Ore Transportation Cost Minimization Model to Determine Optimum Processing Plant Location

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***Abstract*— The main objective of this study is to formulate a mathematical model to minimize the ore transportation cost. The mathematical model will be applicable to serve any ore deposits, especially that consists of many localities such as Aljalamaid phosphate ore deposits. The mathematical model aims to find the location of the intended processing plant. The adopted mathematical model takes into consideration number of locations, tonnage in each location, and the distance from the geometric center of each location to an unknown optimum location of the processing plant. To solve the considered mathematical model, FORTRAN program was developed. The coordinates of the optimum location of processing plant were obtained. The model is applicable to serve any other ore deposit especially that consists of many locations.**

Keywords*— *Optimization, Ore Transportation Cost, Minimization Cost, Processing Plant Location.

I. INTRODUCTION

The ore transportation cost is one of the main important operating costs of any mining project. Transportation cost depends on a number of factors such as distance, quantity, and method of transportation. Many models were formulated such as Larwood and Benson Model [1], Anderson Model [2], Bechtel Model [3], and Zimmerman Model [4]. There were many trials to minimize the transportation cost [5-10]. Fjellstrom [11] determined transportation cost of ore and waste material to the crusher and backfilling rooms in the underground Renstrom mine by using the software package "AutoMod". AutoMod is a simulation program that can simulate all the processes not only in the mine but also in other industries. Main results from the model indicate that the mine truck is 23% more effective than the highway trucks, the usage of only highway trucks are 34% more expensive than the usage of only mine trucks and for combination alternatives, if most of the transportation is done by mine trucks instead of highway trucks, the transportation cost per ton decreases by 10-20%. Wegener [12] suggested recent developments in the field of transportation models. Brazil *et al.* [13] considered the case of optimizing the construction and transportation costs of underground mining roads. The authors focused on the model of underground mine networks. This model consisted of ramps and their relations with maximum gradient. They stated that the cost is affected by ramps lengths, the ore quantity transported through the ramps and their gradients.

Dharma and Ahmad [14] investigated two models for a real-world application of a transportation problem that involved transporting iron ore from two iron ore mines to three steel plants using both linear and integer programming methods. Their models were then further applied to generate an optimized solution that minimized the transportation cost. The authors compared their results to determine the most practical model for a real-world situation and how significant the difference would be.

Shephard [15] suggested a model based on the transportation cost factors such as quantity, distance, shipment delay, transport technology, and route. Inwood and Keay [16] used modern compiled evidence on effective transport costs of iron trade to investigate the relationship between trade costs and trade volumes. Reeb and Leavengood [17] used linear programming to minimize transportation cost. Ali and Sik [18] presented a method according to linear programing to minimize the transportation cost in mining. Chen *et al.* [19] proposed an organization optimization model based on the mathematical model of classical transportation problem and transport path. Optimization method for imported iron ore transportation from the perspective of integrated transportation was built which focused on the optimal transport spatial distribution and path for single freight flow in multi-transportation network without the demand matrix.

Ahmed *et al*. [20] used Linear Programming Problem to minimize the transportation cost. Joshi [21] optimized a technique to reduce transportation problem cost. Transportation problem was formulated as a linear programming problem and was solved by using four methods (northwest corner, least cost, Vogel, and Modi). Ahmad [22] presented a description for solution technique called Best Candidates Method (BCM) for solving optimization problems. His proposed technique aimed to get the optimal solution. The previous transportation models did not present any trials to determine the optimum processing plant location according to ore transportation cost minimization from different locations, Hence this study aims to develop a mathematical model to find an optimum location of the processing plant based on ore transportation cost.

II. THEORETICAL CONSIDERATION

A. General

Selection of optimum location of processing plant depends upon the total transportation cost. Transportation cost is dependent mainly upon the reserves of each ore deposit and the distances of ore transportation from each ore deposit to the suggested optimum location. To select the optimum location, the following mathematical model is suggested.

B. Mathematical Model

The main idea of the suggested mathematical model depends on the minimum sum of weighted distances. This can be graphically illustrated as shown in Figure 1, which shows an ore deposit scattered into different locations (say n). Each location has an ore tonnage of Qi which is considered to be concentrated at a single point that is the center of gravity having coordinates xi, yi, zi. All of the ore tonnages are to be transported to a location where the mineral processing plant is to be built so that the transportation cost should be minimized. Hence, this issue can be mathematically expressed as in equations from 1 to 7.

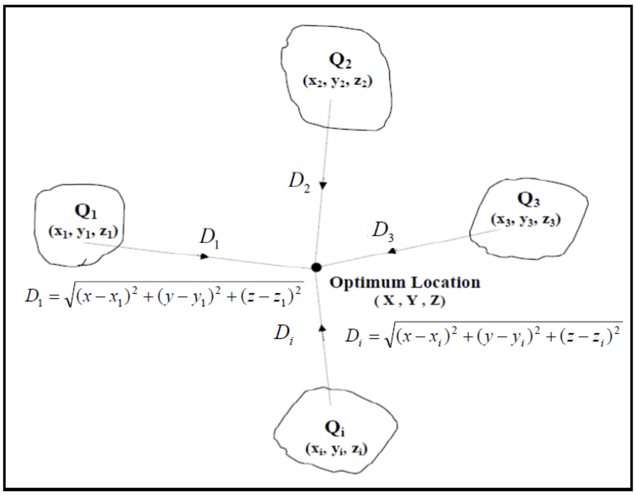

Fig.1 *Schematic diagram for optimum location of the processing plant.*

According to Figure 1 the main idea of the suggested mathematical model depends on the minimum sum of weighted distances as given in equation (1)

$$\sum_{i=1}^{n} Q_i D_i = Minimum \qquad (1)$$

Where:

n: Number of ore deposit locations.

Q_i: Reserves of each ore deposit location.

D_i: Distance between any ore deposit location and the processing plant optimum location, D_i can be mathematically expressed as follows:

$$D_i=\sqrt{(x-x_i)^2+(y-y_i)^2+(z-z_i)^2} \qquad (2)$$

x, y, z: The coordinates of the gravity center of the processing plant optimum location.

x_i, y_i, z_i : The coordinates of the gravity center of different ore deposit locations.

The sum of weighted distances is:

$$S=\sum_{i=1}^{n} Q_i D_i \qquad (3)$$

Or.

$$S=\sum_{i=1}^{n} Q_i \sqrt{(x-x_i)^2+(y-y_i)^2+(z-z_i)^2} \qquad (4)$$

For this sum to be minimum, the partial differentiation in regards to x, y and z should equal zero, which means that the following conditions have to be satisfied:

$$\frac{\partial s}{\partial x}=\sum_{i=1}^{n} Q_i \frac{x-x_i}{\sqrt{(x-x_i)^2+(y-y_i)^2+(z-z_i)^2}}=0 \qquad (5)$$

$$\frac{\partial s}{\partial y}=\sum_{i=1}^{n} Q_i \frac{y-y_i}{\sqrt{(x-x_i)^2+(y-y_i)^2+(z-z_i)^2}}=0 \qquad (6)$$

$$\frac{\partial s}{\partial z}=\sum_{i=1}^{n} Q_i \frac{z-z_i}{\sqrt{(x-x_i)^2+(y-y_i)^2+(z-z_i)^2}}=0 \qquad (7)$$

Where:

S : The sum of weighted distances

D_i : The distance from any gravity center of ore deposit to the processing plant optimum location.

$\frac{\partial s}{\partial x}, \frac{\partial s}{\partial y}, \frac{\partial s}{\partial z}$: The partial differentials.

n : The number of ore deposit locations.

Q_i : The reserves of each ore deposits.

x, y, z :The coordinates of optimum location of processing plant.

x_i, y_i, z_i :The coordinates of the geometric center of different ore deposits.

III. RESULTS AND DISCUSSIONS

A. Determination of Optimum Processing Plant Location

The optimum location of processing plant requires the number of ore deposit locations(n), tonnage (Qi) and center of gravity for each location (x_i,y_i,z_i). The Equations (5, 6& 7) of the mathematical model can be used in the form of 8, 9 and 10 as shown in the following.

$$\frac{\partial s}{\partial x}=\frac{Q_1*(x-x_1)}{\sqrt{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2}}+\frac{Q_2*(x-x_2)}{\sqrt{(x-x_2)^2+(y-y_2)^2+(z-z_2)^2}}+\ldots+\frac{Q_n*(x-x_n)}{\sqrt{(x-x_n)^2+(y-y_n)^2+(z-z_n)^2}}=0 \tag{8}$$

$$\frac{\partial s}{\partial y}=\frac{Q_1*(y-y_1)}{\sqrt{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2}}+\frac{Q_2*(y-y_2)}{\sqrt{(x-x_2)^2+(y-y_2)^2+(z-z_2)^2}}+\ldots+\frac{Q_n*(y-y_n)}{\sqrt{(x-x_n)^2+(y-y_n)^2+(z-z_n)^2}}=0 \tag{9}$$

$$\frac{\partial s}{\partial z}=\frac{Q_1*(z-z_1)}{\sqrt{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2}}+\frac{Q_2*(z-z_2)}{\sqrt{(x-x_2)^2+(y-y_2)^2+(z-z_2)^2}}+\ldots+\frac{Q_n*(z-z_n)}{\sqrt{(x-x_n)^2+(y-y_n)^2+(z-z_n)^2}}=0 \tag{10}$$

The unknowns in the equations 8, 9 and 10 are x, y & z (The coordinates of the optimum location of the processing plant). To solve these equations, FORTRAN program was developed. The flow chart of the FORTRAN program is shown in Figure 2.

B. Model Validation

The hitherto derived mathematical model depends on equations 8, 9 and 10 which are physically derived equations. Hence validation will be carried out according to the following steps:

1. Assume number of locations (n)
2. Assume tonnage for each location Qi
3. Assume coordinates of C.G for each location (xi,yi,zi)
4. Enter these assumptions in program
5. Run the program and get the output i.e processing plant optimum location coordinates (x, y, z).
6. Substitute in equations (8, 9 and 10) using the optimum location coordinates (x ,y, z) resulted from step 5, and assumed values of Qi (step 2) and its corresponding C.G coordinates (step 3) and calculate the values of the equations.
7. The model will be valid if the calculated values in step 6 were zero or within the permissible errors
8. Steps from 1 to 7 are to be repeated for different cases with varied assumptions of the program inputs (variables), n, and Q, x, y, z for each location.

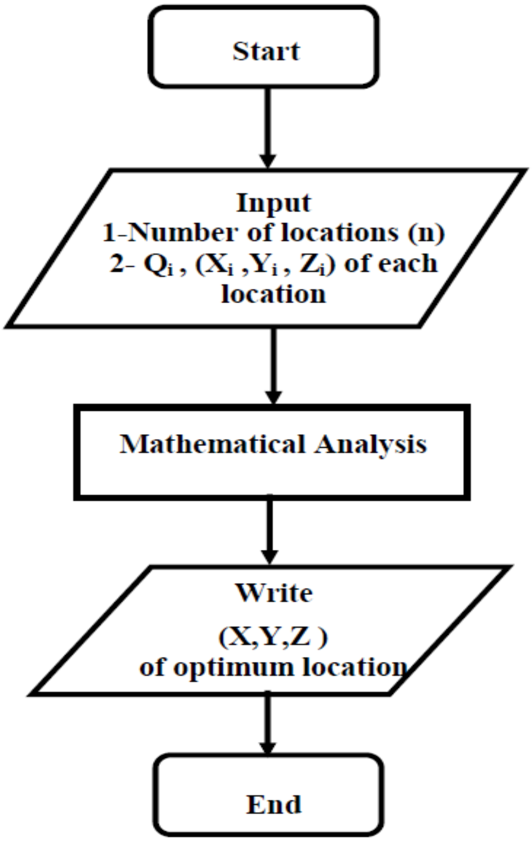

Fig. 2. *Flow chart of the FORTRAN program.*

Table 1 shows the obtained results for 10 validation cases together with their corresponding errors. As an example, case no 1 is shown in Figures 3 and 4.

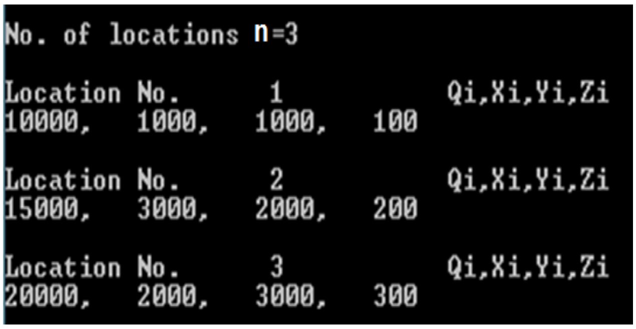

Fig.3. *A snapshot of the program with case no.1 variables entered*

Fig. 4. *A snapshot of the program with optimum location coordinates for case no.1*

TABLE 1. VALIDATION TABLE

Case No.	No. of Locations	Location No.1				Location No.2				Location No.3				Location No.4				Optimum Location			$\frac{\partial s}{\partial x}$	$\frac{\partial s}{\partial y}$	$\frac{\partial s}{\partial z}$
		Q	X	Y	Z	Q	X	Y	Z	Q	X	Y	Z	Q	X	Y	Z	X	Y	Z	Eq.12	Eq.13	Eq.14
		ton	m	m	m	ton	m	m	m	ton	m	m	m	ton	m	m	m	m	m	m			
1	3	10000	1000	1000	100	15000	3000	2000	200	20000	2000	3000	300	_	_	_	_	2034.29257	2895.39495	289.53949	0.0	0.0	0.0
2	4	5000	500	500	50	8000	1000	1500	70	7000	2000	1000	60	9000	2500	500	80	1997.62097	998.13330	60.12598	0.0	0.0	0.0
3	3	60000	800	700	90	75000	1100	950	110	90000	1300	800	115	_	_	_	_	1126.63943	885.40152	109.49180	0.0	0.0	0.0
4	4	75000	1200	1000	150	80000	1500	1800	140	9000	2000	1600	175	68000	2500	900	130	1580.09806	1368.02748	143.46178	0.0	0.0	0.0
5	3	150000	29400	25000	1100	165000	33500	39000	1140	173000	38600	28200	1210	_	_	_	_	35624.57185	29567.31558	1172.70062	0.0	0.0	0.0
6	4	258000	15300	11230	276	195800	17200	13394	245	188600	18100	12850	264	210000	19100	10960	239	17591.21462	12530.08223	256.78175	0.0	0.0	0.0
7	3	310700	36970	11230	320	344270	30900	15610	339	442870	41830	17330	361	_	_	_	_	37298.58255	14357.96841	338.66142	0.0	0.0	0.0
6	4	570280	55370	61700	1230	390900	68300	81370	1270	410380	54800	73230	1290	87100	83120	95419	1320	56407.47650	71864.64699	1276.39557	0.0	0.0	0.0
9	3	51200	52300	41100	360	63400	59420	70900	345	69500	65310	45600	355	_	_	_	_	63703.61335	46721.67314	354.86492	0.0	0.0	0.0
10	4	95800	53120	71225	317	83400	57340	85340	335	76800	77600	86460	350	79200	81300	70250	345	63945.42062	79182.78477	334.47552	0.0	0.0	0.0

C. Optimum Processing Plant Location of Aljalamid Phosphate Ore.

Aljalamid phosphate ore was chosen as a case study. The ore reserves and gravity center of different locations of Aljalamid phosphate ore deposits were calculated and shown in Table 2.

TABLE 2. RESERVES AND COORDINATES OF CENTER OF GRAVITY OF DIFFERENT LOCATIONS OF ALJALAMID PHOSPHATE ORE DEPOSIT.

Ore deposit location	Reserves (Tons)	Center of Gravity		
		x (m)	y(m)	z (m)
Fish Area	450,765,474	291095	245985	756
Southern Area	361,831,423	290119	191354	712
Western Area	338,756,461	242008	216954	723

The mathematical model was used to calculate the optimum location of the processing plant. The required data to apply the mathematical model are the reserves and coordinates of the gravity centers of the different Aljalamid phosphate ore deposits. These data are given in Table 2. According to the output of the FORTRAN program as shown in Figure 5, the coordinates of the optimum location of the processing plant are approximately (278842, 223149, 735) in the east, north, and elevation directions, respectively. Figure 6 shows that the calculated optimum location of the processing plant related to the different ore deposit locations of Aljalamid phosphate ore.

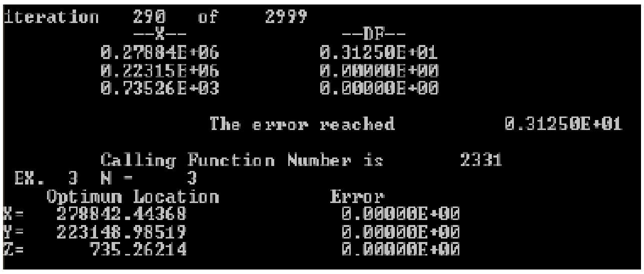

Fig.*5. A snapshot of the end step and required optimum location of processing plant*

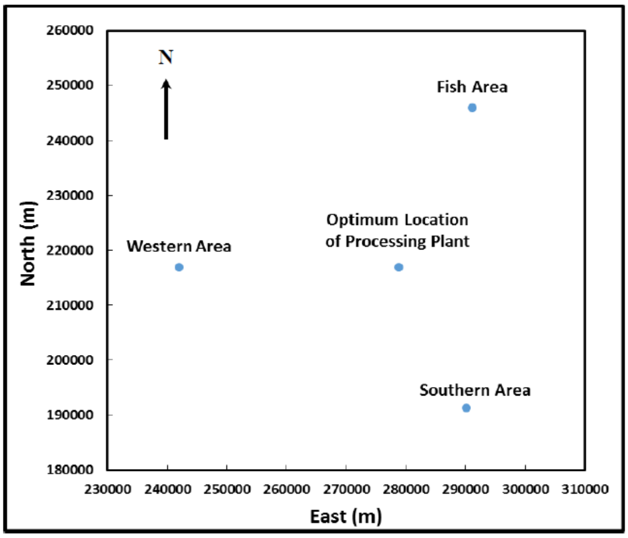

Fig.6. *Calculated optimum location of the processing plant related to the different ore deposit locations of Aljalamid phosphate ore.*

D. Effect of Processing Plant Location Deviation from Optimum

The hitherto presented results showed the optimum mineral processing plant location in an ideally theoretical case. Due to a reason or another, it may be impossible to install the mineral processing plant at the determined optimum location. Now, it is of importance to investigate the transportation of ore to any location somehow around the optimum location of processing plant. Off course this deviation from the optimum plant location will be reflected on the overall transportation cost. Total transportation cost of the ore to processing plant location can be calculated by the following equation.

$$C = c * \sum_{i=1}^{n} Q_i \sqrt{(x-x_{.i})^2 + (y-y_i)^2 + (z-z_i)^2} \qquad (11)$$

Where:

C: is total transportation cost of the ore for the different locations to the processing plant location in US$.

c : is transportation cost of one ton for one kilometer distance, in (US$ / ton.Km).

n : The number of ore deposit locations.

Q_i : The reserves of each ore deposit location.

x, y & z : The coordinates of the processing plant location.

x_i ,y_i & z_i :The coordinates of center of gravity of each ore deposit locations.

The percent additional cost can be calculated as the cost difference related to the cost to the optimum location and it can be mathematically expressed as follows:

$$Additional\ Cost\ Percent = \frac{C - C_{opt}}{C_{opt}} * 100 \qquad (12)$$

Where:

C is total transportation cost of the ore for the different locations to the processing plant location in US$. (see Equation 11).

Copt is the total ore transportation cost to the optimum processing plant location, It can be calculated from Equation 11, when x, y and z coordinates refer to optimum processing plant location i.e C_{opt} is a special case of C when transportation of ore is going to be to the optimum mineral processing location.

Figure 7 shows a contour map of the percentage additional cost compared to the minimum for different selected mineral processing plant locations. It shows that there may be a 50 % increase in the ore transportation cost due to an incorrect selection of the processing plant location.

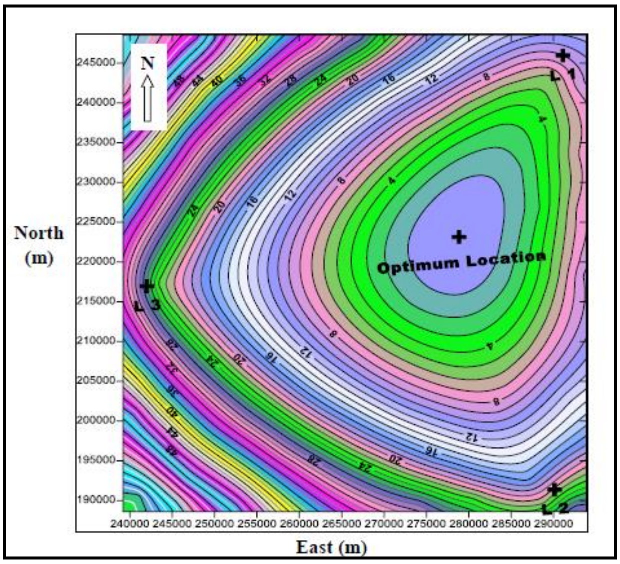

Fig. 7. *Contour map showing percentage of additional cost related to that of the optimum processing plant location. (L_1 = Fish Area, L_2 =Southern Area, and L_3 refers for Western Area)*

IV. CONCLUSIONS

The phosphate ore in Aljalamid area is a scattered deposit with three main locations that are: Fish area, southern area and western area having ore deposits of 451, 362, and 339 million tons of phosphate ore, respectively. The gravity centers of the different ore deposit locations were determined and found to be of approximately 55 km far from each other.

From this study, the following conclusions can be drawn:

1. The suggested mathematical model satisfies a minimum cost of ore transportation to a certain (optimum) location from different ore deposit locations.
2. The FORTRAN computer program that was developed to solve the proposed mathematical model was successfully validated.
3. Results of the FORTRAN program showed that the coordinates of the optimum location of processing plant are (278842, 223149, 735) in the east, north, and elevation directions, respectively.
4. Deviation of processing plant location from optimum location may cause up to 50% increase in the ore transportation cost.
5. The suggested mathematical model can be applied in similar ore deposits.

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