A Review Of Bubble Rise Phenomena In Various Fluids

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Abstract-The study of bubble rise characteristic is vital for the design of heat and mass transfer operations in chemical, biochemical, environmental, and food processing industries. The rate of heat and mass transfer is affected by the bubble size, pressure inside the gas phase, interaction between bubbles, rise velocity and rising trajectory. Study on bubble rise phenomena in non-Newtonian fluids is limited and there is an increasing demand for advance research in this area since most of the industrial fluids are non- Newtonian in nature. This survey is to review existing study on multiple bubble rise. The review found insufficient number of studies on multiple bubble rise phenomena, especially in non-Newtonian fluids.

Keywords—bubble flow; non-Newtonian fluids; heat transfer

1. INTRODUCTION

Bubble flows are frequently applied in many industrial processes which can aid heat and mass transfer. Bubbles play an important role in various applications such as; in fermentation processes where beer, wine, bread, kimchi, yogurt and other foods are produced, in cooking processes, in pipeline transport applications, in polymer and sludge processes as well as many others. The overall heat and mass transfer is affected by the size of bubble, pressure inside the gaseous phase, bubble-bubble interaction, rise velocity and trajectory [1].

The applications of bubbles play a vital role in many industrial devices, for example, vacuum pan operation (shown in Figure 1.1) in the sugar industries [2] . Cane or beet juice in sugar industries is evaporated into concentrated syrups. The syrup is then concentrated and crystallized into sugar by boiling in large vessels called vacuum pans. Seeds with tiny sugar crystal are sent to vacuum pans and the solution of sugar is maintained supersaturated to encourage the seed crystals to grow to a desired size by using additional syrup during controlling the boiling conditions. This entire process is known as crystallization process. Vapour bubbles are created in the sugar solution and rise to the surface during the boiling process. The purpose of steam bubbles is to mix the solution to keep homogeneity and suspend the sugar seed crystals in solution, so that the seeds stay unsettled on the bottom of the vacuum pan [2-4].

The mixture, known as massecuite, is discharged when the crystals reach the desired size in the vacuum pan. Massecuite is the crystal-suspension mixture that comprises sugar crystals surrounded by mother syrup (known as molasses). Crystallization separates molasses from the massecuite in centrifugals and then this molasses is boiled again to recover more sucrose. It gives output when sucrose is crystallized out of the syrup/juice solution. There has been an effect of crystallization process on concentrating the impurities in the molasses. The impurities includepolysaccharides, waxes, gums, etc enter the sugar factory as small concentrations in the juice [2-4].

Massecuites and molasses usually have non-Newtonian flow behaviour characteristics where the viscosity depends on the rate of shear which is well described by the Power-Law Model. The molasses and massecuites demonstrate almost similar shear thinning behaviour in terms of Power-Law index [2-4]. The value of power law index (n) of massecuites lies between 0.5 and unity. Higher consistency value (K) indicates a more viscous fluid while the deviation from unity of the flow behaviour index (n) can be used for measuring non-Newtonian behaviour [2-4].

Massecuites are not transparent and it is not realistic to experimentally investigate bubbly flows through this non-Newtonian fluid. So, it is necessary to produce massecuite-equavalent which are optically clear and have similar rheological properties. Furthermore there has been a non-industrial environmental problem during investigation of massecuite due to the degradation during storage and varying rheological properties under different temperature conditions.

Generally, the dynamics of bubble rise characteristics are complex and the degree of the complexity increases with the bubble size [5]. The bubble rise characteristics for single bubbles of large size (d_b > 5.76 mm) in water, crystal suspensions and non-Newtonian polymeric have extensively been studied [6-13]. The rise behaviour of multiple bubbles can be significantly isolated from that of a single bubble. The wake effects and bubble-bubble interactions greatly dominance the bubble shapes and their velocities and consequently the inter-phase coupling forces [6]. The velocity of the bubble depends on the velocity of the liquid ahead of it, along with other physical and geometrical parameters [11]. The behaviour of multiple bubbles depends on the relative configuration of individual bubbles and is governed by bubble-bubble interactions. When two or more bubbles rise simultaneously in a liquid column, the wake of the leading bubble can affect the velocity of the trailing one and under certain conditions induces coalescence [14]. The velocity of multiple bubble configurations are increased by a factor of 1.5 to 3 times the velocity of a single isolated bubble showing the importance of the bubble-bubble interactions [10]. In a factory environment, where gas-liquid distributions are involved, bubbles are constantly colliding with each other. Bubbles may separate from each other or coalesce, depending on the bubble size, their velocity and the frequency of such collisions [8].

The above mentioned literature indicates several studies on single bubble rise phenomena which dealt with either bubble rise or heat transfer in Newtonian and some non-Newtonian fluids. There are studies of multiple bubble rise-mostly numerical- in Newtonian fluids, however, only a limited study exist in non-Newtonian fluids. In addition, these studies provide inadequate information in relation to the bubble-bubble interactions and bubble coalescence rates. No or little experimental research on multiple bubbles rise in non-Newtonian fluids, are available. Therefore, there is a need for further research

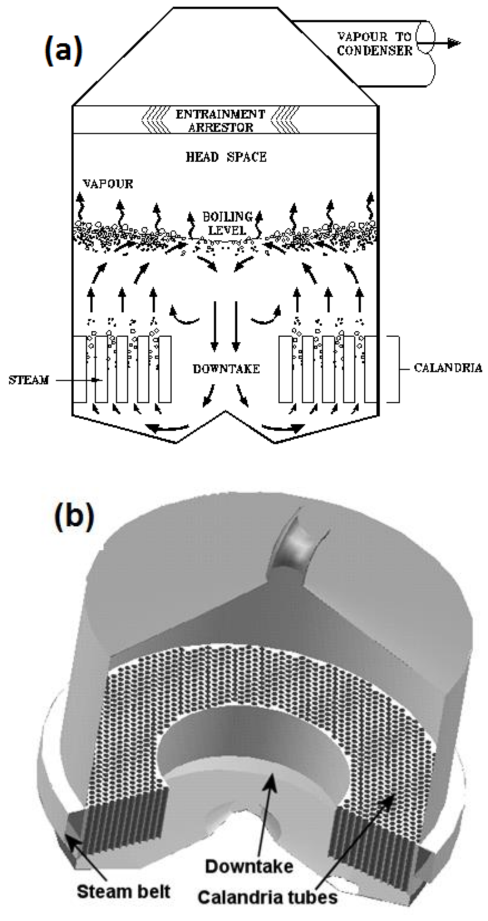

Fig. 1 Batch vacuum pan (a) Circulation patterns and (B) Sectioned view [2]

to investigate motion and interactions of multiple bubbles in non-Newtonian fluids like polymer and crystal suspension since most industrial fluids in food industries are non-Newtonian in nature. There is a demand in process industries to undertake further research to investigate flow patterns of the bubble-bubble interactions and their wake structures and coalescences and break up rates during their rise. The ability to predict heat and mass transfer of bubbly flows in industrial processes requires an adequate knowledge of bubbly dynamics. This knowledge could be utilized to optimize process design and hence maximise productivity of industrial equipment. For efficient design and unique operations of process equipment, it is necessary to get a sound knowledge of bubble-bubble interactions and their wake structure and coalescences and break up rates for maximising heat and mass transfer performance.

2. LITERATURE REVIEW

Bubble rise characteristics in Newtonian liquid such as water have received considerable attention and are understood [2]. However, research on bubble rise phenomena in non-Newtonian fluids is very limited and there is an increased demand for further research in this area since most of the industrial fluids are non-Newtonian in nature. Non-Newtonian fluids exhibit complex flow properties and Newtonian fluids show deformation dependent non-constant viscosity. A number of literatures on single and multiple bubbles are reviewed separately in the next section.

A. Single Bubble

The most significant dynamic behaviours of bubble are the bubble trajectory, velocity and the drag coefficient. The drag coefficient correlates the drag force created on a moving bubble to its terminal velocity and projected surface area. The terminal velocity of an air bubble is known as the velocity achieved at steady state conditions when applied forces are balanced [2]. The bubble rise characteristics for instance velocity and drag coefficient of an air bubble mainly depend on the properties of the liquid and the bubble.

Generally, in stagnant or stationary liquids, terminal rise velocity is actually bubble rise velocity and it is denoted as slip velocity for moving fluids [5]. Bubble rise velocity greatly depends on bubble shape. Large bubbles have a higher amount of drag present in comparison with the smaller bubbles. Viscous effects decrease with shear rate in a shear thinning liquid. A fluid having lower viscosity has less drag force to restrain the bubble to rise faster [15]. When bubbles rise inside liquid, the travelling way is different from the travel way of rigid particles rising due to buoyancy. Solid particles tend to stay rigid and do not circulate internally; however the shape of the solid particle could be same as a shape of bubble. But air bubbles would have repetitive motion internally. The bubble tends to track the path of smallest resistance during the motion of bubble. When bubble tends to rise upwards inside liquid, the most resistance works right on the top. However, when bubble travels slightly to one side, small number of total resistance is experienced. It is seen that bubbles begin a spring or helical shaped path as they rise in a liquid column. The dynamics of bubble rise are, normally, nonlinear and the degree of the nonlinearity rises with the size of the bubble. The values of drag coefficient are dependent on physico-chemical properties of the system and the dimension of bubble. The viscous forces, in low-Reynolds number flows, are large comparing with internal terms and the viscous shear stresses transmit the bubble motion far into the flow. So the forces of viscosity dominate the terminal motion, and terminal rise velocity rises with the rise in bubble diameter at very low Reynolds numbers. In the intermediate region of Reynolds number, surface tension and inertia forces determine the terminal rise velocity. Bubbles are no longer spherical in this region as their size increases and the terminal velocity may increase, remain constant or decrease with equivalent diameter of the bubble. At very high Reynolds number, the shape of bubbles forms spherical-cap or mushroom and the bubble motion is controlled by the force of inertia.

Böhm, Kurita [16] studied rising behaviour of single bubbles in a confined geometry having rectangular cross section and they found a significant influence of the rheology of working fluids. They also summarized published experimental articles in the table 1 on single bubble rise where experimental setup, equipment, bubble size, varied parameters, measured quantities have been clearly described.

B. Multiple Bubbles

Since most of the bubble flows are multiple bubbles in process industries, understanding the dynamics of the multiple bubbles is very important for assessing the effect of heat and mass transfer of bubbly flows in industries. Buoyant rise of multiple bubbles under gravitational force has been a common area of interest for experimental and numerical researchers

The dynamics of bubbles rising in viscous liquids due to buoyancy is relevant to many industrial processes, such as oil/gas transport, ink-jet printing [17, 18] , spray cooling [19, 20], carbon sequestration [21] , soil-vapor extraction [22-24] , nuclear waste management [25, 26], bubble columns and boiling flows. Most of these bubbly flow systems consist of multiple bubbles with different sizes [27]. The interactions among these bubbles impose important effects on the flow behaviour. Especially, the effects of small dispersed bubbles on the rising behaviour of a large bubble can be significant, and these effects are not understood well so far.

TABLE1. ARTICLES DELT WITH SINGLE BUBBLE RISING [28]

Authors	High speed camera setup	Setup geometry (cyl.: i.d. x h, rect.: w x d x h)	Bubble size	Varied parameters	Measured quantity	Comment/brief results
Acharya et al.	-/-	Rectangular 165 x 165 x 245 mm3	-	Injector type	Bubble velocity, shape, deformation	-/-
Clanet et al.	–/25 Hz	Several	Taylor bubbles	Geometry, viscosity	Bubble velocity	Taylor bubbles/–
Dekee et al.	–/6 Hz	Rectangular 230 x 230 x 772 mm3	2.6–26.7 mm	Bubble size, viscosity	Bubble velocity	Investigation of coalescence/–
Drews et al.	752 x 582 pixel2/350 Hz	Rectangular 3-11 x 160 x 700 mm3	3–24 mm	Bubble size, viscosity	Bubble velocity	-/-
Ellingsen et al.	–/1 kHz	Rectangular 150 x 150 x 650 mm3	2.48 mm	-	Rising path, shape	-/-
Zenit et al.	–/–	Rectangular 3.6–4.7 x 200 x 400 mm3	<1.4 mm	Re	Rising path	–/Drag coefficients
Tokuhira et al.	768 x 493 pixel2/shadowgraphy	Rectangular 100 x 100 x 1000 mm3	9.12 mm	Investigation of a bubble and a solid particle	Shape, rising path	Counter current flow cell/–
Fujiwara et al.	-	-	8 mm	Viscosity	-	One side of the channel is a movable belt/influence of the shear flow field on the rising path
Fujiwara et al.	+2nd shadowgraphy camera for 3D bubble shape reconstruction	-	2–6 mm	Bubble size	-	+3D bubble shape
Li et al.	–/–	Rectangular 60 x 60 x 500 mm3	<12 mm	Viscosity	Shape, bubble velocity,	–/Bubble interaction

Frank et al. Li et al.		Cylindrical 300 x 500 mm2	3–14 mm	-	coalescence behaviour	
Li et al		Cylindrical 300 x 1500 mm2	6.5–7.2 mm			
Brücker et al	512 x 512 pixel2/shadowgraphy	Rectangular 100 x 100 x 1200 mm3	5–7 mm	Bubble size	Rising path, oscillation frequency, shape, bubble velocity	Counter current flow cell/ physical description of the bubble behaviour during its ascent
Ortiz-Villafuerte et al.	640 x 480 pixel2/shadowgraphy, 2D hybrid particle tracking and 3D reconstruction	Cylindrical 12.7 x 1300 mm2	3 mm	-	Rising path, bubble velocity	Stagnant water/physical description of the bubble behaviour during its ascent
Hassan et al.	–/–	Rectangular 300 x 300 x 240 mm3	1.5–33 mm	Re number	Bubble velocity	–/–
Liu et al.	1200 x 1600 pixel2/shadowgraphy	Rectangular 68 x 88 x450 mm3	6 mm	Viscosity, gas flow rate	Rising path, shape	Bubble train/influence of the viscosity on the rising path
Maneri et al.	–/–	Rectangular 9.5–1.3 x 63-86 x 914 mm3	<55 mm	Channel inclination, viscosity	Bubble velocity	/Viscosity influences bubble
Miyahara et al.	–/–	Cylindrical 10 x 1000 mm2	2–30 mm	Viscosity	Shape, bubble velocity, oscillation	–/–
Roudet et al.	1280 x 1024 pixel2/500 Hz	Rectangular 1 x 400 x 800 mm3	2.6–8.3 mm	Bubble size, channel inclination	Shape, bubble velocity, oscillation	–/–
Sanada et al.	2 times 512 x 512 pixel2/1 kHz	Rectangular 150 x 150 x 400 mm3	0.66–0.93 mm	Bubble size, viscosity	Shape, rising path	–/–
Sakakibra et al.	960 x 960 pixel2/shadowgraphy	Rectangular 150 x 150 x 270/ 500 mm3	2.9 mm	-	Rising path, shape	–/–
Saito et al.				Surfactant concentration		–/Influence of surfactants on bubble motion
Yoshimoto et al.	1024 x 1024 pixel2/ shadowgraphy	Octagon, 160 x 160 x 230mm3	-	-	-	–/Relation between bubble shape, velocity and path
Sathe et al.	2048 x 2048 pixel2/ shadowgraphy	rectangular 200 x 15 x 500 mm3	0.1–15 mm	Bubble size, liquid velocity	Shape	Bubbly flow/comparison between single bubble behaviour and bubble swarms
Sathe et al.	-	+Cylindrical 150 x 650 mm2		Geometry		Bubbly flow/–
Joshi et al.	-	Rectangular 200 x 15 x 1000 mm3	2–35 mm	Bubble size	Bubble diameter	Bubbly flow/–
Takagi et al.	-					Review/influence of surfactants
De Vries et al.	-	Rectangular 15 x 15 x 500mm3	0.8–1.8 mm	Bubble size	Rising path	–/Wall interaction
Schouten et al.	–/955 Hz	Rectangular 15 x 300 x 2000 mm3	15–80 mm	Bubble size	Shape, bubble velocity	Comparison with CFD/–
Lucas et al.	2 times 1280 x1024 pixel2/ 500 Hz	Rectangular 50 x 50 x 1300 mm3	1–4 mm	Bubble size	Rising path	With superimposed liquid velocity/–
Zhang et al.	752 x 582 pixel2/–	Rectangular 210 x 210 x 600 mm3	2.7–5.2 mm	Bubble size, viscosity	Bubble velocity, bubble size	–/–

***d:** diameter; **h**: height; **i.d**.: inner diameter; **w**: width*

Many researchers have investigated both experimentally [29-31] and numerically [32-35] on multiple bubble rise to understand effects of bubble-bubble interaction and their coalescences with a limited number of bubbles of similar size but using Newtonian fluids, such as water. Stewart [29] performed the experimental studies focused on the interactions of freely rising ellipsoidal bubbles. A video camera following the rising bubbles was used to record the dynamics of the bubble interactions during bubble approach, contact, coalescence or break up. It was found that a bubble contacts another by following its wake, which leads to an overtaking collision. Several interacting bubbles may form a dynamic cluster, which accelerates the overall bubble rise speed. Garnier, Lance [30] studied the rise of a homogeneous dispersion of gas bubbles in water. A special design was adopted in the experiments to limit bubble coalescence and maintain mono-distribution of bubble size. They found that the mean bubble rise velocity decreases when the void fraction increases, due to bubble interactions. This is well known as the hindrance effect. Similarly, the experiments by Simonnet, Gentric [31] studied the influence of the void fraction on the relative velocity of a swarm of gas bubbles (multiple bubbles in water) in a square bubble column with a 0.1 × 0.1 m2 cross-sectional area and a total height h of 1 m as shown in fig. 2. The bubble diameters varied from 2 to 10 mm, and local void fraction could be as high as 35%. It was found that when the local void fraction is smaller than 15%, the relative bubble velocity is determined by the hindrance effect, and consequently decreases with the local void fraction. On the other hand, when the local void fraction is higher than 15%, the relative bubble velocity begins to be dominated by the acceleration of bubbles in the wake of the leading ones, increasing suddenly with the local void fraction. It should be noted that the bubble size increases with the void fraction in the experiments.

However, many researchers [36-42] also make use of several numerical methods such as Volume of Fluid method (VOF), Level Set method (LS), Lattice Boltzmann method (LB), and Front Tracking method (FT) to study bubble dynamics or interactions. In general, there was reasonable agreement in their results with the existing experimental data.

Fan and YIN [36] investigated two bubbles rising dynamics side-by-side in concentrated

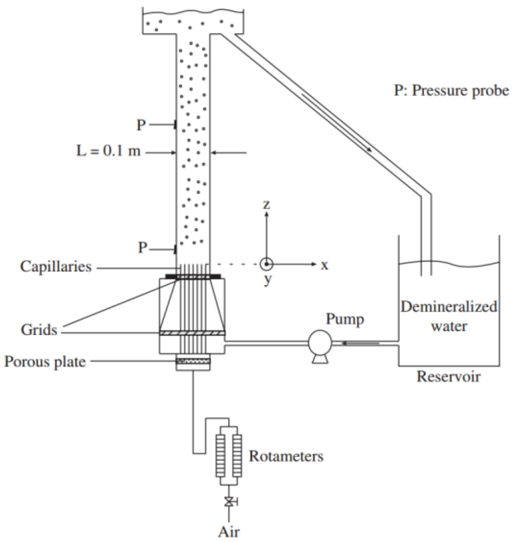

Fig. 2 Experimental setup [42]

carboxymethyl cellulose (CMC) solution by using VOF method. The results exhibited a decent agreement with the experimental data. The authors found that the repulsive effect between two bubbles decreases with increasing the initial center-to-center distance of bubbles and increase of the oblique angle between them. Similarly, Yu, Yang [35] analysed two parallel bubbles rising behaviour in viscous fluid using adaptive LB method. They found that the repulsive behaviour of two spherical bubbles happens at lower Reynolds number, but cohesive behaviour and finally coalescence of the bubbles at higher Reynolds numbers.

Zhang, Yang [37] Investigated movement of a single bubble rising freely through CMC sodium salt, sodium hydroxyl-ethyl cellulose (HEC) and xanthan gum (XG) solution using a level set method for tracking the bubble interface. They studied the shear rate and viscosity distribution and shape of a bubble rising in CMC, HEC, XG solution and compare to sodium acrylate polymer (SAP) shear-thinning solution.

Yeoh and Tu [43] used different numerical methods in his study to simulate multiphase flow. Fuster, Agbaglah [44] used a volume of fluid (VOF) method, balanced-force surface tension and quad/octree adaptive mesh refinement (AMR) to simulate bubble dynamics. van Sint Annaland, Deen [45] presented an interface reconstruction technique based on piecewise linear interface representation in volume of fluid (VOF) method to simulate co-axial and oblique coalescence of two gas bubbles. van Sint Annaland, Dijkhuizen [46] used a 3-D front tracking method employing a new surface tension model to simulate single and multiple bubble dynamics in dispersed fluid. Several researchers [47-49] used a level set method to simulate bubble dynamics. They used COMSOL Multiphysics [50] which is a commercial software that applies level set method to simulate multiphase flow system. The numerical simulation of multiphase flow is a challenging class of problems because of the inherent difficulty in tracking the fluid interfaces, mass conservation, and the correct treatment of the surface tension forces [51]. In recent years, the lattice Boltzmann method (LBM) has emerged as a very promising numerical approach for simulation of complex multiphase flow [52-58]. Vélez-Cordero, Sámano [59] studied the interaction of two bubbles rising in shear-thinning inelastic fluids which is a significant step toward understanding multiphase flow systems.

However, bubble flow increases the heat transfer performance in industrial proceses. Experimental investigations recommend that the presence of moving bubbles usually augments heat transfer. Tamari and Nishikawa [60] found bubbles are applied for addition of a buoyancy force to increase convection. Several studies [61-63] found that the presence of the bubbles amends the local flow and the turbulence structure. Deckwer [64] studied the mechanism of heat transfer in bubble column reactors and he suggested that the occurrence of bubbles can increase the heat transfer in a gas–liquid bubble column by more than an order of magnitude.

Tanaka [65] used the experimental setup almost similar to that used by Lu and Tryggvason [66] where Tanaka [65] shown results for bubbles and drops in a turbulent channel flow. The results found the heat transfer is enhanced for both bubbles and drops but the wall friction also is increased due to their presence. On the other hand, Dabiri and Tryggvason [67] also examined the heat transfer in turbulent bubbly up flows in vertical channels. The results indicated that the presence of bubbles enhances the heat transfer performance.

A review of literature indicates that the experimental studies on multiple bubbles flow are very few but a large number of numerical studies are found. Most of all existing experimental studies dealt with Newtonian fluids such as water.

3. Conclusion

This review concludes that there are several studies on single bubble rise in Newtonian and some non-Newtonian fluids which dealt with either bubble rise movement or heat transfer mechanism. There are very limited studies, mainly numerical, which deal with the rise of multiple bubbles in some non-Newtonian fluids. They, however, provide inadequate information in relation to the bubble-bubble interactions, the bubble coalescences rate and temperature effect. Little or no research works on the multiple bubbles rise in crystal suspended non-Newtonian fluids are available. So further experimental studies are required using Particle Image Velocimetry (PIV) and Thermal Imaging technology for multiple bubbles in non-Newtonian since most industrial fluids, for instance, in food processing industries are non-Newtonian in nature. PIV gives visualization of bubble movement and Thermal Imaging gives heat transfer phenomena among bubbles.

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