bodies whose surfaces cannot be assigned to the class of coordinate systems with separated variables in the scalar Helmholtz equation. We study such a non-analytical scatterer in the form of a finits length circular cylindrical elastic shell bounded at the ends by the two halves of a prolate spheroidal shell (Fig. 8).

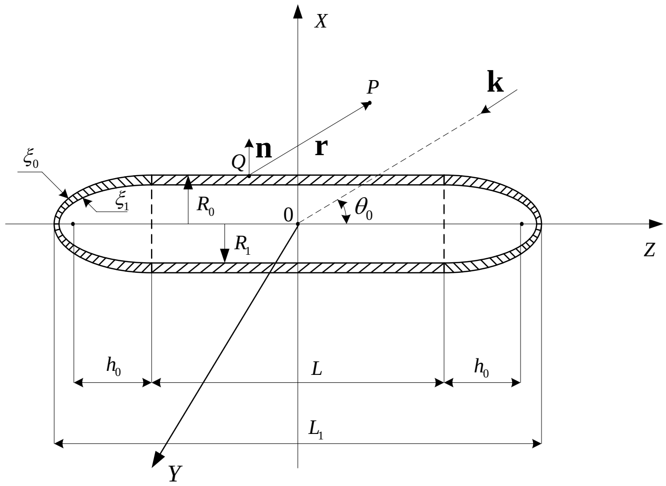


Fig. 8. Non-analytical elastic shell consisting of cylindrical and spheroidal parts.

First, we consider the diffraction problem for the case of oblique incidence of a sound wave on an infinite hollow cylindrical elastic shell [9, 11, 17]. The geometry of the problem is shown
in fig. 4. We transform the representation (8) given in [11] for the vector potential $\vec{A}$ by introdu-cing an additional curl operator so that in automatically satisfies the calibration div $\vec{A}$=0 [16].
The three-dimensional diffraction problem is solved using the Debye potentials *U* and *V* [the ex-pression (6)] [9, 14].

For the model shown in fig. 8, we calculated the absolute value of the angular characteristic $|D(\theta)|$ for $\theta=\theta_0=90^0$ within the wave interval $kR_0=0.053-0.581.$ In the calculations, the model given in fig. 8 was assumed following parameters $L_1=200.51$ m.; $L=100.0$ m.; $h_0=50.0$ m.; $R_0=5.04$ m.; $R_1=5.01$ m.; $\xi_0=1.005075;$ $\xi_1=1.005.$ The shell material was as-sumed in the steel. Under these conditions, $|D(90^0)|$ within 0.49 – 18.46 [16].

Along with the non-analytical scatterer shown in fig. 8 we consider a compound elastic shell formed by a finite cylindrical shell whose ends are closed by two hemispherical shells of the sa-me diameter (Fig. 9).

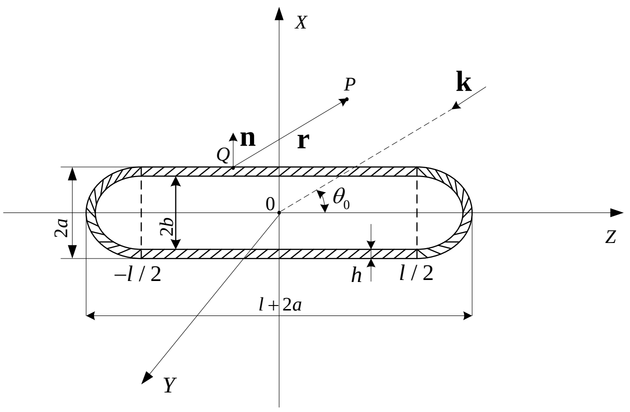


Fig. 9. Non-analytical elastic shell consisting of cylindrical and spherical parts.

To apply the Green's function method, it is necessary in take the solution to the axisymmetric problem of plane wave diffraction by an elastic spherical shell in terms of dynamic elasticity theory [18 – 20] and transform this solution to the three-dimensional version. The resulting so-lution little differs from the obtained above for the three-dimensional problem of diffraction by a spheroidal elastic shell [9, 14, 17, 21]. In the caseunderconsideration the vector finction $\vec{A}$ is expressed through the Debye potentials *U* and *V* [expression (6)] and its spherical compo-nents represented in the spherical coordinate system have the form [22 – 24]:

$$A_R = \left(\frac{\partial^2}{\partial R^2} + k_2^2\right)(RU); \qquad (9)$$

$$A_{\theta_1} = R^{-1}\frac{\partial^2}{\partial R \partial \theta_1}(RU) + ik_2(\sin\theta_1)^{-1}\frac{\partial V}{\partial \varphi}; \qquad (10)$$

$$A_\varphi = (R\sin\theta_1)^{-1}\frac{\partial^2}{\partial R \partial \varphi}(RU) - ik_2\frac{\partial V}{\partial \theta_1}. \qquad (11)$$

Figures 10 and 11 show the absolute values of the angular characteristic $|D(\varphi)|$ (in the XOY plane, by $\theta_0 = 90^0$) for a non-analytical elastic scatterer in the form a cylindrical shell connected with two spherical shells (Fig. 9) with following parameters: $ka = 0.523$ (Fig. 10) and $ka = 0.941$ (Fig. 11) and $l/2a = 11.9$.

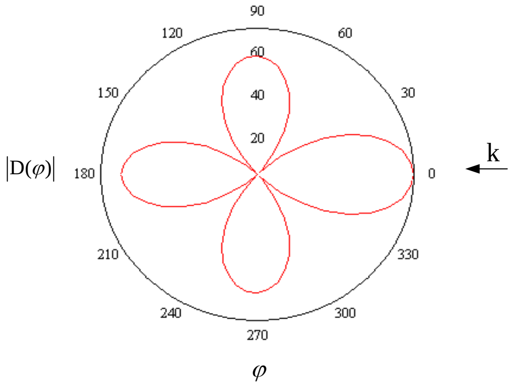


Fig. 10. Absolute values of the angular scattering characteristic.

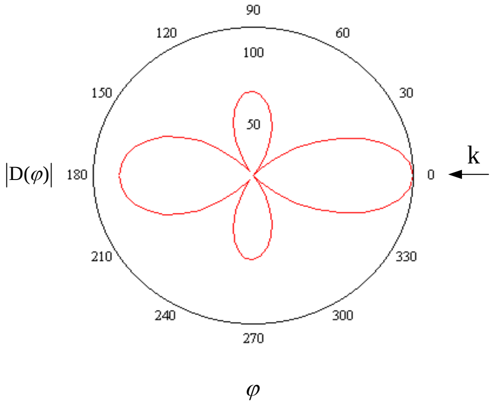


Fig. 11. Absolute values of the angular scattering characteristic.

4. A CALCULATION OF PHASE VELOCITIES OF THREE-DIMENSIONAL ELASTIC WAVES IN CYLINDRICAL BARS AND SHELLS

First Debye potentials were applied [25] in the study of a flexural waves in an isotropic cylindrical bar. The novelty of this approach is calculation of the phase velocities of isotropic bars and shells with the use of a rigorous method based on equations of dynamic elasticity theory and Debye or Debye-type potentials. Using Debye potentials [expression (6)] or Debye-type po-tentials [expression (8)] [25, 26], arrive at a third-order determinant [17, 21, 25, 26]:

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0 \qquad (12)$$

Determinant (12) proves to be a same for a cases of using Debye potentials and Debye-type potentials. This result indirectly confirms a correctness of a method chosen for solving a problem of interest. Setting determinant (12) equal to zero, which ensures a nontrivial solution , and assuming that a bar radius is *a* = 1,0, we obtain a characteristic equation for determining a wavenumbers of three-dimensional flexural waves. Figure 12 shows phase velocities of first three modes of a flexural wave.

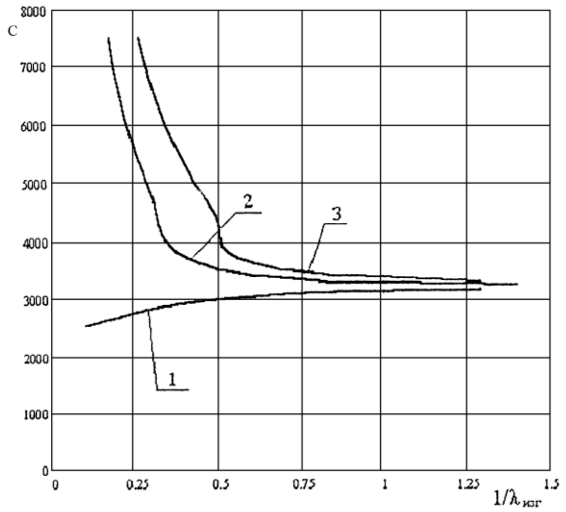
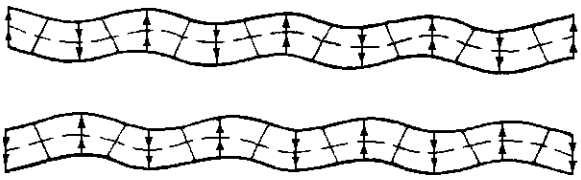


Fig. 12. Phase velocities of first three modes of flexural wave.

Untile a case of a bar a flexural wave propagat-ing in a cylindrical shell can be three-dimen-sional or two-dimensional (axisymmetric). Figure 13 schemati-cally illustrates a deformation of a cylindrical shell for cases of propagation of (a) flexural and (b) longitudi-nal axisymmetric waves. To study a three-dimensional flexural wave propagating in an isotropic cylindrical shell , we apply a same mathematical ap-proach (with an use of Debye potentials or Debye-type po-tentials) as that used for studyng flexural waves in a bar. However, in this case, an inclusion of a second (inner) boundary surface leads to a greate r number of unknowns and a greater number of boundary conditions. Figures 14 and 15 represent a splution to the characteristic equation for

(a)



(b)

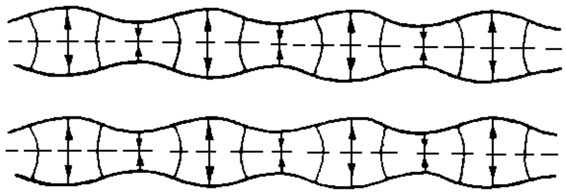


Fig. 13. Deformations of cylindrical shell for propaga-tion of (a) flexural and (b) longitudinal axi-symmetric waves.

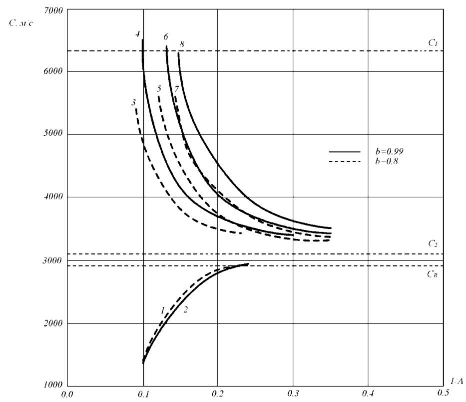


Fig. 14. Phase velocities of three-dimensional flexural waves in steel shell.

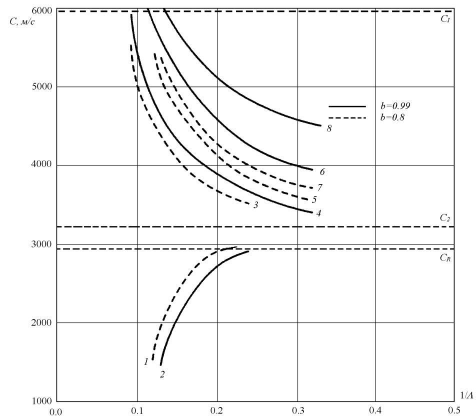


Fig. 15. Phase velocities of three-dimensional flexural waves in aluminum shell.

steel and aluminum shells of different thicknesses.

Debye-type potentials [expression (7)] (with a transformation of Cartesian coordinates to circular cylindrical ones) were used in [27] to study phase velocities of elastic waves in a trans-versely isotropic cylindrical bar. Figures 16 and 17 represent phase velocities of longitudinal

waves (Fig. 16) and flexural waves (Fig. 17) [27].

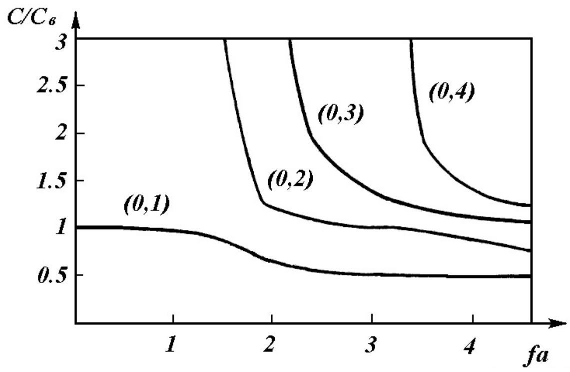


Fig. 16. Phase velocities of longitudinal waves in transversally isotropic bar.

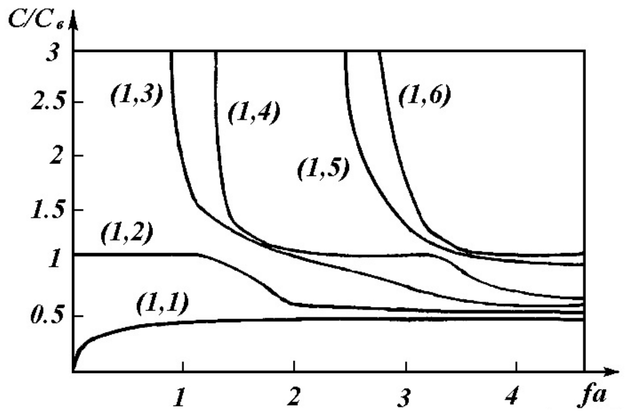


Fig. 17. Phase velocities of flexural waves in transversally isotropic bar.

5. CONCLUSIONS

The paper is a review of articles and monographs on the use of Debye potentials and De-bye-type potentials in three-dimensional problems on diffraction and propagation of elastic waves in isotropic and anisotropic mediums and bodies.

Acknowledgments

The work was supported as part of research under State Contract no P242 of April 21, 2010, within the Federal Target Program “Scientific and scientific - pedagogical personnel of innovative Russia for the years 2009 – 2013”.

REFERENCES

1. Debye, P. (1906) Das Verhalten von Lichtwellen in der Nahe Brennpunktes oder Brennline. Journal Annals Physics. 30, 775-776.
2. Frank, P. and von Mises, R. (1937) Differential and Integral Equations of Mechanics and Physics. ONTI, Moscow.
3. Franz, W. (1957) Theorie der Beugung Elektromagnetischer Wellen. Springer, Berlin-Gotingen-Heidelberg.
4. Meixner, J. (1948) Strenge Theorie der Beugung elektromagnetischer Wellen an der vollkommen leitenden Kreischeibe. Journal Zeitschrift fur Naturforschung. 3a, 501-518.
5. Fock, V. A. (1957) Theorie of Diffraction by a Paraboloid of Revolution. In book “Dif-fraction Electromagnetic Waves by Some Bodies of Revolution. Soviet Radio, Moscow.
6. Fock, V. A. (1970) Electromagnetic Diffraction and Propagation Problems. Soviet Radio, Moscow.
7. Honl, H., Maue, A. and Westpfahl, K. (1961) Theorie der Beugung. Springer, Berlin.
8. Gutman, T. L. and Kleshchev, A. A. (1974) Diffraction of Waves in Elastic Medium on Elastic Spheroid. Proceedings LKI. № 91, 31-37.
9. Kleshchev, A. A. (2012) Hydroacoustics Scatterers. Prima, St.-Petersburg.
10. Buchwald, V. T. (1961) Rayleich waves in transversely isotropic media. Quarterly Journal Mechanics and Applied Mathematics. XIV, 293-317.
11. Fan, Y., Sinclair, A. N. and Honarvar, F. Scattering of a phase acoustic wave from a transversely isotropic cylinder encased in a solid elastic medium. Journal of Acoustical Society of America. 106, 1229-1236.
12. Kleshchev, A. A. (1974) Sound Diffraction on Bodies with mixed boundary conditions. Acoustical Journal. 20, 632-634.
13. Kleshchev, A. A. (1984) Accuracy of Green’s Functions Method. Proceedings LKI “Acoustics of Ships and Oceans”, 19-24.
14. Kleshchev, A. A. and Klukin, I. I. (1987) Principles of Hydroacoustics. Shipbuilding, Leningrad.
15. Il’menkov, S. L. and Kleshchev, A. A. (2014) Solution of Problems of Sound Scattering on Bodies of Non-analytical Form with Help of Method of Green’s Functions. Journal Advances in Signal Processing. 2, 50-54.
16. Il’menkov, S. L., Kleshchev, A. A. and Klimenkov, A. S. (2015) The Green’s Functions Method in the Problem of Sound Diffraction by an Elastic Shell of Non-canonical Shape. Journal Acoustical Physics. 60, 617-623.
17. Kleshchev, A. A. (2002) Diffraction and Propagation of Waves in Elastic Mediums and Bodies. Vlas, St.-Petersburg.
18. Kleschev, A. A. and Klimenkov, A. S. (2013) Sound Diffraction on Elastic Isotropic Bodies of Spherical Form (Strict Solution). Journal Marine Gazette. 2, 74-76.
19. Kleshchev, A. A. and Klimenkov, A. S. (2013) Sound Diffraction on Elastic Bodies of Spherical Form (Strict Solution). Proceedings 26-th Session ian Russian Acoustical Societe. Moscow, 130-133.
20. Kleshchev, A. A. and Klimenkov, A. S. (2013) Diffraction of Sound Impulses on Isotro-pic Bodies of Spherical Form (Strict Solution). Journal Advances in Signal Processing. 1, 68-77.
21. Kleshchev, A. A. (2006) Diffraction, Radiation and Propagation of Elastic Waves. Prof-print, St.-Petersburg.
22. Kleshchev, A. A. (2012) Debye and Debye-Type Potentials in Diffraction, Radiation and Elastic Waves Propagation Problems. Journal Acoustical Physics. 58, 308-311.
23. Fock, V. A. (1970) Problems of Diffraction and Propagation of Electromagnetic Waves. Soviet Radio, Moscow.
24. Kleshchev, A, A, and Klukin, I. I. (1976) About Flexural Waves in Elastic Circular Cylindrical Bar. Proceedings LKI. № 109, 3-5.
25. Kleshchev, A. A. (2011) Scattering of Low-Frequency Pulsed Sound Signals from Elastic Cylindrical Shell. Journal Acoustical Physics. 57, 375-380.
26. Il’menkov, S. L., Kleshchev, A. A. and Klubina, K. A. (2015) Calculation of the Phase Velocities of Three-Dimensional Flexural Waves in Isotropic Cylindrical Bars and Shells Using The Debye and Debye-Type Potentials. Journal Acoustical Physics. 61, 1-7/
27. Ahmad, F. (2001) Guided Waves in aTransversely Isotropic Cylinder Immersed in a Fluid. Journal of Acoustical Society of America. 109, 886-890.