Journal of Multidisciplinary Engineering Science and Technology (JMEST)
ISSN: 3159-0040
Vol. 2 Issue 12, December - 2015

# Investigation of Electromagnetic Wave Propagation in Si-$SiO_2$ Embedded Waveguide

**Nurettin Bilgili**
Eskişehir Osmangazi University
Graduate School, Physics Department
Eskişehir, Turkey
nurettinbilgili@gmail.com

**Ali Çetin**
Eskişehir Osmangazi University
Faculty of Arts and Sciences, Physics Department
Eskişehir, Turkey
acetin@ogu.edu.tr

***Abstract*—In this paper, we propose analysis and simulate few modes of electrical fields $TE_1$, $TE_2$, $TE_3$, $TE_4$ of the electromagnetic waves in Si-$SiO_2$ embedded planar waveguide. Embedded waveguide is evaluated as a boundary value problem by using Marcatili method.**

***Keywords——Embedded waveguide, Marcatili method, TE modes***

## I. Introduction

Optical waveguide is the fundamental element that interconnects the various devices in integrated optics. Therefore, this element has attracted much attention recently and investigated extensively because of its potential applications in photonics.

The propagation mode analysis and simulation of planar dielectric waveguide is an important subject area. There are numerous papers that discuss numerical methods that are based on finding the roots of the characteristic equation. The characterization of the mode is achieved in dielectric waveguides by an algebraic equation.

Modeling techniques of the light propagation in waveguides are divided as analytical and numerical methods. The numerical techniques, various approaches which include the Finite Difference Method (FDM) [1-5], Finite Element Method (FEM) [6-9], Perturbation Method (PM), Beam Propagation Method (BPM) [10-13], Marcatili [14] and Effective Index Method (EIM) [15].

By solving the Helmholtz wave equation in the boundary of planar dielectric waveguide one can find the fundamental transfer electric field modes.

## II. THEORY

The embedded planar waveguide is shown in Figure 1. The guiding layer of the waveguide is core with $n_1$ refractive index and the cladding layer of the waveguide with $n_2$ refractive index provided that $n_1 > n_2$.

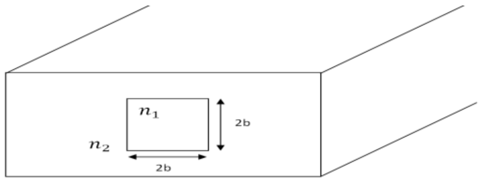



Figure 1. The embedded planar waveguide

In classical electromagnetism, Maxwell equation for Faraday law can be written as

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

If we take the rotation of both sides,

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = \vec{\nabla} \times (-\frac{\partial \vec{B}}{\partial t})$$

and using $\vec{\nabla} \times \vec{\nabla} \times \vec{E} = \vec{\nabla}(\vec{\nabla}.\vec{E}) - \vec{\nabla}^2\vec{E}$ and $\vec{B} = \mu_0\vec{H}$ equations for homogeneous media $(\vec{\nabla}\varepsilon = 0)$ homogeneous wave equation is obtained as

$$\vec{\nabla}^2\vec{E} - \mu_0\varepsilon\frac{\partial^2\vec{E}}{\partial^2 t} = 0$$

where $\vec{E}$ is the electric field, $\vec{H}$ is the magnetic field, $\vec{B}$ is the magnetic flux density, $\varepsilon$ is the permitivity of the medium and $\mu_0$ is the permeability of free space. The propagation wave electric field vector in z-direction is shown below

$$\vec{E}(x,y,z,t) = \vec{E}(x,y)e^{j(\omega t - \beta z)}$$

where $\omega$ is the angular frequency of wave and $\beta$ is the propagation constant of wave.

If we use this equation in homogeneous wave equation, Helmholtz equation for the electric field is written as

$$\vec{\nabla}^2\vec{E} + k^2\vec{E} = 0$$

where k is the wave number in medium.

In this study, we investigate the TE modes of electromagnetic wave which propagate in z-direction and polarized in x-direction so that electric field expression can be write as below [16]

$$E_y(x, y, z) = E_y(x, y)e^{j(\omega t - \beta z)}$$

Writing the electric field expression in Maxwell equation leads to

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} - \beta^2 E_y = -\omega^2 \mu \varepsilon E_y$$

.

If we define $k^2 = \omega^2 \mu \varepsilon$, the scalar wave equation is obtained

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + (k^2 - \beta^2)E_y = 0$$

The scalar wave equation for TE mode is obtained by writing the $k$ in medium and by writing $k_0$ in free space as a function of $n$ [17] as

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + (k_0^2 n^2 - \beta^2)E_y = 0$$

By using Marcatili method in Figure 1 and from the tangential field components in the close media interfaces we obtain following characteristic equation for TE mode [17]

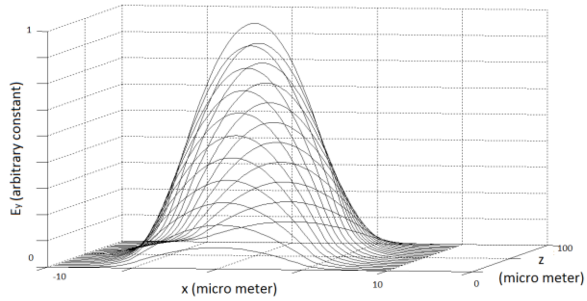
$$\mathrm{k_y} b = tan^{-1}(\gamma_y / \kappa_y) + \frac{1}{2}(s-1)\pi$$

where $2b$ is the embedded waveguide height and width, $\gamma$ is the attenuation coefficient and $\kappa$ is the transverse component of wave vector.

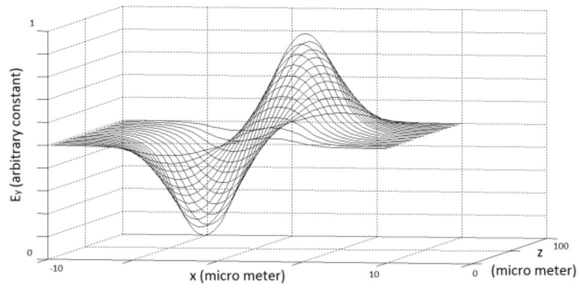
## III. MODELLING AND SIMULATION

In this work, we modeled the embedded waveguide with two different materials. The refractive index of cladding is 1.43 which made of $\mathrm{SiO_2}$ and the core refractive index is 3.44 which made of Si. The wavelength $\lambda = 1.55 \times 10^{-6}\, m$ and propagation constant $k = 6{,}08 \times 10^{6}\, m^{-1}$ were used [16]. We have taken core part of waveguide in which mode was transmitted, height and width $2b = 20\, \mu m$. Given the states in this work, TEM waves simulations were shown in following figures. The graphs associated with four modes supported by waveguide are plotted in Figures 2, 3, 4 respectively.

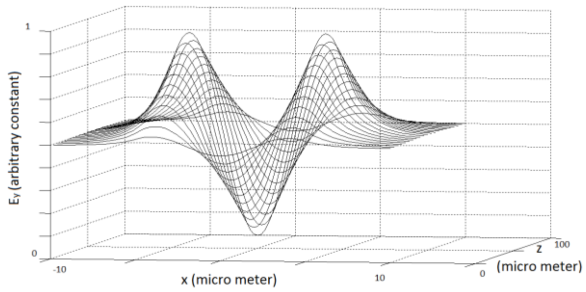
Two dimensional simulations of TE modes which trapped in the core layer are shown in the below graphs. Refractive index of core is $n_1$ and length is $20\, \mu m$. In Figure 1, $\mathrm{TE_1}$, $\mathrm{TE_2}$, $\mathrm{TE_3}$, $\mathrm{TE_4}$ modes are confined in certain z-distance within the waveguide.



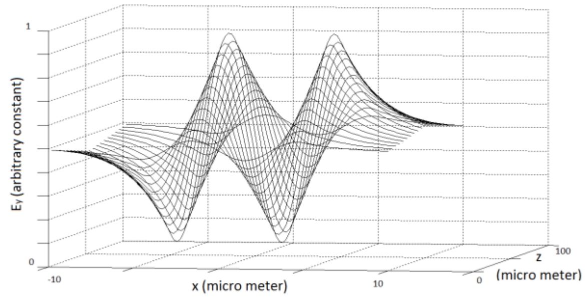

$TE_1$ mode




$TE_2$ mode



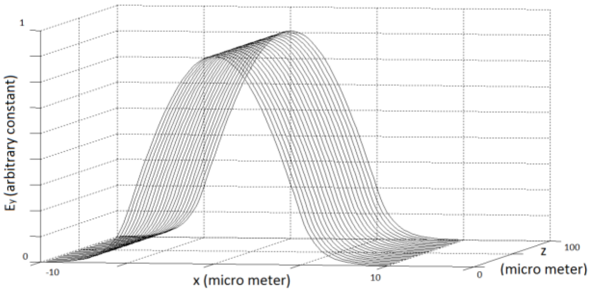

$TE_3$ mode



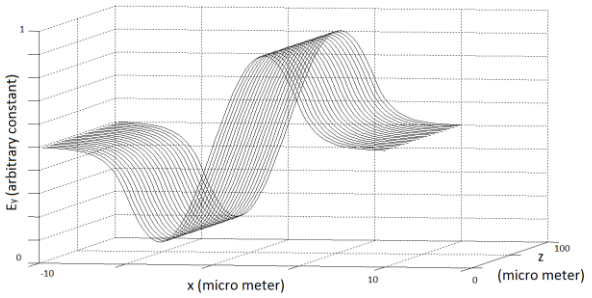

$TE_4$ mode

Figure 2. Confined TE modes in certain z-distance.

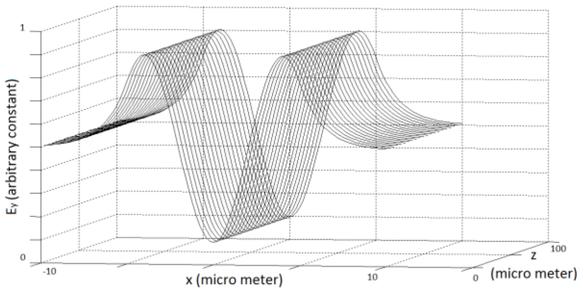
In Figure 3, TE modes which propagate in the core layer and without energy loose simulates below graphs. Because of TE modes do not loose energy, amplitude propagate along the z-direction do not chance.



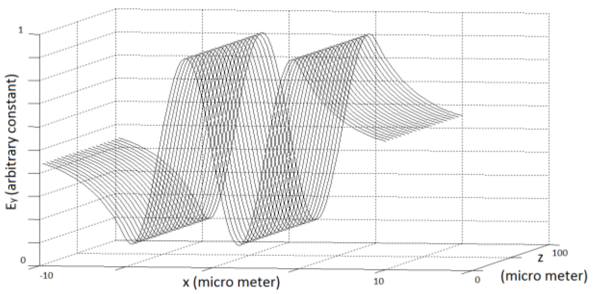

$TE_1$ mode




$TE_2$ mode




$TE_3$ mode

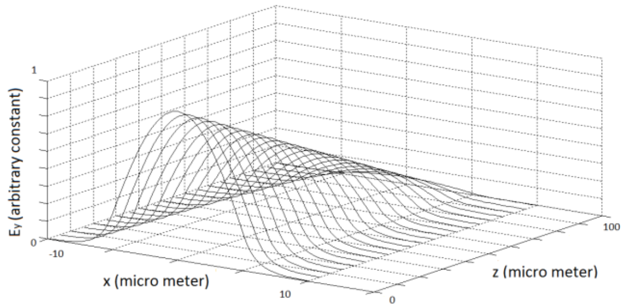



$TE_4$ mode

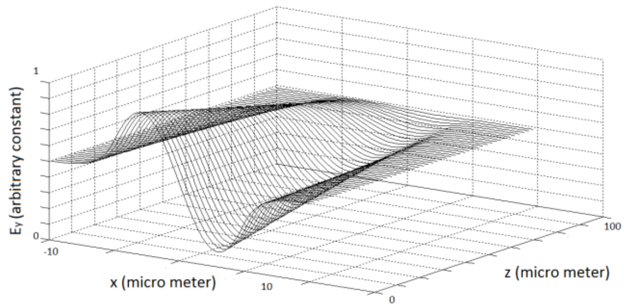
Figure 3. Propagating TE modes without losing energy along the z axis.

If the core layer, which TE modes propagate in, damped the modes simulated in the below graphs.

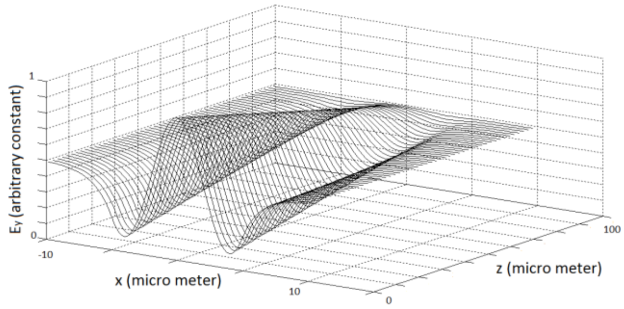
Because of core layer is damped modes are losing energy and amplitudes are decreased until run out by time. Therefore, amplitudes of modes decrease along the z-direction.



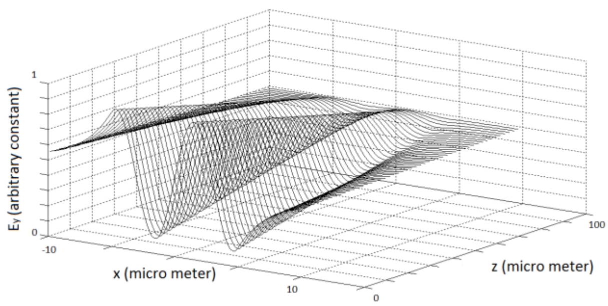

$TE_1$ mode




$TE_2$ mode




$TE_3$ mode




$TE_4$ mode

Figure 4. Evanescent TE modes along the z axis.

## IV. CONCLUSIONS

In this paper, we simulated the TE modes in dielectric embedded waveguide by using Marcatili method. In Figure 2, TE modes trapped in the medium having $n_1$ refractive index. In Figure 3, TE modes amplitude do not change in propagation medium. In

Figure 4, because of guiding media is damped TE modes energy and amplitudes were diminished by time. In graphs, we have achieved peak values of modes in the center of core region and evanescent modes in the core-cladding interface along x-axis.

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