Journal of Multidisciplinary Engineering Science and Technology (JMEST)
ISSN: 3159-0040
Vol. 2 Issue 8, August - 2015

Claims reserving for a DMTPL portfolio using Double Chain Ladder

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***Abstract*— The methodologies for the estimation of claims reserve for a non-life insurance company are based on triangles run-off data. Putting together a payment data triangle and an incurred count triangle, we obtain Double Chain Ladder, a method for the estimation of Reported But Not Settled reserve claims and for the estimation of Incurred But Not Reported reserve claims.**

Keywords—Double Chain Ladder (DCL) Method; Chain Ladder (CL); Reported But Not Settled (RBNS) reserve;Incurred But Not Reported (IBNR) reserve; Aggregate Payments Triangle, Incurred Count Triangle

I. INTRODUCTION

Reserving claims methods are based on run-off triangles [2].

The chain ladder method is based on

- Paid claims triangle
- Reported claims triangle
- Incurred claims triangle
- Incurred counts triangle

The Munich Chain Ladder method is based on incurred claims triangle + paid claims triangle.

The Double Chain Ladder method is based on paid claims triangle + incurred counts triangle.

We applied DCL method to an Albanian DMTPL portfolio and we compared the results with Chain Ladder Method. The paid claims triangle and the incurred counts triangle cover ten development years and ten accident years. All values are in Albanian currency.

II. DOUBLE CHAIN LADDER METHOD

A. *General structure and data used*

We consider two run-off triangles, the aggregated paid triangle and the aggregated incurred counts triangle [3]

We define the Aggregated Payments:

$$\Delta_m = \{X_{ij}: (i,j) \in I\}$$

X_{ij} is the total payments of claims incurred in year i and paid with j period delay from year i; and the Aggregated incurred counts:

$$\aleph_m = \{N_{ij}: (i,j) \in I \}$$

N_{ij} is the total number of claims incurred in year i and reported in year i+j, with j periods delay from year i; I={(i,j): i=1,…,m; j=0, …,m-1; i+j≤m}

The payments and the counts triangles $\Delta_m, \aleph_m$ are real data, but the RBNS and the settlement delay are stochastic components modelled considering the unobserved variables N_{ijl}^{paid}, that is the number of the future payments originating from the N_{ij} reported claims, paid with l periods delay, l=0, 1,……, m-1.

The individual settled payments arise from N_{ijl}^{paid} are $Y_{ijl}^{(k)}$ where $(k = 1 ,\dots, N_{ijl}^{paid}$, $(i,j) \in I, \ell = 0, \dots, m-1)$.

We can estimate the RBNS reserve using these components. It is necessary to model IBNR delay for estimating IBNR reserve.

Aggregated Count Incurred

i/j	0	1	2	3	4	5	6	7	8	9
2005	298	30	2	0	0	0	0	0	0	0
2006	447	14	1	0	1	0	0	0	0	
2007	439	27	0	2	0	0	0	0		
2008	542	41	5	2	1	2	0			
2009	569	54	5	3	0	0				
2010	531	34	9	1	1					
2011	462	37	10	1						
2012	507	45	8							
2013	427	39								
2014	395									

Aggregated Paid

i/j	0	1	2	3	4	5	6	7	8	9
2005	13,247,635	6,624,159	2,642,654	911,253	0	4,609,220	0	500,000	1,092,696	0
2006	18,545,283	9,551,339	408,400	1,879,844	5,825,431	0	0	0	0	
2007	22,805,548	9,871,739	379,680	1,487,431	356,650	5,000,000	0	0		
2008	25,061,272	16,388,840	12,082,016	1,100,000	0	849,872	0			
2009	27,800,292	12,084,832	7,851,518	11,090,317	400,000	0				
2010	28,171,203	18,313,884	11,392,441	14,156,137	2,649,184					
2011	24,747,420	20,303,903	6,483,741	3,295,964						
2012	22,311,896	22,367,649	10,962,560							
2013	16,811,300	20,170,086								
2014	21,903,227									

B. *Assumptions*

Assumption 1 – Independence

- N_{ij} are independent
- The claims are paid with one payment or as a zero –claim
- The variables $Y_{ij}^{(k)}$ are independent of the counts N_{ij} and independent of reporting and payment delay

Assumption 2 – RBNS delay

- N_{ij} random variables having a multiplicative parametrization $E\left[N_{ij}\right] = \alpha_i \beta_j$ and identification $\sum \beta_j = 1$

- The delay parameters are $\pi = \{\pi_0,, \pi_{m-1}\}$
- The mean of RBNS delay variables is

$$E\left[N_{ijl}^{paid}\middle|\aleph_m\right] = N_{ij}\tilde{\pi}_l$$

Assumption 3 – Claim size distribution

- The individual payments $Y_{ij}^{(k)}$ are independent with mean μ_i and variance σ_i^2

$$E\left[Y_{ij}^{(k)}\right] = \mu\gamma_i \equiv \mu_i \text{ and } V[Y_{ij}^{(k)}]=\sigma^2\gamma_i^2 = \sigma_i^2$$

μ and σ^2 are mean and variance factors, γ_i is the inflation in the accident year.

III. DERIVATION OF DCL ESTIMATE

Double Chain Ladder derives through comparison of conditional means of claims counts and claims payments calculated from the underlying DCL assumptions and ordinary Chain Ladder [3].

Using the assumptions we have:

$$E\left[\sum_{k=1}^{N_{i,j-l,l}^{paid}} Y_{i,j-l,l}^{(k)}\,|\aleph_m\right] = E\left[\sum_{k=1}^{N_{i,j-l,l}^{paid}} E[Y_{i,j-l,l}^{(k)}\,|\aleph_m, N_{i,j-l,l}^{paid}]|\aleph_m\right]$$

$$= E[N_{i,j-l,l}^{paid}\tilde{\mu}_l\gamma_i|\aleph_m] = N_{i,j-l}\tilde{\pi}_l\tilde{\mu}_l\gamma_i$$

The aggregate payments can be written as:

$$X_{ij} = \sum_{l=0}^{j}\sum_{k=1}^{N_{i,j-l,l}^{paid}} Y_{i,j-l,l}^{(k)} \quad for\ each\ (i,j) \in I$$

For the conditional and unconditional means we have

$$E[X_{ij}|\aleph_m] = \sum_{l=0}^{j} N_{i,j-l}\tilde{\pi}_l\,\tilde{\mu}_l\gamma_i = \sum_{l=0}^{j} N_{i,j-l}\pi_l\mu\gamma_i$$

$$E[X_{ij}] = \alpha_i\mu\gamma_i\sum_{l=0}^{j}\beta_{j-l}\pi_l$$

$$\mu = \sum_{l=0}^{m-1}\tilde{\pi}_l\tilde{\mu}_l$$

$$\pi_l = \frac{\tilde{\pi}_l\tilde{\mu}_l}{\mu}$$

Parameters α_i and β_j can be estimated using Chain Ladder applied on the triangle of claims counts. It is possible to use conditional and unconditional mean to estimate RBNS reserve and it is possible to use unconditional mean to estimate IBNR reserve. We must estimate μ,γ_i and π_l.

A. Estimation of parameters

Using the standard chain – ladder assumptions applied on the payment triangles, we know there exist parameters $\tilde{\alpha}_i, \tilde{\beta}_j$ that it is satisfied

$$E[X_{ij}] = \tilde{\alpha}_i\tilde{\beta}_j$$

From the formula

$$E[X_{ij}] = \alpha_i\mu\gamma_i\sum_{l=0}^{j}\beta_{j-l}\pi_l$$

we can deduct that

$$\tilde{\alpha}_i = \alpha_i\mu\gamma_i \text{ and}$$

$$\tilde{\beta}_j = \sum_{l=0}^{j}\beta_{j-l}\pi_l$$

We denote the estimates of $\alpha_i, \beta_j, \tilde{\alpha}_i, \tilde{\beta}_j$ by $\hat{\alpha}_i, \hat{\beta}_j, \hat{\tilde{\alpha}}_\iota, \hat{\tilde{\beta}}_j$. They are used to estimate μ,γ_i and π_l.

We can estimate π_l using the estimators of the other coefficients $\hat{\beta}_j, \hat{\tilde{\beta}}_j$ and for this, it is necessary to solve the linear system where the solution is $\hat{\pi}_l$.

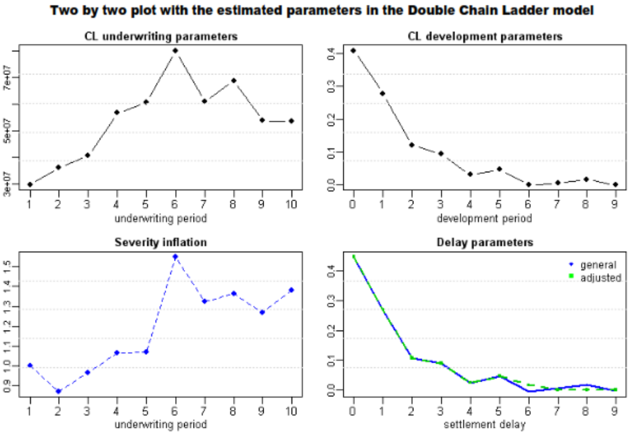
$$\begin{pmatrix} \hat{\tilde{\beta}}_0 \\ \vdots \\ \hat{\tilde{\beta}}_{m-1} \end{pmatrix} = \begin{pmatrix} \hat{\beta}_0 & 0 & \cdots & 0 \\ \hat{\beta}_1 & \hat{\beta}_0 & \ddots & 0 \\ \vdots & \ddots & \ddots & 0 \\ \hat{\beta}_{m-1} & \cdots & \hat{\beta}_1 & \hat{\beta}_0 \end{pmatrix} \begin{pmatrix} \pi_0 \\ \vdots \\ \pi_{m-1} \end{pmatrix}$$

Using the $\hat{\alpha}_i, \hat{\tilde{\alpha}}_\iota$ estimator we got the estimator for γ_i and μ

$$\hat{\gamma}_i = \frac{\hat{\tilde{\alpha}}_\iota}{\hat{\alpha}_i\hat{\mu}}$$

and to ensure identifiability we put $\gamma_1 = 1$

$$\hat{\mu} = \frac{\hat{\tilde{\alpha}}_1}{\hat{\alpha}_1}$$



B. Estimates for RBNS and IBNR

One of the two estimates for RBNS component is:

$$\hat{X}_{ij}^{RBNS(1)} = \sum_{l=i-m+j}^{j} N_{i,j-l}\hat{\pi}_l\hat{\mu}\hat{\gamma}_i$$

And the second one is:

$$\hat{X}_{ij}^{RBNS(2)} = \sum_{l=i-m+j}^{j} \hat{N}_{i,j-l}\hat{\pi}_l\hat{\mu}\hat{\gamma}_i$$

The estimate for IBNR is:

$$\hat{X}_{ij}^{IBNR} = \sum_{l=0}^{i-m+j-1} \hat{N}_{i,j-l}\hat{\pi}_l\hat{\mu}\hat{\gamma}_i$$

where $\hat{N}_{i,j} = \hat{\alpha}_i\hat{\beta}_j$

We can also estimate the tail:

$$\hat{R}^{tail} = \sum_{(i,j)} \sum_{l=0}^{\min(j,d)} \hat{N}_{i,j-l}\hat{\pi}_l\hat{\mu}\hat{\gamma}_i$$

where d is the maximum delay.

C. DCL Calculations

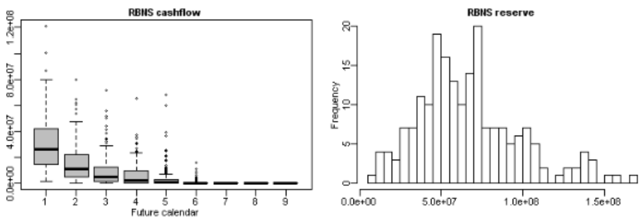
Estimates by calendar years and rows of the outstanding liabilities compared to standard Chain Ladder Method

Future years	RBNS	IBNR	TOTAL	CLM
1	32923044.1	2030773.81	34953817.95	34309555.4
2	16561954.8	1570470.67	18132425.44	17604387.6
3	10423063.3	798767.34	11221830.60	11209514.0
4	5047128.2	596156.38	5643284.53	5755111.6
5	3487774.3	248792.46	3736566.73	3834310.6
6	886542.6	276889.81	1163432.40	1394760.7
7	0.0	129006.74	129006.74	1143349.4
8	0.0	30102.14	30102.14	887649.8
9	0.0	9496.02	9496.02	0.0
10	0.0	3587.12	3587.12	0.0
11	0.0	697.88	697.88	
12	0.0	0.00	0.00	
13	0.0	0.00	0.00	
14	0.0	0.00	0.00	
15	0.0	0.00	0.00	
16	0.0	0.00	0.00	
17	0.0	0.00	0.00	
18	0.0	0.00	0.00	
Tot.	69329507.2	5694740.37	75024247.55	76138639.1

The distribution of RBNS reserves by calendar years and rows using bootstrapping [1]

period	rbns	mean.rbns	sd.rbns	Q1.rbns	Q5.rbns	Q50.rbns	Q95.rbns	Q99.rbns
1	32923044.1	30310997.0	21000341	2470831.32	5235622.06	26009745.65	68006441	100387542
2	16561954.8	16095025.4	14734258	221629.05	855282.45	11403482.96	44981685	62171084
3	10423063.3	9478535.6	11608703	9535.83	165217.30	5152131.67	33630485	51959551
4	5047128.2	6256265.6	9008953	8.62	274.58	2351969.62	24242767	35530812
5	3487774.3	3844128.1	8720026	0.60	9.70	533507.38	18282955	39580296
6	886542.6	526321.3	1903836	0.00	0.00	25.09	3649100	9339174
7	0.0	0.0	0	0.00	0.00	0.00	0	0
8	0.0	0.0	0	0.00	0.00	0.00	0	0
9	0.0	0.0	0	0.00	0.00	0.00	0	0
10	0.0	0.0	0	0.00	0.00	0.00	0	0
11	0.0	0.0	0	0.00	0.00	0.00	0	0
12	0.0	0.0	0	0.00	0.00	0.00	0	0
13	0.0	0.0	0	0.00	0.00	0.00	0	0
14	0.0	0.0	0	0.00	0.00	0.00	0	0
15	0.0	0.0	0	0.00	0.00	0.00	0	0
16	0.0	0.0	0	0.00	0.00	0.00	0	0
17	0.0	0.0	0	0.00	0.00	0.00	0	0
18	0.0	0.0	0	0.00	0.00	0.00	0	0
Tot.	69329507.2	66511273.0	31690927	14460100.11	21831172.09	62308914.07	134994311	154962470

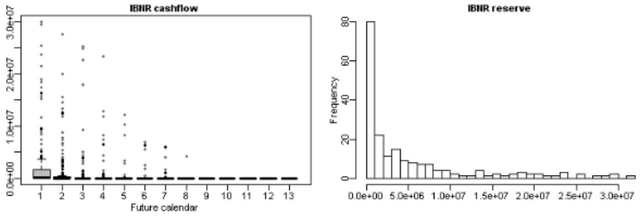
The boxplot of the distribution of the outstanding liabilities in the future calendar periods and the histogram of the distribution of RBNS



The distribution of IBNR reserves by calendar years and rows using bootstrapping

period	ibnr	mean.ibnr	sd.ibnr	Q1.ibnr	Q5.ibnr	Q50.ibnr	Q95.ibnr	Q99.ibnr
1	2030773.81	2258545.49	5279413.0	0.00	0.00	61818.50	15746843.23	25403513.75
2	1570470.67	1210444.53	3591506.9	0.00	0.00	1137.30	7736374.84	15637313.29
3	798767.34	735556.63	3328182.0	0.00	0.00	0.23	3433924.25	22716040.44
4	596156.38	450353.85	2300878.6	0.00	0.00	0.00	1001055.89	11854205.50
5	248792.46	191703.42	1293161.4	0.00	0.00	0.00	51970.71	8258638.05
6	276889.81	155813.40	904878.1	0.00	0.00	0.00	130694.03	6287408.47
7	129006.74	101510.89	668064.4	0.00	0.00	0.00	45873.87	3985549.55
8	30102.14	21372.57	301472.8	0.00	0.00	0.00	0.00	189.34
9	9496.02	0.00	0.0	0.00	0.00	0.00	0.00	0.00
10	3587.12	0.00	0.0	0.00	0.00	0.00	0.00	0.00
11	697.88	0.00	0.0	0.00	0.00	0.00	0.00	0.00
12	0.00	0.00	0.0	0.00	0.00	0.00	0.00	0.00
13	0.00	0.00	0.0	0.00	0.00	0.00	0.00	0.00
14	0.00	0.00	0.0	0.00	0.00	0.00	0.00	0.00
15	0.00	0.00	0.0	0.00	0.00	0.00	0.00	0.00
16	0.00	0.00	0.0	0.00	0.00	0.00	0.00	0.00
17	0.00	0.00	0.0	0.00	0.00	0.00	0.00	0.00
18	0.00	0.00	0.0	0.00	0.00	0.00	0.00	0.00
Tot.	5694740.37	5125300.78	7244983.2	21.07	2579.25	1789533.21	22675740.78	29517415.18

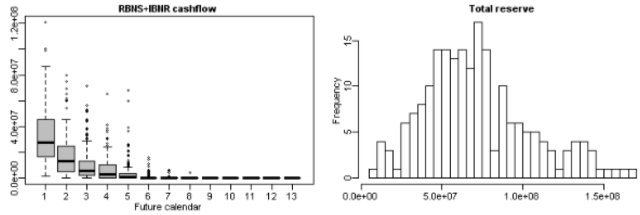
The boxplot of the distribution of the outstanding liabilities in the future calendar periods and the histogram of the distribution of IBNR



The distribution of total RBNS+IBNR reserves by calendar years and rows using bootstrapping

period	total	mean.total	sd.total	Q1.total	Q5.total	Q50.total	Q95.total	Q99.total
1	34953817.95	32569542.51	21829695.1	3201777.03	6273082.52	27639943.20	68094765.73	100666514.81
2	18132425.44	17305469.98	15667458.6	221629.21	1016646.99	13377496.11	45213841.73	69628863.93
3	11221830.60	10214092.22	12461369.7	40261.27	219572.96	5440510.66	34779521.75	53114828.14
4	5643284.53	6706619.48	9524411.8	8.62	424.82	2526293.89	26722785.91	38446694.59
5	3736566.73	4035831.49	8880742.1	0.60	33.27	555771.22	20599163.67	39580295.54
6	1163432.40	682134.67	2188590.9	0.00	0.00	126.21	4863438.29	11149601.87
7	129006.74	101510.89	668064.4	0.00	0.00	0.00	45873.87	3985549.55
8	30102.14	21372.57	301472.8	0.00	0.00	0.00	0.00	189.34
9	9496.02	0.00	0.0	0.00	0.00	0.00	0.00	0.00
10	3587.12	0.00	0.0	0.00	0.00	0.00	0.00	0.00
11	697.88	0.00	0.0	0.00	0.00	0.00	0.00	0.00
12	0.00	0.00	0.0	0.00	0.00	0.00	0.00	0.00
13	0.00	0.00	0.0	0.00	0.00	0.00	0.00	0.00
14	0.00	0.00	0.0	0.00	0.00	0.00	0.00	0.00
15	0.00	0.00	0.0	0.00	0.00	0.00	0.00	0.00
16	0.00	0.00	0.0	0.00	0.00	0.00	0.00	0.00
17	0.00	0.00	0.0	0.00	0.00	0.00	0.00	0.00
18	0.00	0.00	0.0	0.00	0.00	0.00	0.00	0.00
Tot.	75024247.55	71636573.80	32971138.8	14601380.73	26381296.16	67804786.40	136469317.60	156163900.12

The boxplot of the distribution of the outstanding liabilities in the future calendar periods and the histogram of the distribution of the total reserve RBNS+IBNR



IV. CONCLUSIONS

The Double Chain Ladder Method like the Standard Chain Ladder Method doesn't require distributional assumptions. Using DCL we can estimate the RBNS, IBNR separately and also the tail. There are two possibilities to estimate RBNS. The first, uses the true observed values, the second one ignoring the tail arrives to the standard chain ladder estimate. The results between DCL and CL are closed each to other, DCL result is around 75 million Albanian Lek, the CL result is around 76 million Albanian Lek (1 Eur=140 ALL), this means that there is no much difference between N_{ij} and $\hat{N}_{ij}$.

The first moment formulation is the more appropriate only for the best estimate. To provide cash flows we used a parametric bootstrapping.

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