Effect Of Fluids Properties On The Velocity Of Single Drop In Spray Column

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Abstract—The rising or falling liquid drops in the solvent extraction equipment determine its capacity, and the relative motion between liquid drops and the fluid phase has a great affects on the convective mass transfer between the phases. Hence the relationship between the drop terminal velocity and down flow of the fluid continuous phase in extraction columns was examined experimentally and a new model is suggested for determination the terminal velocity of liquid drop including the effect of down flow. Experiment were performed using different chemical systems of fluid and different needles diameter to obtain drops of different drop size and a general correlation has been used to emphasis the close relation between the terminal velocity and the continuous phase velocity. Such information is required in estimating drop size distribution which are usually expected to change along the extraction columns. The resulting data were used to check the available relations in the literature. It found that the previously published correlation's of terminal velocity ignored the effect of down flow and a modification has been made in this work to include the influence of continuous phase on drop velocity.

Keywords—Terminal velocity, continuous phase velocity, single liquid drop, spray column.Introduction A great advances have been made in the past few years towards an understanding of basic principles of extraction by single drop technique. The hydrodynamic behavior of the drops is a very important factor in liquid-liquid extraction processes. The falling or rising drops determine the capacity of the solvent extraction equipment, and the relative motion between the drops and the continuous phase affects the convective mass transfer between the phases. The relationships between mass transfer and hydrodynamic performance is complex and there are many types of column each requiring a special understanding. Spray, packed bed and perforatedplate towers, etc. are used commercially in liquidliquid extraction systems. In these towers many drops are formed simultaneously and rise (or fall) in a swarm. yet, no much work has been undertaken to study the effect of continuous phase velocity on the rate of drop formation, terminal velocity and the mass transfer coefficients.

The movement of the dispersed phase as a droplet swarm is controlled by factors that affect the distribution of drop sizes such as hinder settling and other interactions caused by the crowding of the droplets in the swarm, the local velocity of the fluid and the eddy motions in the continuous phase [1]. The efforts to improve the efficiency through increasing or decreasing the size of the dispersed phase drops have met with very little success; smaller drops results in limited throughput due to their low rising velocities whereas larger drops imply a reduction in interfacial area.

Knowledge of drop velocity is of fundamental importance for the description of mass transfer process. In order to estimate mass transfer to or from droplets, it is necessary to consider some aspects of the fluid mechanics of a single particle in free motion through a continuous fluid phase. Aspects which are directly related to mass transfer are the drop velocity relative to the continuous phase, external surface area and internal motion if present, [2, 3].

The movement of the drop with respect to the continuous phase has been assumed constant with time while the mass transfer phenomena are intrinsically transient. hydrodynamic Any consideration about a fluid or solid particle moving through a continuous fluid phase starts principally with the Navier-Stokes equations of motion coupled to the continuity equation [4, 5, 6]. For a fluid particle, due to the presence of internal circulation motions, the equations of motion must be extended also to the internal phase. Two additional boundary conditions are then required at the interface. These are obtained by stating that the normal and shearing stress balanced at the interface. It is noticed that since the shearing stress balance at the interface involves the interfacial tension between the fluids in contact, the internal motion are highly affected by all those variables which affect the interfacial tension.

In this work single drop velocities in a section of 0.15 m diameter column under conditions of the presence and absence of continuous phase flow have been measured. The values of the terminal velocity has been compared to that proposed by the Grace et al. [5]. The effects of down flow of the continuous phase have been investigated with the object of obtaining a method of determining the terminal velocity in the presence of down flow of the continuous phase. Comparison of the predicted values of the terminal velocity using the measured drop size data and those predicted from Grace has been made to show the validity of single liquid drop data for use in column design work.

The results of this study and the lack of data published about the hydrodynamic of freely rising liquid drop in the presence of down flow were the main reason for the present work.

I. Equipment and Experimental Procedure

A. Equipment

The experiments for determination of the free-rising velocity of single liquid drop were conducted in a glass column of 0.15 m diameter and a height 0.5 m filled with water (Fig. 1). Inlet and exit column flow were measured by calibrated rotameters and all experiment were conducted with the aqueous phase continuous and the organic dispersed at laboratory temperature. The diameter of column was large enough to avoid wall effects [7, 8]. The wall effect calculated using an equation recommended by Grace et al. [5] was found to be negligible.

In all experiments, the lighter phase was dispersed (organic), solvent drops passed through the continuous phase, usually water. Different binary systems have been used and the physical properties of these systems studied are summarized in Table (1). Investigation of so many systems allowed the work to be carried out over different range of physical properties relevance to liquid extraction processes. To some system (system 8), Oil Red O dye, non surface active, was added since the drop formed were so small and their refractive index was close to that of water [5, 9, 10]. Some of the systems used (system 1, 2, 3, 4) were recommended for test work by the European Federation of Chemical Engineering Working Party on Distillation, Absorption and Extraction [9].



| ~ | ρ_c | ρ_{d} | μ_c | μ_d | γ |
|--------------------------|----------|------------|----------------------|----------------|-------|
| Systems | (kg/m³) | (kg/m³) | $(kg/ms.10^{\circ})$ | $(kg/ms.10^3)$ | mN/m) |
| 1- Toluene / water | 998 | 863 | 0.98 | 0.59 | 34 |
| 2- MIPK / water | 997 | 896 | 0.98 | 0.55 | 10.6 |
| 3- Butyl acetate / water | 998 | 877 | 0.98 | 0.75 | 13.4 |
| 4- Ethyl acetate / water | 998 | 895 | 0.98 | 0.48 | .5 |
| 5- Benzene / water | 998 | 873 | 0.98 | 0.67 | 32.8 |
| 6- Kerosene / water | 998.5 | 807 | 1.08 | 1.47 | 42 |
| 7- Xylene / water | 998 | 863 | 1.08 | 1.24 | 33.4 |
| 8- Butanole / water | 986 | 846 | 1.43 | 2.4 | 1.75 |

 Table 1: Physical properties of the fluid systems used

B. Experimental Procedure

The column was filled with water as the aqueous phase before the solvent (dispersed phase) commenced. Rotameters on the lines to and from the column enabled accurate flow settings and adjustments to be made thus ensuring that balanced flows were rapidly achieved and then maintained.

A positive displacement pump (Razel Scientific Instrument type A-99) was used to introduce the organic phase into the bottom of the column. When the dispersed phase enters into the top section of the glass needle the pump flow rate setting was reduced so it almost equals the atmospheric pressure ensuring the dispersed phase level inside the glass needle remains constant. The syringe pump was kept running while the column was filled with distilled water [10]. The pump and syringe were mounted on a higher level in order to avoid back flow of solvent in the syringe because the actual flow rate value given for each setting would no longer be precise and there was also the possibility of damaging the pump due to the back flow of solvent. After each experiment the column were thoroughly cleaned. Experiment were performed using different needles to obtain drops of different size. The equivalent spherical drop diameter was calculated, knowing the flow rate from the syringe pump and counting the number of drops formed and measuring the time of formation using an electronic stop watch. At least 300 drops were timed for each run, after a period allowed to reach to steady state conditions.

II. Theoretical Aspect and Background

A. Drop Terminal Velocity

The terminal velocity of a liquid drop in a liquid medium is the free fall or rise, depending on the relative density, velocity of a single isolated drop in the gravitational field. The terminal velocity of small drops, which are essentially spherical are larger than those of the solid spheres of the same diameter and density owing to the mobility and the internal circulation within the drop. The surface velocity is not zero, as it is for a solid, with increasing diameter, there occurs a transition drop size, beyond which the drop shape is no longer spherical and the drop oscillates and distorts [11]. The terminal velocity of the transition size is a maximum and for larger size the velocity fall slowly with increased diameter [12,13,14,15,16]. Dimensionless analysis shows Re = f (CD,We), where Re is the drop Reynolds number at the terminal velocity, We, is the drop Weber number and CD the usual drag coefficient. For very pure liquid, i.e. no surface active agent and no mass transfer and the continuous phase viscosity less than 0.005 kg/ m s the fundamental relation for terminal velocity [26] is defined as

$$V_t = 4 \text{ Re}^4 / 3C_D \text{ We}^3 = \rho_c 2 (g c \gamma)^2 / g \mu_c^4 \Delta \rho (1)$$

B. Drag Coefficients

The drag over a drop immersed in a fluid flow is caused by two stresses, the first is the shearing stresses and the second is the pressure stresses. The first is the result of both the fluid viscosity and the spatial gradients of the velocity components. Since velocity gradients increase in magnitude as the boundary of the drop is approached, the stresses reach a maximum at the surface. At the boundary the shear stresses must maintain the fluid flow to equal the velocity of the boundary. Outside of this boundary the fluid layers can slip, but at the boundary the lamina next to the surface may not slip. The pressure stress are the results of both stationary and dynamic forces. They largely occur in the wake, which is the region downstream of the separation point, i.e. shear stresses is zero [17,18].

For a particle moving with steady terminal velocity in a gravitational field, there are two forces acting on a drop of diameter ,d, which moves in a stationary field. The first is the apparent buoyancy force

Fb =
$$(\pi d^3 / 6) g \Delta \rho$$
 (2)

and the second is resistance force

$$Fr = C_D * (\pi d^2 / 4) \rho_c V_t^2 / 2$$
 (3)

Under steady state conditions, the buoyancy force is balanced by the resistance force, then, i.e.

$$(\pi d^{3}/6) g \Delta \rho = C_{D}^{*} (\pi d^{2}/4) \rho_{c} V_{t}^{2}/2$$
 (4)

From which

$$C_{D}$$
 = (4/3)($\Delta \rho / \rho_{c}$)(d g/ V_{t}^{2} /2) = 4 $\rho \Delta \rho g$ d³ / 3 μ^{2} Re² (5)

III. Present Applications, Estimation of terminal velocity

Knowledge of particle velocity is of fundamental importance for the description of mass transfer process. Recommended relationships for the evaluation of the terminal velocity will be given for the two different situations: low and high Reynolds number [19]. At low Re number the drag coefficient for a drop can be evaluated from the Hadamard-Rybczynski model [20]. This predicts values of the terminal velocity up to 50% higher than those given by Stokes for a rigid drop. However from most experimental measurement it appears that the terminal velocity of fluid particles particularly of small diameter, is very close to the value predicted by the Stokes law. This is most probably due to the presence of surface active substance which tends to accumulate at the interface between the two fluids. For a given fluid with intermediate but unknown contaminant content, the value of terminal velocity is bounded from above by the pure system value (Hadamard-Rybczynski: 1.5 Vt stokes) and bounded from below by the fully contaminated system (rigid drops). Also at a high Re number contaminant play a major role. The most convenient approach for developing terminal velocity correlations valid for contaminated fluid is to empirically correlate the large body for various fluids. Mekasut et at. [21, 22, 23] found that experimental results agree well with [24] only for the pure or slightly contaminated system. Large deviation are found when the system is highly contaminated.

Grace et al., [5] publishes three types of correlations to a large body of experimental data: the form suggested by [25]: that suggested by Hu and Kinter [26] and its extension by Johnson and Braida [4]: and a wave analogy suggested for bubbles by Mendelson [27] and extended to drops by Marrucci [28]. The resulting correlation is

$$J = 0.94 H^{0.757} \text{ for } 2 \le H \le 59.3$$
 (6)

$$J = 3.42 H^{0.411} \text{ for } H > 59.3$$
 (7)

Where
$$H = 4/3 E M^{-0.149} (\mu / \mu_w) 0.14$$
 (8)

$$J = \text{Re M}^{0.149} + 0.857$$
 (9)

The terminal velocity may be expressed explicitly as

$$V_t = (\mu_c / \rho_c d) M^{-0.149}$$
 (J- 0.857) (10)

This correlation is recommended for calculations of bubble and drop terminal velocities when the criteria outline above are satisfied and where some surface active contamination is inevitable.

Many other correlations for calculating the terminal velocity of bubbles and drops are available [6, 28,29, 30 32, 33, 35, 36]. Johnson and Braida [4, 37, 38, 40] .Non covers such a broad range of data as equation (10). Moreover, a number of the earlier correlation required that values be read from graph or that iterative procedure be used to determine the terminal velocity.

In view of the limited data available, Grace et al. [5] modified the previous correlation rather than proposing an entirely different correlation. For pure bubbles and drops, it can be shown that the terminal velocity is related to that for the same system under contaminated conditions by:

Vpure =
$$V_t (1 + (1 / (2 + 3 k)))$$
 (11)

IV. Discussions of the Results

A. Terminal Velocity and Drop size

The size of a drop formed in an immiscible liquid is dependent upon the physical properties of the system and the formation conditions. The motion of drop is characterized by the gross terminal velocity. A plot of the terminal velocity versus the equivalent drop diameter is shown in Fig. 2. Also shown in Fig. 3, the terminal velocities for these drops rising singly as predicated by the correlation of Grace e. al., [5]. It is observed that the terminal velocity of a liquid drop increases as the diameter is increased and thereafter decreases to some asymptotic value. When a drop is released from rest into another immiscible liquid it will accelerate to its terminal velocity and rise through the liquid at this rate. Klee and Trybal [42, 43, 44] noted a peak in the terminal velocities versus drop diameter, corresponding to the beginning of drop oscillation,

after which the terminal velocity becomes constant and is dependent only upon the physical properties of



B. Drag Coefficients

Drag on drop motion in low viscosity field liquid has been studied in terms of drag coefficient versus drop Reynolds number [42, 56]. The experimental results for drag coefficients are shown in Fig. 4. Deformation of drops from the spherical shape appears to cause an increase in drag coefficient. The variation of drag coefficient with Reynolds number ag*ree* with the work



C. Effect of Continuous Phase Velocity

the system involved.



of [45, 46, 47].Fig. 5 shows the plots of Reynolds number against Eotovs numbers, the Morton number range $3*10^{-9}$ to 10-10, but widely different values of the viscosity ratio. Such a plot will give an indication of the dependence of the terminal velocity on the viscosity ratio [48, 49, 50]. The data exhibt some scatter, this could be attributed to the differences in viscosity ratio, thus inferring the presence of internal circulation inside the drops [51 to 57].

Since liquids was initially circulate over the top cross sectional area of the column using a liquid distributor. This circulation has a significant effect on increasing the rising time of drop and hence decreasing the terminal velocity. Fig. 6 shows the profile of down flow of the continuous phase against the terminal velocity under various flow rates conditions.

As mention earlier, to use Grace correlation one use the system properties and drop equivalent diameter to

When we compared the experimental results with values calculated based on the above equation, a good agreement was found as shown in Fig. 7. The terminal velocity decreases as the down flow rate increases, this due to the increase on the residence time (rising time) of the drop inside the column. On account of this drag and skin friction increase which

V. Conclusions

Observation of liquid - liquid extraction from single liquid drops has the advantage that the extraction can be carried out under known conditions of interfacial area and time of contact. Results confirmed that the factors determining terminal velocity are drop size, calculate H., J is then found from equation (9) and the terminal velocity is given by equation (10).

By introducing an additional parameter such as the continuous phase velocity to the Grace equation to correlate data covering continuous phase velocity up to 0.022 m / s the terminal velocity in the presence of down flow can be calculated from the following equation:

$$V_t$$
 = (μ_c / d ρ_c) M- 0.149 (J- 0.857) - V_c (12)

apparently cause a decrease in the terminal velocity. As a conclusion, it can be stated that suggested equation can be used for the correlation of the observed data when the continues phase velocity greater than or equal zero. i.e. in the presence of down flow of the continuous phase.

shape and physical properties of drop and continuous phase.

The correlation of Grace et al. [5] was found to be best predict terminal velocities and by introducing the continuous phase velocity terms to Grace equation, it is possible to extend the applicability of the equation to represent the terminal velocity whether there are down flow or not. The obtained results should be directly applicable and should help to explain the mechanism of extraction in other types of extraction equipment.

Symbols used

- d Volume equivalent sphere diameter for drop
- Eo Eötvos number, g d2 $\Delta \rho / \gamma$
- g Acceleration due to gravity
- H Dimensionless group defined by equation (8)
- J Dimensionless group defined by equation (9)
- k Viscosity ratio, $\mu d / \mu c$
- M Modified Morton number, g μ c 4 $\Delta\rho$ / ρ c2 γ 3
- Re Drop Reynolds number, V d $\rho c / \mu c$

Subscripts

- c Continuous
- d Dispersed

Greek Symbols

- γ Interfacial tension
- μ Viscosity
- ρ Density
- $\Delta \rho$ Phase density difference

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Further work is recommended at a large scale columns with other chemical systems and with a wider range of physical properties relevance to liquid extraction processes.

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