

# On the Form-formation, Transformations of Meshes and Some Applications to the Industrial Design

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**Abstract**—The paper discusses some fundamentals of the form-formation of shapes and their applications to the industrial design. The emphasis is on studying surfaces by using geometrical transformations of meshes, continuous and non-continuous transformations, and structural combinatorial geometry.

**Keywords**—*industrial design; form-formation; structural combinatorics; transformations of meshes*

## I. INTRODUCTION

It is well-known that the fundamentals of the form-formation and structures of shapes are of a great importance in the design practice, and particularly in the industrial design.

The study of different classifications of forms and their form formation has been an active area of research for decades. There exists a variety of attempts to classify basic forms that are used in different areas of knowledge about the world and in the creation of works, objects, products and intellectual solutions in architecture, art, science, technology, culture and design projects. From the middle of the XIX century there is a particularly noticeable increase in the number of studies on the morphology of forms.

Significant theoretical and applied contributions to the area of classifications forms in architecture and design have researchers as Burrell [2], Ching [4], Iten [6], Williams [13], Zitzmann and Schultz [15], etc.

One basic method of investigating material forms considered as surfaces uses geometrical transformations of meshes (Fig. 1).

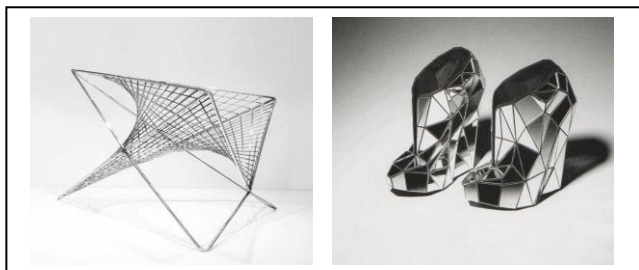


Fig. 1. Using of meshes in the industrial design

There are several excellent works devoted to mesh-representations of 3D surfaces related mainly to computer graphics, medical image analysis research and surface parametrizations. See, for example [1], [3], [8], [10], [14] and the references therein. The book written by R. Raitchev [11] is one of the first and played an outstanding role in the development of the techniques of structural combinatorial geometry. It is entirely devoted to a thorough systematic presentation of the basic ideas and methods of form formation of surfaces using geometrical transformations of meshes.

In this paper, some applications of the transformation techniques introduced in [11] are proposed and used in the vehicle design [7], [12] and more generally, in the industrial design [5], [9]. Indeed, the methods studied by Raitchev in [11] are relevant in any context for a description of the shapes of products.

## II. STUDY OF SURFACES THROUGH GEOMETRICAL MESHES

### A. Structures of the Forms

Knowledge of the basic structural relationships between the forms, their comparison and classification, and the development of logical dependencies between them leads to the systematization of the construction of forms. This systematization examines all material forms as surfaces.

In his study, Raitchev [11] proposed division of a 3D surface of the parts, i.e. laying the grid or a mesh, portions of which are geometrically measurable. Everything that is valid for the construction of the whole form or its parts is expressed by the structural connections of the individual components of this mesh.

Using the criteria of orientation and community can systematize surfaces following four categories:

- Bilateral open surfaces
- Bilateral closed surfaces
- Unilateral open surfaces
- Unilateral closed surfaces

The first category includes all surfaces that have edges and such that theoretically do not close the

space. The second category covers bilateral closed surfaces. Category fourth derived from logical combinations of the division criteria - if there are bilateral closed surfaces, they may also exist unilateral (one-sided) closed surfaces.

The next part of this section is devoted to the first of four separate categories, which is of particular interest in terms of technical design and industrial design.

*B. Using of Geometrical Meshes in the Description of Bilateral Open Surfaces*

The method introduced in [11] for the study and classification of bilateral open surfaces is based on building geometric meshes. Three main types of meshes are defined: regular, irregular and semi-regular.

When the construction of the meshes is considered the condition that regular systems are only triangular, square and hexagonal built by the same right polygons is assumed, The semi-regular meshes the author defined as those formed from various different regular polygons. The sort order, however, depends on the number of polygons in the mesh and the smallest regular polygon participating in the mesh, or the smallest number of connections on each node of the mesh.

Not all cases can be realized in a geometric way. Possible groups meshes adopted as basic are 11 (total regular and semi-regular meshes) that can be divided into three groups.

The construction of the regular meshes is determined by the existing options of systems of points equidistant from a given center – a point *i* (Fig. 2)

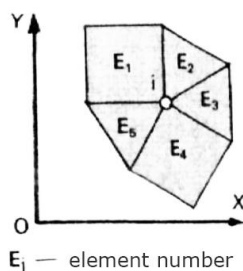


Fig. 2. Example of notations of the elements

For the sake of clarity the individual meshes are called systems, and are denoted by C1, C2 etc.

The first group of meshes (systems C1 to C4) is with three figures of regular polygons in each node of the mesh (*i* = 3). System C1 (Fig.3.1) consists of triangles and dodecagons (12-sided polygons). C2 system (Fig. 3.2) is composed of squares, hexagons and dodecagons. C3 system (Fig. 3.3) is composed of squares and octagons. System C4 (Fig. 3.4) is composed of hexagons. It defines the case of the highest degree of repeatability of regular polygons in each vertex of the meshes so far and has the most widely used in the practice of modular design.

The second group meshes with four regular polygons in each node of the net consists of three systems (C5, C6, C7). System C5 (Fig. 3.5) is composed of triangles and hexagons. It is considered in two types differing in the successive link between the figures at the vertices of the grid. In each embodiment, the connection between the figures developed differently in the two arms of vertex points. In the first case the triangles and hexagons arranged with the shift, and the second figures are arranged in pairs - two triangles, two hexagons. The next system C6 is constructed of triangles, squares and hexagons. It has three options, one of which - regular and two semi-regular (Fig. 3.6). Note that this system can be considered as a part of the system C2 and is obtained by development of the field of dodecagons in which both differ in the mode of construction. The last system from this class, system C7 is the most regular system (Fig. 3.7). The fact that this group allows the positions of the vertices to be uniquely determined with the least trouble, defines this system as fundamental to the analytic geometry.

The third group networks (five regular polygon adjacent to each vertex of the network) consists of two systems: C8 and C10. The C8 systems are constructed of triangles and squares whose four options are divided into two categories. In the first category the same connections are used, and the second - different. So they are divided into two classes. The first of these (Fig. 3.8) has expressed layered character - C8; second - C9 has a regular and two semi-regular options. The semi-regular options (Fig. 3.9) have two kinds of links. The network C10 is obtained from a triangular network, each node in the network occurs at four regular triangles and a hexagon (Fig. 3.10).

Finally, the most densely building surface regular mesh is C11. It is composed of equilateral triangles only, occurring six in each node of the network (Fig. 3.11).

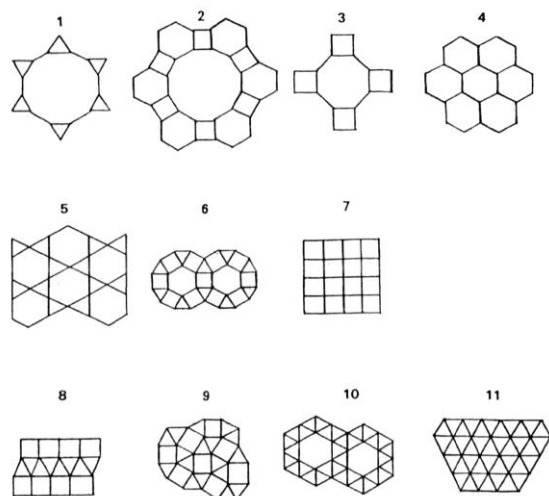


Fig. 3. Systems C1-C11

When comparing all systems and combinations we can draw the following conclusions:

- System C2 is a part of system C6;
- Systems C4, C5 and C10 can be reduced to one triangle mesh – system C11.

So eleven systems of combinations of regular polygons only seven have fully separable features.

### C. Geometrical Transformations of Meshes

In the structural combinatorial geometry all types of meshes (regular, irregular and semi-regular) can be transformed by geometric transformations in the plane or space. Such transformations are used in studying of shapes of the objects.

In the process of transformation the regular meshes preserve, though modified, their elements and links (connections).

Key elements of the systems of meshes that can be used in the plane (for 2D design) are right polygons (triangles, squares and hexagons) and in space (for 3D design) - tetrahedrons, cubes, octahedrons or rhomboidal dodecahedrons (Fig. 4).

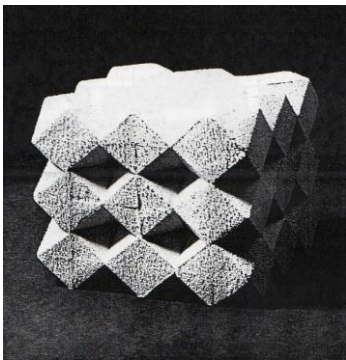


Fig. 4. Design of a cube using pyramidal elements on the space

The moving of a figure from one type of shape to another can be made by means of two types of geometric transformations of the elements - continuous and discontinuous. In the discontinuous transformations a system of elements is re-ordering by decomposition and re-assembling. In the continuous transformation a system passes in consecutive states of forms through the modification of the structure of the face of units themselves elements without breaking the links between them.

### III. APPLICATIONS TO THE INDUSTRIAL DESIGN

The method of investigating of surfaces by using of meshes and their geometric transformations can be applied to the industrial design.

All three types of meshes can be applied: regular, irregular and semi-regular. The description of the regular meshes is the same as above. Some of the semi-regular meshes also can be described in a similar manner. Irregular meshes can be obtained by using geometric transformations of regular and semi-regular ones.

Two main types of geometric transformations can be applied: discontinuous and continuous:

#### A. Discontinuous Transformations

Discontinuous transformations of the forms can be considered as a practical application of combinatorial methods in different areas of human activity, and particularly in the vehicle design. Modular solutions elements of the investigated shape are used. By the application of combinations, permutations or variations of these elements converting the original shape without distortion of the modular elements is applied.



Fig. 5. Constructing of a mesh using only one element

In the simplest discontinuous transformation uniform items, uniform connections and a constant number of elements in all spheres of transformation are used (Fig. 5).

#### B. Continuous Transformations

The process of continuous conversion is considered as a continuous transformation of surfaces - plastic modifications of the surface in space through continuous conversion of regular and semi-regular meshes. These changes supposed that links at the vertices of the grid become constants.

Depending on the nature of the movement of the networks it is divided into two groups:

- movement of meshes in the plane;
- movement of meshes in space.

In fact, the movement of the right networks should be seen as the basis for heterogeneous activity in the physical environment. Forms with variable structure in terms of functionality and purpose are very easy to be found in the forms intended to serve for rapid adaptation in a given environment.

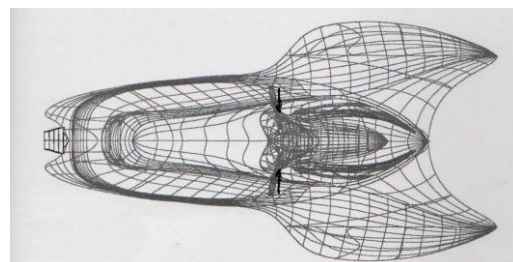


Fig. 6. Using of the system C7 in the 3D design

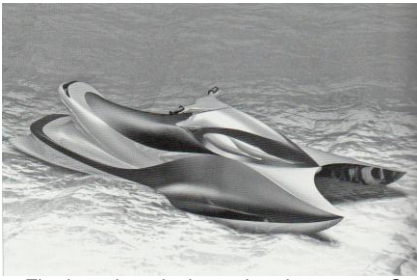


Fig. 7. Final product design using the system C7

In the design of meshes such conversion is used in the design of a device. Wire frame models represent the underlying design structure of a 3D model. Thus creating model can easily be manipulated to the development of the final production decision. Currently, the most commonly used networks are designing C7 and C11, but in the future it is possible to use other kinds of legal and semi-regular meshes (Fig.6 and Fig. 7)

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