Journal of Multidisciplinary Engineering Science and Technology (JMEST)
ISSN: 3159-0040
Vol. 3 Issue 2, February - 2016

Wave Propagation In A Thermo-Magneto-Electro-Elastic Solid

Baljeet Singh[1]
[1]Department of Mathematics, Post Graduate Government College, Sector-11, Chandigarh, 160011,India(E-mail: bsinghgc11@gmail.com).

Aarti Singh[2] **and Nidhi Sharma**[2]
[2]Department of Mathematics, Maharishi Markandeshwar *University*, *Mullana*, Haryana, India

***Abstract*— In the present paper, the governing equations of generalized thermo-magneto-electro-elastic solid are formulated in one-dimension. The time-harmonic plane wave solution of these equations leads to a velocity equation, which shows the existence of two plane waves namely longitudinal and thermal waves. A particular example of $LiNbO_3$ is taken for numerical computation of the speeds of longitudinal and thermal waves. The effects of electric, magnetic and thermal parameters are shown graphically on the speeds of these plane waves.**

Keywords—Thermo-magneto-electro-elasticity, plane waves, speed

I. INTRODUCTION

Biot (1956) developed the coupled theory of thermoelasticity. In the classical theory of theromoelasticity, when an elastic solid is subjected to a thermal disturbance, the effect is felt at a location far from the source, instantaneously. This implies that the thermal wave propagates with an infinite speed, a physically impossible result. In contrast to the conventional thermoelasticity, nonclassical theories came into existence during the last four decades. These theories, referred to as generalized thermoelasticity, were introduced into the literature in an attempt to eliminate the shortcomings of the classical dynamical thermoelasticity. For example, Lord and Shulman (1967), by incorporating a flux-rate term into Fourier's law of heat conduction, formulated a generalized theory which involves a hyperbolic heat transport equation admitting a finite speed for thermal signals. Green and Lindsay (1972), by including temperature rate among the constitutive variables, developed a temperature-rate-dependent thermoelasticity that does not violate the classical Fourier law of heat conduction, when the body under consideration has a centre of symmetry and this theory also predicts a finite speed for heat propagation. Chandrasekharaiah (1986) referred to this wave like thermal disturbance as 'second sound'. Lord and Shulman theory of generalized thermoelasticity was further extended by Dhaliwal and Sherief (1980) for anisotropic case. A survey article of representative theories in the range of generalized thermoelasticity was presented by Hetnarski and Ignaczak (1999). Wide literature on generalized thermoelasticity is available in books by Suhubi (1975), Iesan and Scalia (1996), Iesan (2004), Ignaczak and Ostoja-Starzewski (2009) and *Encyclopaedia of Thermal Stresses* (2013) edited by R. B. Hetnarski.

Wave propagation in thermo-magneto-electro-elastic solid is of much importance due to wide use of piezoelectric and piezomagnetic materials in aerospace and automobile industries. Thermo-magneto-electro-elastic solid are extensively used as electric packaging, sensors and actuators. Kaliski (1965), Coleman and Dill (1971), Amendola (2000), Li (2003) and Aouadi (2007) contributed towards the development of theory of thermo-magneto-electroelasticity. Wave propagation in a generalized thermo-magneto-electro-elastic solid has not been studied so far. In the present paper, one-dimensional governing equations of generalized thermo-magneto-electro-elastic solid are formulated. These equations are solved to show the existence of two plane waves namely longitudinal and thermal waves. The speeds of longitudinal and thermal waves are computed for a particular example $LiNbO_3$. The effects electric, magnetic and thermal parameters are shown graphically on the speeds of these plane waves.

II. FUNDAMENTAL EQUATIONS

We consider a body that occupies the region V of the Euclidean three-dimensional space at some instant and is bounded by the piecewise smooth surface ∂V. The motion of the body is referred to the reference configuration V and a fixed system of rectangular Cartesian axes Ox_i (i = 1, 2, 3). Following Lord and Shulman (1967), Coleman and Dill (1971), Amendola (2000), Li (2003) and Aouadi (2007) the field equations governing the generalized theory of thermo-magneto-electro-elasticity are:

The equations of motion

$\sigma_{ji,j} + F_i = \rho \ddot{u}_i$, (1)

The equations of the electric and magnetic fields

$D_{i,i} = \rho_0, B_{i,i} = \sigma$, (2)

The energy equation

$\rho T_0 \dot{\eta} = q_{i,i} + \rho h$, (3)

The constitutive equations

$\sigma_{ij} = c_{ijkl} e_{kl} + F_{ijk} \zeta_k + \lambda_{ijk} E_k - a_{ij} T$, (4)

$D_k = -\lambda_{kij}e_{ij} + \alpha_{ki}\zeta_i + \gamma_{ki}E_i + p_k T,$ (5)

$B_k = -F_{kij}e_{ij} + A_{ki}\zeta_i + \alpha_{ki}E_i + m_k T,$ (6)

$\rho\eta = a_{ij}e_{ij} + m_k\zeta_k + p_k E_k + c_e T,$ (7)

$K_{ij}T_{,j} = q_i + \tau_0 \dot{q}_i,$ (8)

The geometrical equations

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}),$$

$E_i = -\psi_{,i}, \zeta_i = \phi_{,i},$ (9)

where F_i, ρ_0 and σ are the body force, electric charge density, and electric current density, respectively; ρ is the mass density; h is the heat supply; u_i, ψ and ϕ are the displacement vector, the electric potential, and the magnetic potential, respectively; σ_{ij}, D_k, B_k and η are stress tensor, the dielectric displacement vector, the magnetic intensity, and the entropy density, respectively; e_{ij}, E_i, ζ_i and T are strain tensor, electric field, magnetic field, and temperature change to a reference temperature T_0, respectively; K_{ij} is the conductivity tensor; $c_{ijkl}, \gamma_{kj}, A_{kj}$ and c_e are constitutive moduli connecting fields like stress and strain; $\lambda_{ijk}, F_{ijk}, \alpha_{kj}, a_{ij}, p_i$ and m_i are coupling coefficients connecting various fields like mechanical, magnetic, thermal and electric fields and τ_0 is relaxation time. Subscripts preceded by a comma denote partial differentiation with respect to the corresponding Cartesian coordinate. The superposed dot denotes the partial differentiation with respect to the time *t*.

The constitutive parameters satisfy the

following symmetry conditions

$c_{ijkl} = c_{klij} = c_{jikl}, \lambda_{ijk} = \lambda_{kij} = \lambda_{kji}, F_{ijk} = F_{kij} = F_{kji}, a_{ij} = a_{ji}, \gamma_{ij} = \gamma_{ji}, \alpha_{ij} = \alpha_{ji}, K_{ij} = K_{ji}, A_{ij} = A_{ji}.$ (10)

III. Governing equations in one dimension

We consider one-dimensional disturbance of the medium. Using equations (1) to (10), we obtain the following equations of motion for generalized thermo-magneto-electroelasticity in one-dimension

$c_{11}\frac{\partial^2 u_1}{\partial x^2} - F_{11}\frac{\partial^2 \phi}{\partial x^2} - \lambda_{11}\frac{\partial^2 \psi}{\partial x^2} - a_1\frac{\partial T}{\partial x} = \rho\frac{\partial^2 u_1}{\partial t^2},$ (11)

$\lambda_{11}\frac{\partial^2 u_1}{\partial x^2} + \alpha_1\frac{\partial^2 \phi}{\partial x^2} + \gamma_1\frac{\partial^2 \psi}{\partial x^2} - p_1\frac{\partial T}{\partial x} = 0,$ (12)

$F_{11}\frac{\partial^2 u_1}{\partial x^2} + A_1\frac{\partial^2 \phi}{\partial x^2} + \alpha_1\frac{\partial^2 \psi}{\partial x^2} - m_1\frac{\partial T}{\partial x} = 0,$ (13)

$a_{11}\frac{\partial^2 u_1}{\partial x \partial t} - m_1\frac{\partial^2 \phi}{\partial x \partial t} - p_1\frac{\partial^2 \psi}{\partial x \partial t} + c_e\frac{\partial T}{\partial t} = \frac{K_1}{T_0(1+\tau_0 \partial/_{\partial t})}\frac{\partial^2 T}{\partial x^2},$ (14)

where

$c_{11} = c_{1111}, \lambda_{11} = \lambda_{111}, a_1 = a_{11}, F_{11} = F_{111},$

$\alpha_1 = \alpha_{11}, \gamma_1 = \gamma_{11}, A_1 = A_{11}, K_1 = K_{11}.$

IV. Plane wave solution and dispersion equation

For time harmonic plane wave propagating in the *x*-direction, we can take

$\{u_1, T, \psi, \phi\} = \{\bar{u}_1, \bar{T}, \bar{\psi}, \bar{\phi}\}e^{i(kx-\omega t)},$ (15)

and using equation (15) in equations (11) to (14), we get

$(-k^2 c_{11} + \rho\omega^2)\bar{u}_1 - ia_1 k\bar{T} + k^2 F_{11}\bar{\phi} + k^2\lambda_{11}\bar{\psi} = 0,$ (16)

$k^2\lambda_{11}\bar{u}_1 + ip_1 k\bar{T} + k^2\alpha_1\bar{\phi} + k^2\gamma_1\bar{\psi} = 0,$ (17)

$k^2 F_{11}\bar{u}_1 + im_1 k\bar{T} + k^2 A_1\bar{\phi} + k^2\alpha_1\bar{\psi} = 0,$ (18)

$ka_1\omega T_0\bar{u}_1 - (ic_e\omega + \frac{K_1}{1-i\tau_0\omega}k^2)\bar{T} - km_1\omega T_0\bar{\phi} - kp_1\omega T_0\bar{\psi} = 0.$ (19)

The homogeneous system of equations (16) to (19) has non-trivial solution if the determinant of coefficients vanishes, i. e.,

$A\,v^4 + B\,v^2 + C = 0,$ (20)

where $v = \omega/k$,

$$A = \rho\bar{c}_e(\gamma_1 A_1 - \alpha_1^2) + \rho(2\alpha_1 m_1 p_1 - \gamma_1 m_1^2 - A_1 p_1^2),$$

$$\begin{aligned} B = {} & \lambda_{11}^2 m_1^2 + F_{11}^2 p_1^2 + \alpha_1^2 a_1^2 + \gamma_1 A_1 a_1^2 + 2\alpha_1 a_1 m_1 \lambda_{11} \\ & - 2p_1 m_1 \lambda_{11} F_{11} - 2p_1 a_1 A_{11}\lambda_{11} \\ & - 2\gamma_1 a_1 m_{11} F_{11} + 2a_1\alpha_1 p_1 F_{11} \\ & - (\gamma_1 A_1 - \alpha_1^2)(c_{11}\bar{c}_e + \rho\bar{\bar{K}}) \\ & - c_{11}(2\alpha_1 m_1 p_1 - \gamma_1 m_1^2 - A_1 p_1^2) \\ & + \bar{c}_e(2\alpha_1\lambda_{11}F_{11} - A_1\lambda_{11}^2 - \gamma_1 F_{11}^2), \end{aligned}$$

$$C = \bar{\bar{K}}c_{11}(\gamma_1 A_1 - \alpha_1^2) - \bar{\bar{K}}(2\alpha_1\lambda_{11}F_{11} - A_1\lambda_{11}^2 - \gamma_1 F_{11}^2),$$

and

$\bar{\bar{K}} = K_1/T_0(\tau_0 + \frac{i}{\omega}), \bar{c}_e = \frac{c_e}{T_0}.$

The dispersion equation (20) is a quadratic equation with complex coefficients. The two roots of equation (20) correspond to longitudinal and thermal waves. Further, $v = Re(v) + i\,Im\,(v)$ is a complex constant so that $Re(v)$ is giving the wave speed.

V. Particular cases

(i) In absence of thermal parameters , i.e for $a_1 = p_1 = \alpha_1 = 0$, the equation (20) reduces to

$$v^2 = \frac{c_{11}}{\rho} + \frac{\lambda_{11}^2 A_1 + c_{11}^2\gamma_1 - 2\alpha_1\lambda_{11}F_{11}}{\rho(\gamma_1 A_1 - \alpha_1^2)},$$

(21)

(ii) In absence of thermal, electric and magnetic parameters, i.e. for $a_1 = p_1 = m_1 = \lambda_{11} = F_{11} = 0$, the equation (20) reduces to

$v^2 = \frac{c_{11}}{\rho}$, (22)

(iii) In absence of thermal and magnetic parameters, i.e. for $a_1 = p_1 = m_1 = F_{11} = 0$, the equation (20) reduces to

$$v^2 = \frac{c_{11}}{\rho} + \frac{\lambda_{11}^2}{\rho\gamma_1}, (23)$$

(iv) In absence of thermal and electric parameters, i.e. for $a_1 = p_1 = m_1 = \lambda_{11} = 0$, the equation (20) reduces to

$$v^2 = \frac{c_{11}}{\rho} + \frac{F_{11}^2}{\rho A_1}, (24)$$

(v) In absence of electric and magnetic parameters, i.e. for $p_1 = m_1 = \lambda_{11} = F_{11} = 0$, the equation (20) reduces to

$$v^2 = \frac{a_1^2+\rho^2+c_{11}\bar{c}_e \pm \sqrt{(a_1^2+\rho^2+c_{11}\bar{c}_e)^2 - 4\rho\bar{c}_e\bar{\bar{k}}c_{11}}}{2\rho\bar{c}_e}. (25)$$

VI. NUMERICAL RESULTS AND DISCUSSIOIN

Following relevant physical constants of $LiNbO_3$ are considered for numerical computation of wave speeds of longitudinal and thermal waves:

$$\rho = 4.647 \times 10^3\ Kg\ m^{-3},$$
$$c_{11} = 2.03 \times 10^{11} N\ m^{-2},$$
$$\lambda_{11} = 1.33\ Cm^{-2}, \gamma_1 = 85.2,$$
$$F_{11} = 0.2 \times 10^{-2}\ Kg,$$
$$A_1 = 0.005,$$
$$a_1 = 13.3 \times 10^{-6} K^{-1},$$
$$K = 4\ W\ m^{-1}K^{-1},$$
$$\alpha_1 = 0.02, m_1 = 0.006,$$
$$p_1 = 0.133 \times 10^5\ N\ C^{-1}K^{-1},$$
$$\omega = 10\ Hz, \tau_0 = 0.05s$$

The speed of longitudinal wave is plotted against frequency (0.01 Hz ≤ ω ≤ 20 Hz) in Figure 1, when

$\lambda_{11} = 1.33\ Cm^{-2},\ F_{11} = 0.2 \times 10^{-2}\ Kg$ $and\ \tau_0 = 0.05s$. The speed of longitudinal wave in thermo-magneto-electro-elastic solid half-space is 7.814 × 10^4 m s^{-1} at ω = 0.01 Hz. It increases slowly with the increase in value of frequency and attains value 11.585 × 10^4 m s^{-1} at ω = 20 Hz. The variation of the speed is shown by solid curve in Figure 1. In absence of electric and magnetic parameters, the solid curve reduces to dotted curve. If we further neglect thermal effects, the dotted curve reduces to dotted line with center symbols, where speed is independent of frequency. The speed of thermal wave is also plotted against frequency (0.01 Hz ≤ ω ≤ 20 Hz) in Figure 2. The speed of thermal wave in thermo-magneto-electro-elastic solid half-space is 0.04 × 10^4 m s^{-1} at ω = 0.01 Hz. It increases sharply with the increase in value of frequency and attains value 1.395 × 10^4 m s^{-1} at ω = 20 Hz. The variation of the speed is shown by solid curve in Figure 2. In absence of electric and magnetic parameters, the solid curve reduces to dotted curve. If we further neglect thermal effects, then the thermal wave does not exist.

The speeds of longitudinal and thermal waves are also plotted against the electric parameter $0 \le \lambda_{11} \le 10$ in Figures 3 and 4, respectively, when $\omega = 10\ Hz,\ F_{11} = 0.2 \times 10^{-2}\ Kg\ and\ \tau_0 = 0.05s$. The speed of longitudinal wave is 9.2928 × 10^4 m s^{-1} at λ_{11}= 0. It first decreases slowly to 9.29 × 10^4 m s^{-1} at λ_{11}= 0.75 and then increases to 9.6 × 10^4 m s^{-1} at λ_{11}= 10. This variation is shown by solid curve in Figure 3. In absence of magnetic and thermal parameters, it reduces to dotted curve. The speed of thermal wave is 1.27356 × 10^4 m s^{-1} at λ_{11}= 0. It first decreases slowly to 1.27332 × 10^4 m s^{-1} at λ_{11}= 0.85 and then increases to 1.29725 × 10^4 m s^{-1} at λ_{11}= 10. This variation is shown by solid curve in Figure 4. In absence of magnetic and thermal effect, this wave does not exist.

The speeds of longitudinal and thermal waves are also plotted against the magnetic parameter $0 \le F_{11} \le 0.5$ in Figures 5 and 6, respectively, when $\lambda_{11} = 1.33\ Cm^{-2},\ \omega = 10\ Hz\ and\ \tau_0 = 0.05s.$ The speed of longitudinal wave is 6.836 × 10^4 m s^{-1} at F_{11}= 0. It increases to 19.69× 10^4 m s^{-1} at F_{11}= 0.5. This variation is shown by solid curve in Figure 5. In absence of electric and thermal parameters, it reduces to dotted curve. The speed of thermal wave is 0.6643× 10^4 m s^{-1} at F_{11}= 0. It increases sharply to 1.3524× 10^4 m s^{-1} at F_{11}= 0.285 and then decreases slowly to 1.268× 10^4 m s^{-1} at F_{11}= 0.5. This variation is shown by solid curve in Figure 6. In absence of electric and thermal effect, this wave does not exist.

The speeds of longitudinal and thermal waves are plotted against the thermal relaxation time $0 \le \tau_0 \le 0.5$ in Figures 7 and 8, respectively, when $\lambda_{11} = 1.33\ Cm^{-2},\ F_{11} = 0.2 \times 10^{-2}\ Kg\ and\ \omega = 10\ Hz$. The speed of longitudinal wave is 9.105 × 10^4 m s^{-1} at $\tau_0 = 0$. It increases to 10.48× 10^4 m s^{-1} at τ_0 = 0.08 and then decreases to 8.816 × 10^4 m s^{-1} $\tau_0 = 0.5$. This variation is shown by solid curve in Figure 7. In absence of electric and magnetic parameters, it reduces to dotted curve. The speed of thermal wave is 1.286 × 10^4 m s^{-1} at $\tau_0 = 0$. It decrease to 0.694 × 10^4 m s^{-1} at τ_0 = 0.5. This variation is shown by solid curve in Figure 8. In absence of electric and magnetic parameters, it reduces to dotted curve.

VII. CONCLUSION

The governing equations of thermo-magneto-electro-elastic solid are formulated in one-dimension. These equations are solved for plane wave solution and a quadratic velocity equation is obtained. The velocity equation shows the existence of two plane waves namely longitudinal and thermal waves. The speeds of longitudinal and thermal waves are computed for $LiNbO_3$. From numerical results, it is observed that the speeds of plane waves are affected significantly by electric, magnetic and thermal parameters.

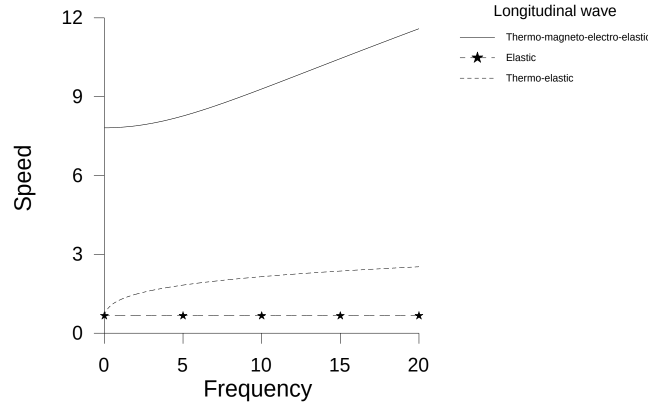

Figure 1. Variations of speed of longitudinal wave against frequency.

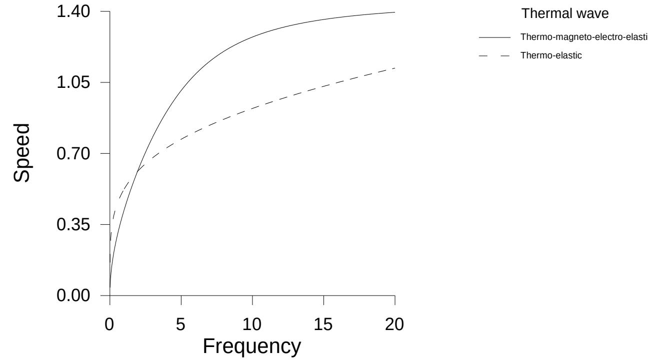

Figure 2. Variations of speed of thermal wave against frequency.

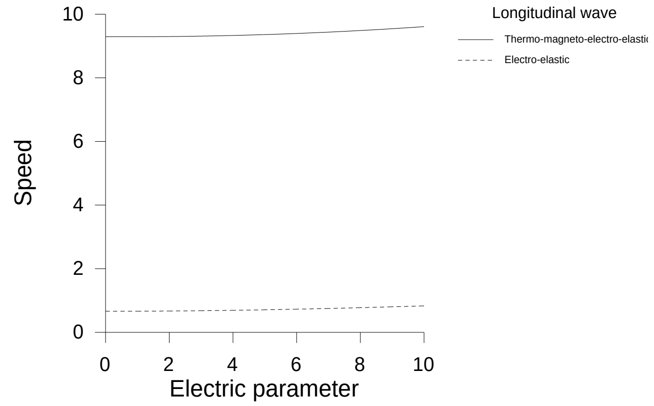

Figure 3. Variations of speed of longitudinal wave against electric parameter λ_{11}.

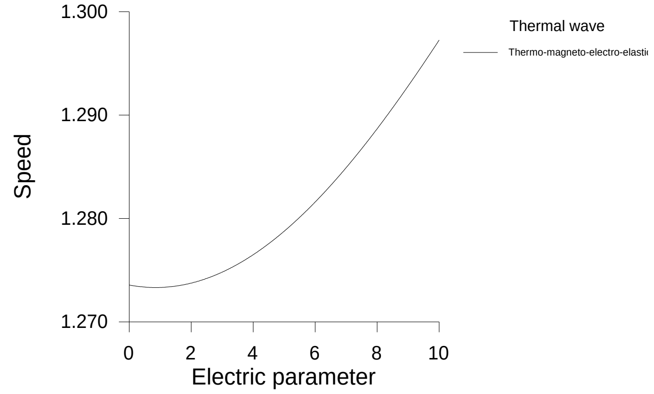

Figure 4. Variations of speed of thermal wave against electric parameter λ_{11}.

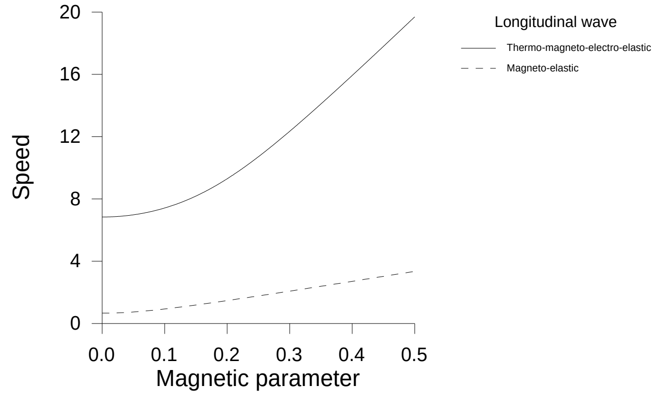

Figure 5. Variations of speed of longitudinal wave against magnetic parameter F_{11}.

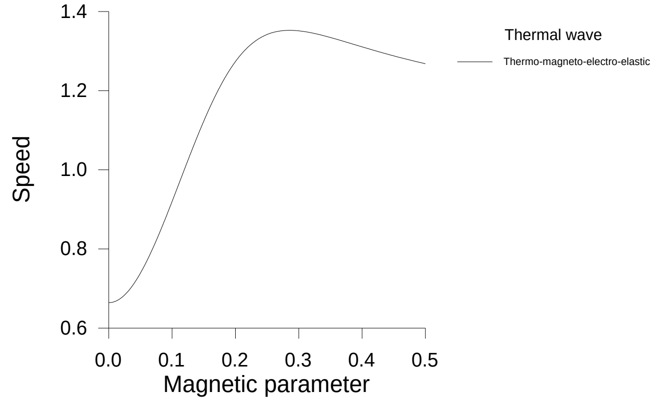

Figure 6. Variations of speed of thermal wave against magnetic parameter F_{11}.

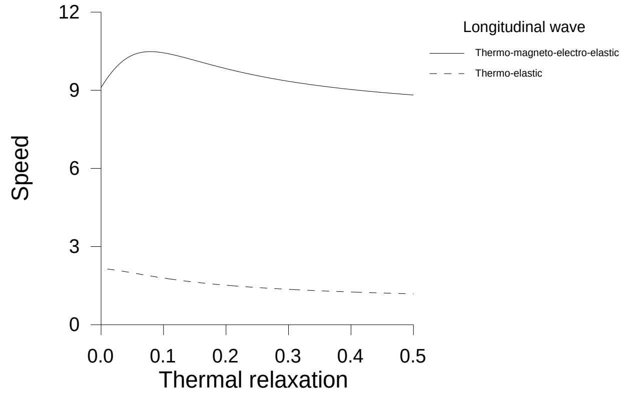

Figure 7. Variations of speed of longitudinal wave against thermal reaxation parameter τ_0.

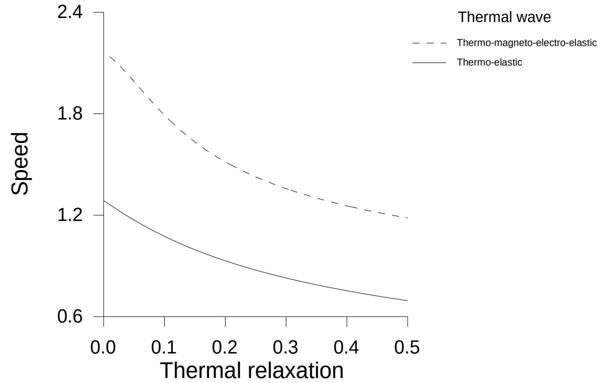

Figure 8. Variations of speed of thermal wave against thermal reaxation parameter τ_0.

REFERENCES

[1] M. Biot, “Thermoelasticity and irreversible thermodynamics”, *J. Appl. Phys.* Vol. 27, pp. 249-253, 1956.

[2] H. Lord and Y. Shulman, “A generalized dynamical theory of thermoelasticity”, *J. Mech. Phys. Solids, vol.*15, pp. 299-309, 1967.

[3] A.E. Green and K.A. Lindsay, “Thermoelasticity”, *J. Elasticity,* vol. 2, pp. 1-7, 1972.

[4] D. S.Chandrasekharaiah, “Thermoelasticity with second sound: A Review”, *Appl. Mech. Rev.* vol. 39, pp. 355-376, 1986.

[5] R.S. Dhaliwal and H.H. Sherief, “Generalized thermoelasticity for anisotropic media” *Quart. Appl. Math. v*ol. 33, pp. 1-8, 1980.

[6] R.B. Hetnarski and J. Ignaczak, “Generalized Thermoelasticity” *J. Thermal Stresses* 22, 451-476, 1999.

[7] S. Kaliski, Wave equations of thermo-electro-magneto-elasticity. *Proc. Vibr. Probl.*,

vol. 6, pp. 231-263, 1965.

[8] E. Suhubi, “Thermoelastic Solids”, in Cont. Physics, ERINGEN A. C. Ed., Acad.

Press, New York, 11, p. 191, 1975.

[9] D. Iesan and A. Scalia, “Thermoelastic Deformations”, Springer, 1996.

[10] D. Iesan, “Thermoelastic Models of Continua” (Solid Mechanics and its

Applications), Springer, 2004.

[11] B.D. Coleman and E.H. Dill, “Thermodynamic restrictions on the constitutive equations of electromagnetic theory” .*Z. Angew. Math. Phys.*, vol. 22, pp. 691-702, 1971.

[12] G. Amendola, “On thermodynamic conditions for the stability of a thermo-electro-magnetic system”. *Math. Meth. Appl. Sci.*, vol. 23, pp. 17-39, 2000.

[13] J.Y. Li, “Uniqueness and reciprocity theorems for linear thermo-electro-magnetoelasticity”. *Q. J. Mech. Appl.Math.*, vol. 56, pp. 35-43, 2003.

[14] M. Aouadi, “The generalized theory of thermo-magnetoelectroelaticity”, Technische Mechanik, vol. 27, pp. 133-146, 2007