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Temperature Distribution In A Circular Cylinder With General Mixed Boundary Conditions

Rahmatullah Ibrahim Nuruddeen
Mathematics and Statistics Department,
King Fahd University of Petroleum & Minerals,
Dhahran 31261, Saudi Arabia.
muhammad975@hotmail.com

F. D. Zaman
Mathematics and Statistics Department,
King Fahd University of Petroleum & Minerals,
Dhahran 31261, Saudi Arabia.
fzaman@kfupm.edu.sa

***Abstract*— we consider an infinite cylinder in which part of the boundary is being heated while the other part is insulated. The resulting mixed boundary value problem is solved using the Wiener-Hopf technique. The temperature distribution and the heat flux are found in some special cases of interest.**

Keywords—Circular Cylinder; Heat Equation; Mixed Boundary Conditions; Wiener-Hopf Technique.

I INTRODUCTION

Heat conduction in cylindrical materials and tubes has been extensively studied due to various industrial applications. Carslaw and Jaeger [2] have discussed the boundary value problems in one and more dimensions arising from the problems of determination of heat distribution in solids having homogeneous or composite structure satisfying different boundary conditions. In engineering application, the cylindrical bars and tubes are extensively used due to various design advantages. The problem of heat transfer and temperature distribution becomes of interest in cooling of such cylindrical bars in rewetting and quenching process. In the nuclear reactors, cylindrical rods are heated internally by fission and are immersed in cooling fluid to produce energy using heat transfer at the surface. If these structures are totally immersed in coolants, the integral transforms can be used to study the temperature distribution and heat transfer rate assuming these to be cylinders of infinite extent. If however, the process by wetting

or immersing in a fluid involves only part of the cylinder, a mixed boundary value problem is formed. In such cases, the Wiener-Hopf technique, based upon integral transform has been used by a number of researchers in the past. The technique was originally proposed to solve certain half-range singular integral equations [9]. In 1952, Jones [7] presented the modified Wiener-Hopf technique to solve the mixed boundary value problems directly. In Noble [10], an excellent exposure to this technique has been presented. Achenbach [1] has also given account of this technique as applied to the wave propagation problems. Mittra and Lee [9] have extensively used this technique in studying wave-guide problems. Evans [5] gave an explicit expression for a steady state temperature distribution within the cylinder at the point of entry into a cooling fluid. In Chakrabarti [2], the explicit solution of the sputtering temperature of a cooling cylindrical rod with an insulated core when allowed to enter into a cold fluid of large extent thereby giving rise to a mixed boundary condition which has been tackled using the Wiener-Hopf method. Georgiadis et al [6] considered infinite dissimilar half-spaces which are joined and brought in contact over half of their common boundary and the other half insulated all along the common boundary (interface). Chakrabarti and Bera [3] studied a mixed boundary-valued problem associated with the heat equation which involves the physical problem of cooling of an infinite slab in a two-fluid medium. An analytical solution is derived for the temperature distribution at the quench fronts. Similarly, Zaman [13] studied a heat conduction problem across a semi-infinite interface in layered plates. Further, Zaman and Al-khairy [14] discussed the cooling problem of a composite layered plate comprising of dissimilar layers of uniform thickness having mixed interface thereby finding the closed forms of both the temperature field and the heat flux. Satapathy [10] considered a two-dimensional quasi-steady heat conduction equation governing conduction controlled by rewetting of an infinite cylinder with heat generation. The analytical solution obtained by Wiener–Hopf technique gave the quench front temperature as a function of various model parameters. Shafei and Nekoo [12], solved the heat conduction problem of a finite hollow cylinder using generalized finite Hankel transform which is based on the use of the integral transform method. Kedar and Deshmukh [8], considered the inverse heat conduction problem in a semi-infinite hollow circular cylinder under some specific assumptions.

In this paper, we present the analytical solution of transient heat conduction in a solid homogenous infinite circular cylinder. The cylinder of uniform cross-section of an isotropic material is subjected to general boundary conditions on both the positive and negative semi-infinite ranges of $z-$axis. The Jones modification of the Wiener-Hopf technique is utilized due to the mixed nature of the boundary conditions.

II FORMULATION OF THE PROBLEM

We consider an infinite cylindrical rod of uniform cross section and finite radius r. The cylinder is assumed to have the general mixed boundary conditions on the boundary; that is, the temperature distribution is given by $f(r,z,t)$ on $-\infty < z < 0$ half-plane while $g(r,z,t)$ stands for the heat flux on $0 < z < \infty$ as shown in figure 1. The temperature distribution $'T'$ satisfies the governing equation

$T_{rr} + \frac{1}{r}T_r + \frac{1}{r^2}T_{\theta\theta} + T_{zz} = \frac{1}{k}T_t$, (1)

and when further assume that the temperature distribution is axially symmetric, that is equation (1) becomes

$T_{rr} + \frac{1}{r}T_r + T_{zz} = \frac{1}{k}T_t$,(2)

where k is the thermal diffusivity, expressed as $k = \frac{\upsilon}{\rho c}$ where υ is thermal conductivity, ρ is density and c is specific heat of the material. The initial and boundary conditions are as follow

(i) The initial condition

$T(r,z,t) = 0;$ at $t = 0$. (3)

(ii) The temperature on $z < 0$,

$T(r,z,t) = f(r,z,t);\ -\infty < z < 0, r = a$. (4)

(iii) The heat flux on $z > 0$,

$T_r(r,z,t) = g(r,z,t);\ 0 < z < \infty, r = a$. (5)

(iv) a. $T \to 0(e^{\tau_+ z})$ as $z \to -\infty$

b. $T \to 0(e^{\tau_- z})$ as $z \to +\infty$(6)

In addition, the functions $f(r,z,t)$ and $g(r,z,t)$ are assumed to be of exponential order as $|z|$ turns to ∞.

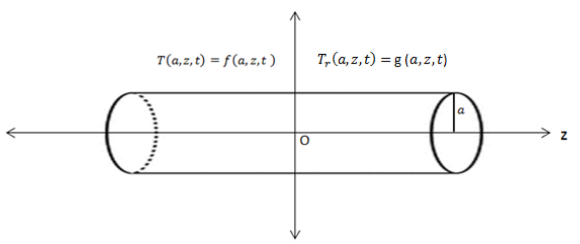

Figure 1: Geometry of the problem

III WIENER-HOPF EQUATION

The Laplace transform in the time variable t and its inverse transform in s are defined by:

$\mathcal{L}\{T(t)\} = \int_0^\infty T(t)e^{-st}dt = \bar{T}(s)$, (7)

and

$\mathcal{L}^{-1}\{\bar{T}(s)\} = \frac{1}{2\pi i}\int_{-i\infty+h}^{i\infty+h}\bar{T}(s)e^{st}ds = T(t)$.(8)

In the same way, we define the Fourier transform in z and its corresponding inverse Fourier transform in α by:

$\mathcal{F}\{T(z)\} = \int_{-\infty}^{\infty} T(z)e^{i\alpha z}dz = T^*(\alpha)$,(9)

and

$\mathcal{F}^{-1}\{T^*(\alpha)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} T^*(\alpha)e^{-i\alpha z}d\alpha = T(z)$,(10)

with $\alpha = \sigma + i\tau$.

Moreover, we also introduce the half range Fourier transforms as

$\int_0^\infty T(z)e^{i\alpha z}dz = T_+^*(\alpha)$,(11)

and

$\int_{-\infty}^0 T(z)e^{i\alpha z}dz = T_-^*(\alpha)$.(12)

So that,

$T^*(\alpha) = T_+^*(\alpha) + T_-^*(\alpha)$.(13)

Thus, equation (13) defines the Fourier transform as $T^*(\alpha) = \int_{-\infty}^{\infty} T(z)e^{i\alpha z}dz$, where $T(z) = 0(e^{\tau_- z})$ as $z \to \infty$ and $T(z) = 0(e^{\tau_+ z})$ as $z \to -\infty$. Hence, $T_+^*(\alpha)$ is an analytic function of α in the upper half-plane $\tau > \tau_-$, while $T_-^*(\alpha)$ is an analytic function of α in the lower half-plane $\tau < \tau_+$ respectively. Thus, $T^*(\alpha)$ defined an analytic function in the common strip $\tau_- < \tau < \tau_+$ with $\tau = Im(\alpha)$.

We now take the Laplace transform in t and Fourier transform in z of equation (2) to get

$\bar{T}_{rr}^* + \frac{1}{r}\bar{T}_r^* - \left(\alpha^2 + \frac{s}{k}\right)\bar{T}^* = 0$.(14)

Therefore, the solution of the transformed equation (14) is given by

$\bar{T}^*(r,\alpha,s) = A(\alpha)I_0(qr) + B(\alpha)K_0(qr)$.(15)

Where $I_0(qr)$ and $K_0(qr)$ are modified Bessel functions of first and second kinds of order zero respectively, and $q(\alpha) = \sqrt{\alpha^2 + s/k}$. Furthermore, for the boundedness of the solution, equation (15) takes the form

$\bar{T}^*(r,\alpha,s) = A(\alpha)I_0(qr)$ for $0 \le r \le a$. (16)

Now, on applying the transformed boundary conditions to equation (16) we get

$\bar{T}_+^*(a,\alpha,s) + \bar{f}_-^*(a,\alpha,s) = A(\alpha)I_0(qa)$,(17)

and

$\bar{T}_-^{*\prime}(a,\alpha,s) + \bar{g}_+^*(a,\alpha,s) = A(\alpha)\frac{I_1(qa)}{q}$. (18)

Thus, from equations (17) and (18), we obtain

$$\bar{T}_-^{*\prime}(a,\alpha,s) + \bar{g}_+^*(a,\alpha,s) = \frac{I_1(qa)}{qI_0(qa)}\{\bar{T}_+^*(a,\alpha,s) + \bar{f}_-^*(a,\alpha,s)\}.$$

(19)

Thus, equation (19) is the Wiener-Hopf equation holding in the strip of analyticity $\tau_- < \tau < \tau_+$, and where T' is the derivative of T with respect to r.

IV SOLUTION OF THE WIENER-HOPF EQUATION

In equation (19), the mixed term $\frac{I_1(qa)}{qI_0(qa)}$ is named $K(\alpha)$.

We then factorize $K(\alpha)$ as

$$K(\alpha) = \frac{I_1(qa)}{qI_0(qa)} = K_+(\alpha)K_-(\alpha), \quad (20)$$

(See Mittra and Lee [9] and Noble [10]). Where $K_+(\alpha)$ and $K_-(\alpha)$ are analytic functions in the upper and lower half planes respectively. The expressions for $K_+(\alpha)$ and $K_-(\alpha)$ are given in appendices D_1 and D_2 respectively. Thus, equation (19) becomes

$$\frac{\bar{T}_-^{*\prime}(a,\alpha,s)}{K_-(\alpha)} + \frac{\bar{g}_+^*(a,\alpha,s)}{K_-(\alpha)} = K_+(\alpha)\bar{T}_+^*(a,\alpha,s) + K_+(\alpha)\bar{f}_-^*(a,\alpha,s). \quad (21)$$

Again, decomposing the mixed terms in equation (21) using decomposition theorem (see Noble [10]) we get as follows

$$M(\alpha) = \frac{\bar{g}_+^*(a,\alpha,s)}{K_-(\alpha)} - K_+(\alpha)\bar{f}_-^*(a,\alpha,s)$$

$$= M_+(\alpha) + M_-(\alpha), \quad (22)$$

where $M_+(\alpha)$ and $M_-(\alpha)$ are given to be

$$M_+(\alpha) = \frac{1}{2\pi i}\int_{-\infty+ic}^{\infty+ic}\left\{\frac{\bar{g}_+^*(a,\zeta,s)}{K_-(\zeta)} - K_+(\zeta)\bar{f}_-^*(a,\zeta,s)\right\}\frac{1}{\zeta-\alpha}d\zeta \quad (23)$$

and

$$M_-(\alpha) = -\frac{1}{2\pi i}\int_{-\infty+id}^{\infty+id}\left\{\frac{\bar{g}_+^*(a,\zeta,s)}{K_-(\zeta)} - K_+(\zeta)\bar{f}_-^*(a,\zeta,s)\right\}\frac{1}{\zeta-\alpha}d\zeta \quad (24)$$

respectively, where c and d are in the analytic region. That is, $\tau_- < c < \tau < d < \tau_+$. Finally, equation (21) becomes

$$\frac{\bar{T}_-^{*\prime}(a,\alpha,s)}{K_-(\alpha)} + M_-(\alpha) = K_+(\alpha)\bar{T}_+^*(a,\alpha,s) - M_+(\alpha). \quad (25)$$

Equation (25) defines an entire function in the whole $\alpha - plane$ by analytic continuation. In which the left hand side is analytic in the lower half-plane $\tau < \tau_+$, while the right hand side is analytic in the upper half plane $\tau > \tau_-$ respectively. In addition, both sides can be shown to be zero by the extended form of Liouville's theorem as $\alpha \to \pm\infty$.

Thus, our unknown functions are found to be:

$$\bar{T}_+^*(a,\alpha,s) = \frac{M_+(\alpha)}{K_+(\alpha)}, \quad (26)$$

and

$$\bar{T}_-^{*\prime}(a,\alpha,s) = -M_-(\alpha)K_-(\alpha). \quad (27)$$

We note that $K_-(\alpha)$ and $K_+(\alpha)$ are given in the appendices D_1 and D_2 while $M_-(\alpha)$ and $M_+(\alpha)$ are given by equations (23) and (24) in terms of known functions respectively. Equations (26) and (27) can then be used together with equations (17) and (18) to determine the overall temperature distribution in the transformed domain. The inverse Laplace transform and the inversion Fourier transform can then be used to obtain the overall temperature distribution $T(r,z,t)$ and the corresponding heat flux $T_r(r,z,t)$ respectively. Hence, on taking these inversions we got the overall temperature distribution of the body under consideration as follows:

$$T(r,z,t) = \frac{1}{4\pi^2 i}\int_{-\infty}^{\infty}\int_{-i\infty+h}^{i\infty+h}\left\{\frac{\bar{T}_+^*(a,\alpha,s) + \bar{f}_-^*(a,\alpha,s)}{I_0(qa)}I_0(qr)\right\}\times e^{st}e^{-i\alpha z}\,ds\,d\alpha \quad (28)$$

However, due to complexity of the double integral in equation (28), we present below some special cases in which this has been evaluated.

V EVALUATION OF THE TEMPERATURE DISTRIBUTIONS IN SOME SPECIAL CASES

In trying to determine the explicit analytical solution of the problem solved, we consider some meaningful boundary conditions. Further, we consider a special case of transient heat conduction, that is, of the form

$$T(r,z,t) = T(r,z)e^{i\omega t}, \quad (29)$$

where ω is the angular frequency. We note that this assumption, together with the Fourier transform in z, gives us equation (14) without having to use the initial condition (3) with the Laplace transform parameter s changed to $i\omega$. This also has been observed by Noble [10] as comparison between transient and steady state problem. In the following, the change $s \leftrightarrow i\omega$ has been used and no Laplace transform in t is taken.

Case 1

$T(r,z,t) = T_o\, e^{i\omega t}, z < 0$ where T_o is constant, and $T_r(r,z,t) = 0, z > 0$ all at $r = a$.

So, by changing s to $i\omega$ we get from equations (23) and (24) we get

$$M_+(\alpha) = \frac{T_o}{2\pi i}\frac{\sqrt{a}}{\omega\sqrt{i}}\int_{-\infty+ic}^{\infty+ic}\frac{L_+(\zeta)}{\zeta(\zeta-\alpha)P_+(\zeta)}d\zeta, \quad (30)$$

with simple poles at $\zeta = 0$, $\zeta = \alpha$ and $\zeta = -i\alpha_m = \beta_m$ where $\alpha_m = i\sqrt{\left(\frac{j_{0,m}}{a}\right)^2 + \frac{i\omega}{k}}$ for $m = 1,2,.$ are simple zeros of $P_+(\zeta)$. Thus, evaluating equation (30) we get

$$M_+(\alpha) = \frac{T_o\sqrt{a}}{\omega\sqrt{i}}\left[\sum_{m=1}^{\infty}\frac{A_m}{\alpha-\beta_m} + \frac{L_+(\alpha)}{\alpha P_+(\alpha)} - \frac{L_+(0)}{\alpha P_+(0)}\right], \quad (31)$$

where $A_m = \frac{-L_+(\beta_m)}{\beta_m P_+'(\beta_m)}$ and $L_+(\alpha)$ and $P_+(\alpha)$ are in appendices (A_3) and (B_3).

Similarly,

$$M_-(\alpha) = -\frac{T_o}{2\pi i}\frac{\sqrt{a}}{\omega\sqrt{i}}\int_{-\infty+id}^{\infty+id}\frac{L_+(\zeta)}{\zeta(\zeta-\alpha)P_+(\zeta)}d\zeta, \quad (32)$$

with simple poles at $\beta_k, \beta_{k+1}, .,$ and $k << m$ and β_k's are the poles laying above the line of integration $-\infty + id$ to $\infty + id$ given that $\tau_- < c < \tau < d < \tau_+$.

Thus,

$M_{-}(\alpha) = \frac{T_o\sqrt{a}}{\omega\sqrt{i}}\sum_{k=1}^{\infty}\frac{B_k}{\alpha-\beta_k}$, (33)

where $B_k = \frac{L_+(\beta_k)}{\beta_k P_+'(\beta_k)}$ and $\{\,'\,\}$ stands for derivative. Now, to evaluate the temperature distribution, from equations (26) and (31) we get

$T_+(a,z) = \frac{T_o}{\omega}\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\sum_{m=1}^{\infty}\frac{A_m P_+(\alpha)}{(\alpha-\beta_m)L_+(\alpha)} + \frac{1}{\alpha} - \frac{L_+(0)P_+(\alpha)}{\alpha P_+(0)L_+(\alpha)}\right]e^{-i\alpha z}d\alpha$,(34)

with simple poles at $\alpha = 0$, $\alpha = \beta_m$ $m = 1,2,.$ and $\alpha = -i\alpha_n = \beta_n$.

Where $\alpha_n = i\sqrt{\left(\frac{j_{1,n}}{a}\right)^2 + \frac{i\omega}{k}}$ $n = 1,2,.$ and

$\beta_m = \sqrt{\left(\frac{j_{0,m}}{a}\right)^2 + \frac{i\omega}{k}}$.

For $z > 0$; we close the contour in the lower half-plane and for $z < 0$; we close the contour in the upper half-plane in such a way that $\alpha = 0$ would be in the upper half-plane. Thus, we get the overall temperature distribution after the evaluation of the complex integral in equation (34) using residue theorem [12] and also through the use of equation (29) as

$$T_+(a,z,t) = \frac{T_o}{\omega}i\left\{\sum_{j=1}^{\infty}\left[A_j\frac{P_+(\beta_j)}{L_+(\beta_j)}e^{-i\beta_j z} + \sum_{n=1,n\neq j}^{\infty}A_j\frac{P_+(\beta_n)}{(\beta_n-\beta_j)L_+'(\beta_n)}e^{-i\beta_n z}\right] - \sum_{n=1}^{\infty}\frac{L_+(0)P_+(\beta_n)}{\beta_n P_+(0)L_+'(\beta_n)}e^{-i\beta_n z}\right\}e^{-i\omega t},$$

(35)

where, $A_j = \frac{-L_+(\beta_j)}{\beta_j P_+'(\beta_j)}$.

In a similar manner, to evaluate the heat flux from equations (27) and (33); we have that

$T^{*\prime}_{-}(a,z) = -\frac{T_o a}{\omega i}\frac{1}{2\pi i}\int_{-\infty}^{\infty}\sum_{k=1}^{\infty}\frac{B_k\alpha L_{-}(\alpha)}{(\alpha-\beta_k)P_{-}(\alpha)}\;e^{-i\alpha z}d\alpha$,(36)

with simple poles at $\alpha = -\beta_m, m = 1,2,.$ and $\alpha = \beta_k, k = 1,2,.$ In the same way as in above for $z > 0$ and for $z < 0$ we finally get the overall heat flux as

$$T^{*\prime}_{-}(a,z,t) = -\frac{T_o a}{\omega}\sum_{j=1}^{\infty}\left\{B_j\frac{\beta_j L_{-}(\beta_j)}{P_{-}'(\beta_j)}e^{-i\beta_j z} - \sum_{m=1}^{\infty}B_j\frac{\beta_m L_{-}(-\beta_m)}{(\beta_m+\beta_j)P_{-}'(-\beta_m)}e^{i\beta_m z}\right\}e^{-i\omega t},$$

(37)

where, $B_j = \frac{L_+(\beta_j)}{\beta_j P_+'(\beta_j)}$.

Case 2

$T(r,z,t) = T_o e^{i\omega t+\lambda z}, \lambda > 0, z < 0$ and $T_r(r,z,t) = 0, z > 0$ all at $r = a$.

So, using the same method as in above, and from equations (26), (29) and (31) we get the temperature distribution as

$$T_+(a,z,t) = \frac{T_o}{\omega}i\left\{\sum_{j=1}^{\infty}\left[A_j\frac{P_+(\beta_j)}{L_+(\beta_j)}e^{-i\beta_j z} + \sum_{n=1,n\neq j}^{\infty}A_j\frac{P_+(\beta_n)}{(\beta_n-\beta_j)L_+'(\beta_n)}e^{-i\beta_n z}\right] - \sum_{n=1}^{\infty}\frac{L_+(i\lambda)P_+'(\beta_n)}{(\beta_n-i\lambda)P_+(i\lambda)L_+'(\beta_n)}e^{-i\beta_n z}\right\}e^{-i\omega t},$$

(38)

where $A_j = \frac{-L_+(\beta_j)}{(\beta_j-i\lambda)P_+'(\beta_j)}$.

Similarly, from equations (27), (29) and (33) we get the heat flux as

$$T^{*\prime}_{-}(a,z,t) = -\frac{T_o a}{\omega}\sum_{j=1}^{\infty}\left\{B_j\frac{\beta_j L_{-}(\beta_j)}{P_{-}'(\beta_j)}e^{-i\beta_j z} - \sum_{m=1}^{\infty}B_j\frac{\beta_m L_{-\beta_m}}{(\beta_m+\beta_j)P_{-\beta_m}'}e^{i\beta_m z}\right\}e^{-i\omega t},$$

(39)

where $B_j = \frac{L_+(\beta_j)}{(\beta_j-i\lambda)P_+'(\beta_j)}$.

Case 3

$T(r,z,t) = T_o e^{i(\omega-\mu)t}, \mu > 0\,, z < 0$ and $T_r(r,z,t) = 0, z > 0$ all at $r = a$.

So, we determine the temperature distribution from equations (26), (29) and (31) as

$$T_+(a,z,t) = \frac{T_o}{\omega-i\mu}i\left\{\sum_{j=1}^{\infty}\left[A_j\frac{P_+(\beta_j)}{L_+(\beta_j)}e^{-i\beta_j z} + \sum_{n=1,n\neq j}^{\infty}A_j\frac{P_+(\beta_n)}{(\beta_n-\beta_j)L_+'(\beta_n)}e^{-i\beta_n z}\right] - \sum_{n=1}^{\infty}\frac{L_+(0)P_+'(\beta_n)}{\beta_n P_+(0)L_+'(\beta_n)}e^{-i\beta_n z}\right\}e^{-i\omega t},$$

(40) where, $A_j = \frac{-L_+(\beta_j)}{\beta_j P_+'(\beta_j)}$.

The corresponding heat flux also can be found from equations (27), (29) and (33) as

$$T^{*\prime}_{-}(a,z,t) = -\frac{T_o a}{\omega - i\mu}\sum_{j=1}^{\infty}\left\{B_j \frac{\beta_j L_-(\beta_j)}{P_-'(\beta_j)} e^{-i\beta_j z} - \sum_{m=1}^{\infty} B_j \frac{\beta_m L_{-\beta_m}}{(\beta_m + \beta_j)P'_{-\beta_m}} e^{i\beta_m z}\right\} e^{-i\omega t},$$

(41)

where, $B_j = \frac{L_+(\beta_j)}{\beta_j P_+'(\beta_j)}$.

VI Conclusion

In this study, a mixed boundary value problem arising from temperature distribution in an infinite homogeneous cylinder has been considered using the Wiener-Hopf technique. The infinite boundary of the cylinder has been subjected to two different boundary conditions. One part of the boundary is held at a prescribed temperature while the other part is insulated. The solution is obtained in a closed from. The temperature and the flux at the surface of the cylinder have been found in some special cases.

REFERENCES

[1] J.D. Achenbach, Wave Propagation in Elastic Solids", North-Holland, Amsterdam 1973.

[2] H. Carslaw and J.C Jeager, The Conduction of Heat in Solids, 2nd edition, Clarendon Press, Oxford 1959.

[3] A. Chakrabarti, The sputtering temperature of a cooling cylindrical rod with an insulated core, Applied Scientific Research 43, 107-113, 1986.

[4] A. Chakrabarti, and R.K Bera, Cooling of an infinite slab in a two-fluid medium, J. Austral. Math. Soc. Ser. B 33, 474-485, 1992.

[5] D.V. Evans, A Note on the Cooling of a Cylinder Entering a Fluid, IMA Journal of Applied Mathematics 33, 49-54, 1984.

[6] H.G. Georgiadis, J.R. Barber and F.B. Ammar, An asymptotic solution for short time transient heat conduction between two dissimilar bodies in contact, Quart. J. Math. 44(2), 303-322, 1991.

[7] D.S. Jones, A simplifying technique in the solution of a class of diffraction problems, Quart. J. Math. (2) 3, 189-196, 1952.

[8] G.D. Kedar and K.C Deshmukh, Inverse heat conduction problem in a semi-infinite hollow cylinder and its thermal deflection by quasi-static approach, International journal of applied mathematics and computation. ISSN: 0974 - 4665 (Print) 0974 - 4673 (Online) Volume 6(2), 2015.

[9] R. Mittra and S.W. Lee, Analytical Technique in the Theory of Guided Waves, The Macmillan Company, New York 1971.

[10] B. Noble, Methods based on the Wiener-Hopf Technique, Second Edition, Chelsea Publishing Company, New York 1958.

[11] A.K. Satapathy, Wiener–Hopf solution of rewetting of an infinite cylinder with internal heat generation, Heat Mass Transfer 45:651–658 DOI 10.1007/s00231-008-0457-6, 2009.

[12] A.M. Shafei and S.R. Nekoo, Heat conduction of a hollow cylinder via generalized Hankel transform, International Research Journal of Applied and Basic Sciences. Vol., 3 (4), 758-769, 2012.

[13] F.D. Zaman, Heat Conduction across a Semi Infinite Interface in a Layered Plate, Utilitas Mathematica 43pp. 89-96, 1993.

[14] F.D. Zaman and R. Al-khairy, Cooling of a Composite Plate, Far East Math. (FJMS), 2(4), 645-663, 2000.

Appendix

Let

$$L(\alpha) = L_+(\alpha)L_-(\alpha), (A_1)$$

where $L(\alpha)$ is given below using infinite product theorem [9] and [13]

$$L(\alpha) = \frac{J_1(iqa)}{q} = \frac{J_0\left(i\sqrt{\frac{s}{k}}a\right)}{\sqrt{\frac{s}{k}}} \prod_{n=1}^{\infty}\{1 - \frac{\alpha}{i\alpha_n}\} e^{-\frac{i\alpha a}{n\pi}} \prod_{n=1}^{\infty}\{1 + \frac{\alpha}{i\alpha_n}\} e^{\frac{i\alpha a}{n\pi}}. (A_2)$$

Where $q(\alpha) = \sqrt{\alpha^2 + \frac{s}{k}}$ and $\alpha_n = i\sqrt{\left(\frac{j_{1,n}}{a}\right)^2 + \frac{s}{k}}$, for $n = 1,2,3,$.are the zeros of $\frac{J_1(iqa)}{q}$.

Hence,

$$L_+(\alpha) = \sqrt{\frac{J_0\left(i\sqrt{\frac{s}{k}}a\right)}{\sqrt{\frac{s}{k}}}} \prod_{n=1}^{\infty}\{1 + \frac{\alpha}{i\alpha_n}\} e^{\frac{i\alpha a}{n\pi} + \chi(\alpha)} (A_3)$$

$$L_-(\alpha) = \sqrt{\frac{J_0\left(i\sqrt{\frac{s}{k}}a\right)}{\sqrt{\frac{s}{k}}}} \prod_{n=1}^{\infty}\{1 - \frac{\alpha}{i\alpha_n}\} e^{-\frac{i\alpha a}{n\pi} - \chi(\alpha)} (A_4)$$

where

$$\chi(\alpha) = i\frac{\alpha a}{\pi}[1 - C + \ln\left(\frac{2\pi}{a}\right) + i\frac{\pi}{2}]$$

with C is the Euler's Constant given by 0.57721.

In the same way,

$$P(\alpha) = P_+(\alpha)P_{(\alpha)}, (B_1)$$

$$P(\alpha) = J_0(iqa) = J_0\left(i\sqrt{\frac{s}{k}}a\right) \prod_{m=1}^{\infty}\{1 - \frac{\alpha}{i\alpha_m}\} e^{-\frac{i\alpha a}{m\pi}} \prod_{m=1}^{\infty}\{1 + \frac{\alpha}{i\alpha_m}\} e^{\frac{i\alpha a}{m\pi}} (B_2)$$

where $q(\alpha) = \sqrt{\alpha^2 + \frac{s}{k}}$; $\alpha_m = i\sqrt{\left(\frac{j_{0,n}}{a}\right)^2 + \frac{s}{k}}$, for $m = 1,2,3,$.are the zeros of $J_0(iqa)$

$$P_+(\alpha) = \sqrt{J_0\left(i\sqrt{\frac{S}{k}}a\right)} \prod_{m=1}^{\infty} \{1 + \frac{\alpha}{i\alpha_m}\} e^{\frac{i\alpha a}{m\pi}} (B_3)$$

$$P_{(\alpha)} = \sqrt{J_0\left(i\sqrt{\frac{S}{k}}a\right)} \prod_{m=1}^{\infty} \left\{1 - \frac{\alpha}{i\alpha_m}\right\} e^{-\frac{i\alpha a}{m\pi}} (B_4)$$

From appendices (A) and (B) above; we obtain

$$K(\alpha) = \frac{J_1(iqa)}{qJ_0(iqa)} = \frac{L_+(\alpha)}{P_+(\alpha)} \frac{L_-(\alpha)}{P_-(\alpha)} = K_+(\alpha)\, K_-(\alpha), (C_1)$$

where,

$$K_+(\alpha) = \frac{\sqrt{J_1\left(i\sqrt{\frac{S}{k}}a\right)}}{\sqrt{i\sqrt{\frac{S}{k}}\, J_0\left(i\sqrt{\frac{S}{k}}a\right)}} \frac{\prod_{n=1}^{\infty}\{1 + \frac{\alpha}{i\alpha_n}\}\, e^{\frac{i\alpha a}{\pi}(\frac{1}{n} - \frac{1}{m}) + \chi(\alpha)}}{\prod_{m=1}^{\infty}\{1 + \frac{\alpha}{i\alpha_m}\}} (D_1)$$

$$K_-(\alpha) = \frac{\sqrt{J_1\left(i\sqrt{\frac{S}{k}}a\right)}}{\sqrt{i\sqrt{\frac{S}{k}}\, J_0\left(i\sqrt{\frac{S}{k}}a\right)}} \frac{\prod_{n=1}^{\infty}\{1 - \frac{\alpha}{i\alpha_n}\}\, e^{-\frac{i\alpha a}{\pi}(\frac{1}{n} - \frac{1}{m}) - \chi(\alpha)}}{\prod_{m=1}^{\infty}\{1 - \frac{\alpha}{i\alpha_m}\}} (D_2)$$