Electromagnetic Scattering By Random Two-Dimensional Rough Surface Using The Joint Probability Distribution Function And Monte Carlo Integration Transformation

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Abstract—Electromagnetic scattering by two dimensional random rough surface is studied. After generating a two-dimensional rough surface, the joint probability density function of the surface height and slope is computed and used to determine the electromagnetic scattering field. The ray-tracing base model is utilized. Also Monte-Carlo method is employed to transform an infinite to a finite integration. In this paper two case studies have been performed and compared with each other. For the first case, we worked on the suggested method on [1] to find the 2D electromagnetic scattering field. In this, the authors for their future work have proposed a way to solve the 2D scattering problem. Their method consisted of computing the PDF of the scattering field in $\underset{x}{\rightarrow}$ and $\underset{y}{\rightarrow}$ direction and use them to determine the scattered field. For second case we discuss our proposed method which is to use the joint PDF of surface height and slope instead of the one proposed in [1] to determine the 2D scattering field. The reason for using the joint PDF of the surface height and slope is that any point on a surface can be only well represented by its height and slope. Both methods are compared each another. And the two methods are then compared to others works. From these comparisons we have shown that our method gives better result.

Keywords—*Electromagnetic scattering, Rough surface, Probability density function, Monte Carlo, Scattering coefficient.*

I. INTRODUCTION

The scattering of electromagnetic by two-dimensional random rough surfaces have been developed in the last century. Nowadays researchers interested in solving the problem have increased because of the impact of this phenomenon in diverse disciplines of sciences. These include communication, geophysics and remote sensing of the earth. The classic methods applied to two-dimensional random rough surface scattering are the small perturbation and Kirchhoff approximation [2], [3]. These two method are limited in their domain of validity. Monte Carlo simulations of direct solutions of electromagnetic scattering problem is popular due to advent of modern computers and the advancement of fast numerical methods. Monte Carlo is also restricted because it requires more powerful numerical approach and the use of long surface. In this paper, the electromagnetic scattering field is studied using the joint PDF of height and slope. We assume that the surface height and slope are uncorrelated.

First, we have generated a 2D random rough surface using MATLAB. Ray tracing technique is employed to investigate the electromagnetic field [4], [5].

Second, referring to the two-dimensional configuration in polar coordinates (see figure 4) the 2D surface slope is calculated. After getting the surface slope we determine the joint PDF of the slope and the height where the PDF of the height is calculated too.

Third, this Joint PDF has been utilized to compute the scattering filed. The scattering coefficient is plotted against the scattering angle in MATLAB. And our result is compared with others works (see figure 6). These include Kirchhoff approximation and Monte Carlo method [12], [16].

II. 2D ROUGH SURFACE GENERATION

In this section we generated a two-dimensional random rough surface. Several methods have been developed to generate a random rough surface. These include convolution method, lines-oriented method for generating inhomogeneous random rough surface and directed Fourier transform [6], [7], [8].

In this paper, the rough surface parameters are derived analytically by using convolution method for its generation.

A. 2D Rough Surface Characteristics

In general a surface is characterized by its spectrum density, correlation length autocorrelation function and its autocovariance. The Gaussian height probability density function is given by [9], [10].

$$P(Z = f(x,y)) = \frac{1}{\rho\sqrt{2\pi}} e^{\frac{(Z=f(x,y))^2}{2\rho^2}} \quad (1)$$

where $f(x,y)$ represent the height function of the 2D random rough surface and ρ^2 its mean square.
For 2D surface the spectrum density function $W(\boldsymbol{K})$ is defined as follows:

$$W(\boldsymbol{K}) = \left(\frac{cl^2\rho^2}{4\pi}\right) e^{-\frac{K^2 cl^2}{4}} \quad (2)$$

where cl is the correlation length. $\boldsymbol{K}$ the spatial angular frequency vector, is defined by:

$$\boldsymbol{K} = (K_x, K_y) \quad , \quad K = \sqrt{K_x^2 + K_y^2} \quad (3)$$

From the spectrum density function we determine auto-correlation function (ACF)

$$ACF = \int_{-\infty}^{+\infty} W(\boldsymbol{K}) e^{iK(x+y)} d\boldsymbol{K} = h^2 e^{\frac{-x^2}{cl_x^2} - \frac{y^2}{cl_y^2}} \quad (4)$$

where h is the rms height and cl_x and cl_y are the correlation lengths in x and y respectively.
As the ACF and the height are both Gaussian distributions then the slope will be also Gaussian.
The slope in x and y give

$$S_x = \frac{\partial f}{\partial x} \text{ and } S_y = \frac{\partial f}{\partial y} \quad (5)$$

Then we get the power spectrum density function of the surface.

$$P_S(S_x, S_y) = \frac{1}{2\pi w_x w_y} e^{\frac{-S_x{}^2}{2w_x^2} - \frac{S_y{}^2}{2w_y^2}} \quad (6)$$

where $w = \frac{\rho\sqrt{2}}{cl}$ represents the rms slope for Gaussian rough surface.

B. Algorithm and Plot for 2D Rough Surface Generated

In convolution method the 2D discrete Fourier transform (DFT) is given by

$$F = DFT(f)$$

then the inverse DFT is $f = DFT^{-1}(f)$

$$F_{\gamma_x \gamma_y} = \sum_{n_y=0}^{N_y-1} \sum_{n_x=0}^{N_x-1} f_{n_x n_y} e^{-j2\pi\left(\frac{n_x \gamma_x}{N_x} + \frac{n_y \gamma_y}{N_y}\right)}$$

with $\gamma_x = 0, 1, \dots \dots . . N_x - 1$
$\gamma_y = 0, 1, \dots \dots . . N_y - 1$ (7)

N_x and N_y represent the numbers of arrays in x and y direction respectively for 2D.
Therefore we have the surface height

$$f_{n_x n_y} = \sum_{\gamma_y=0}^{N_y-1} \sum_{\gamma_x=0}^{N_x-1} F_{\gamma_x \gamma_y} e^{j2\pi\left(\frac{n_x \gamma_x}{N_x} + \frac{n_y \gamma_y}{N_y}\right)}$$

with $n_x = 0, 1, \dots \dots . . N_x - 1$
$n_y = 0, 1, \dots \dots . . N_y - 1$ (8)

We use MATLAB to perform the generation of the 2D surface (see figure 1).

Table 1. 2D rough surface parameters values

Parameters	Number of surface points	rL- length of surface	h - rms height	Clx and cly - correlation length (in x and y)
values	200	1λ	$0.1\,\lambda$	$0.08\,\lambda$

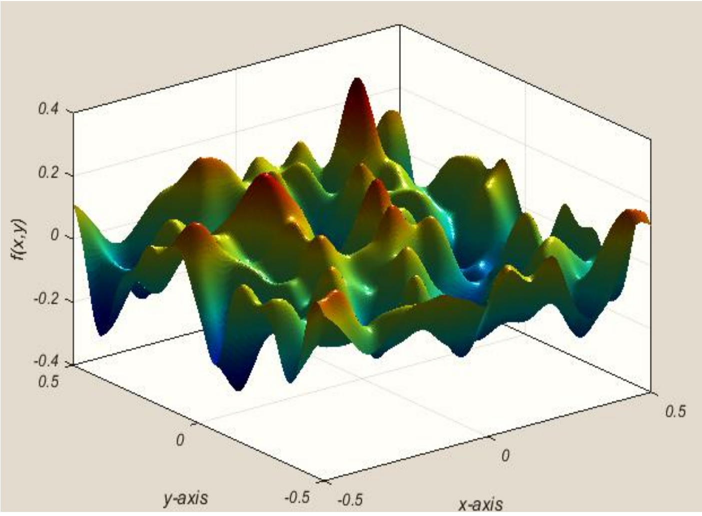

Figure 1. Two-dimensional rough surface generated.

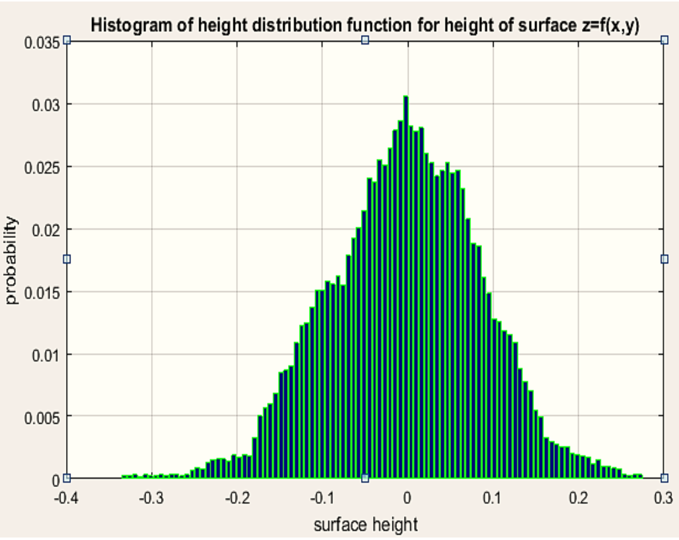

Figure 2. Probability distribution function (PDF) of 2D rough surface height.

III. THE SCATTERING FIELDS COMPUTATION AND PLOTS

In this part of this paper we report our work on two proposed methods. We study first the proposed method in [1]. Second our proposed method is studied and these two methods are compared with each other. Last, both methods are compared with others works existing the literature.

A. *Case Study 1 Using $P_{2D}(\theta_s)$ for Scattering Field Computation*

Here we worked on the proposed method described in [1] to calculate the electromagnetic scattering field. In their paper the authors have worked on the 1D surface problem where the probability density function of the scattering field in θ_s direction has been used to calculate the electromagnetic scattering. The PDF is given by

$$P_{1D}(\theta_s) = \frac{cl}{2\sigma_h cos^2(\alpha)\sqrt{\pi}} \exp\left(-\left(\frac{tan(\alpha)\, cl}{2\sigma_h}\right)\right) \quad (9)$$

where $tan(\alpha)$ represents the surface inclination.

For the study of 2D rough surfaces, they mentioned that there is no a simple relationship that links the local normal orientation and the scattering direction.

Therefore for the future work on 2D case, they proposed to use the PDF of the scattering field in $\underset{x}{\rightarrow}$ and $\underset{y}{\rightarrow}$ direction where the orientations of all the microfacets of rough surface are computed. As the two PDFS are independent then the joint probability is given by

$$P_{2D}(\theta_s) = P_h(\theta_s)_{\vec{x}} * P_h(\theta_s)_{\vec{y}} \quad (10)$$

We need to mention that in [1] the 2D rough surface case has not been studied but only 1D rough surface has been done. The authors have just proposed a method to solve the 2D case.

In this paper we have studied their proposed method and computed the scattering field [12]

$$E_s(\theta_s) = E_0 * F_s(\theta_i, \theta_s, n) * P_{2D}(\theta_s) \int exp\left(-j\varphi(Z, \theta_i)\right)\, dZ \quad (11)$$

Here $F_s(\theta_i, \theta_s, n)$ is Fresnel coefficient in function of incident angle θ_i, scattering angle θ_s and index of refraction n

Now let us apply Monte Carlo method to transform infinite integration to finite. Then the integral becomes

$$\int exp\left(-j\varphi(Z, \theta_i)\right) * P(Z)\, dZ = \int \frac{exp\left(-j\varphi(Z, \theta_i)\right) * P(Z)\, dZ}{P(Z)}$$

$$= \frac{1}{k}\sum_{1}^{k} e^{-j\varphi_k}$$

Therefore

$$E_s(\theta_s) = E_0 * F_s(\theta_i, \theta_s, n) * P_{2D}(\theta_s) * \frac{1}{k}\sum_1^k e^{-j\varphi_k} \quad (12)$$

Thus the electromagnetic scattering field for 2D is

$$E_s(\theta_s) = E_0 * F_s(\theta_i, n) * P_h(\theta_s)_{\vec{x}} * P_h(\theta_s)_{\vec{y}} * \frac{1}{k}\sum_{1}^{k} e^{-j\varphi_k}$$

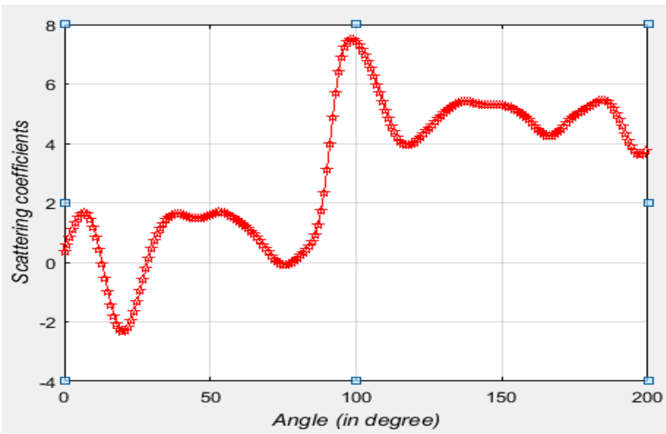

Figure 3. Scattering coefficients for case study 1 using $P_{2D}(\theta_s)$.

B. *Case Study 2 Using $P_{sz}(S, Z)$ for Scattering Field Computation*

For our study of electromagnetic scattering by two-dimensional rough surface we propose to use the PDF of the 2D surface height and slope. This is used because any point of the surface is described by its height Z and slope S. That shows the surface is well represented by its slope and height. Therefore we believe here that the use of $P_{sz}(S,Z)$ to calculate the scattering field would be more suitable and appropriate compared to the one proposed in [1] ($ie\ P_{2D}(\theta_s)$).

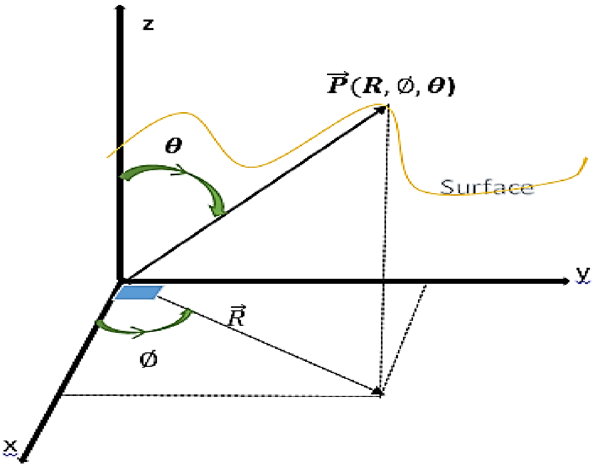

Figure 4. 2D surface configuration in polar coordinates.

It assumes that the surface height and slope are uncorrelated. There are two independent Gaussian variables and the probability density function PDF [11]

$$P_{sz}(S,Z) = P_s(S) * P_z(Z) = \frac{1}{2\pi\sigma_s\sigma_z} e^{\frac{-S^2}{2\sigma_S^2} - \frac{Z^2}{2\sigma_Z^2}} \quad (13)$$

From Figure 4, we have

$S = S_x cos\emptyset + S_y sin\emptyset$ with S_x and S_y represent the surface slopes in the (Ox), (Oy) direction (see equation 5). According to (Ox), $\emptyset$ is the azimuthal direction.

The covariance matrix: $Cov = \begin{bmatrix} E(Z^2) & E(ZS) \\ E(SZ) & E(S^2) \end{bmatrix}$,

where for example $E(Z^2)$ represent the expected value of the surface height.

Assume that the surface height and slope are uncorrelated then $E(ZS) = E(SZ) = 0$

where $E(Z^2) = \sigma_z^2$ and $E(S^2) = E\big((S_x cos\emptyset + S_y sin\emptyset)^2\big)$

$$= (\omega_x cos\emptyset)^2 + (\omega_y sin\emptyset)^2 + E(2S_xS_y sin\emptyset cos\emptyset)$$

As S_x and S_y are uncorrelated therefore $E(S_xS_y) = 0$

Thus $\sigma_S^2 = (\omega_x cos\emptyset)^2 + (\omega_y sin\emptyset)^2$ with ω_x^2 and ω_y^2 are the variance of the slope in the Ox and Oy.

Therefore the matrix becomes: $Cov = \begin{bmatrix} \sigma_Z^2 & 0 \\ 0 & \sigma_S^2 \end{bmatrix}$

$$E_s(\theta_s) = E_0 * F_s(\theta_i,\theta_s,n) * P_{sz}(S,Z) * \frac{1}{k}\sum_{1}^{k} e^{-j\varphi_k}$$

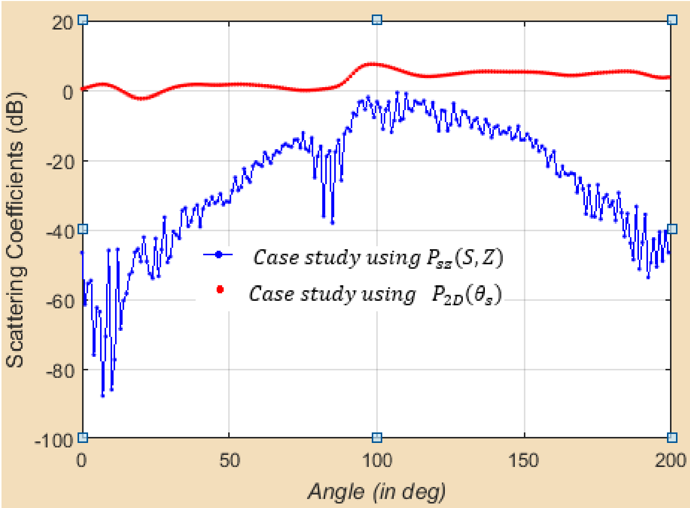

Figure 5. Comparison scattering coefficients for Case study 1, 2 using $P_{sz}(S,Z)$ and $P_{2D}(\theta_s)$.

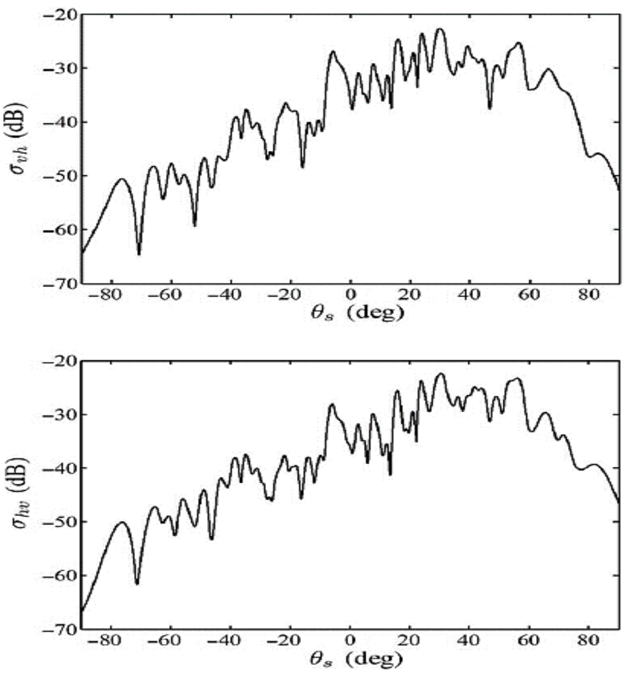

Figure 6. NRCS of a Gaussian rough surface and area of 66λ $\times$ 66λ Ax =λ/20. (h_{rms} , l_c) = $(0.16\lambda, 0.95\lambda)$, θ_i = 30 °, ε_r= 4−j, fc = 0.3 GHz and r_a = 7.69λ, averaged over two realizations using DD-FDTD with 5 X 5 subdomain, Le = 3λ [15].

C. Results Comparison

Figure 5 shows the comparison of scattering coefficient for case study using $P_{sz}(S,Z)$ and that of $P_{2D}(\theta_s)$. Both cases studied have been realized on 2D rough surface with the same surface characteristics and parameters defined on Table 1. For our case study of $P_{sz}(S,Z)$, we can notice from the plot that the scattering field increase rapidly with scattering angle and at around 80 degree it got an inflection point. From that point it starts increasing by 20dB until it reaches a critical point 100 degree scattering angle where it starts decreasing. In contrast to our method, the proposed method for case study using $P_{2D}(\theta_s)$ the scattering field is almost constant(0dB) and at 80 degree of scattering angle it gains around 10dB before it decrease by around 5dB and from that is almost constant (5dB) again. There is an agreement between our method and those presented in the literature. These include Monte Carlo Simulation [13], Kirchhoff Approximation [14] and the method use in [15] (see figure 6). This has demonstrated the accuracy and effectiveness of our method.

IV. CONCLUSION

In this paper we have proposed a new method to compute the electromagnetic scattering field using the joint probability distribution function of the height and slope of 2D rough surface. This method has been compared with the proposed method in [1] for the future 2D study by plotting their scattering coefficient. And the contrast between the two methods can be clearly notified. But our method is more accurate because it has shown its agreement with others works in the literature. electromagnetic wave.

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