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# Fixed Point Theorems In Fuzzy Menger Cone Metric Spaces

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***Abstract*—We define Fuzzy Menger Cone metric space and find some fixed point results for weak contraction condition. In support an example is furnished.**

***Keywords*—*Fuzzy Probabilistic Cone Metric Space, Cauchy Sequence, Fixed point.***

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## 1. INTRODUCTION

There have been a number of generalizations of metric space. One such generalization is Menger space introduced in 1942 by Menger [3] who used distribution functions instead of nonnegative real numbers as values of the metric, the notion of probabilistic metric space correspond to situations when we do not know exactly the distance between the two points but we know probabilities of possible values of this distance. A probabilistic generalization of metric spaces appears to be interest in the investigation of physical quantities and physiological threshold. It is also a fundamental importance in probabilistic functional analysis. Schweizer and Sklar [5] studied this concept and then the important development of Menger space theory was due to Sehgal and Bharucha-Reid [6]. The development of fixed point theory in PM-spaces was due to Schweizer and Sklar [4, 5].

In [1] Huang and Zhang generalized the concept of metric spaces, replacing the set of real numbers by an ordered Banach space, hence they have defined the cone metric spaces. They also described the convergence of sequences and introduced the notion of completeness in cone metric spaces. They have proved some fixed point theorems of contractive mappings on complete cone metric space with the assumption of normality of a cone. Subsequently, various authors have generalized the results of Huang and Zhang and have studied fixed point theorems for normal and non-normal cones. There exist a lot of works involving fixed points used the Banach contraction principle. This principle has been extended kind of contraction mappings by various authors [2, 8]. Further Rajesh Shrivastav, Vivek Patel and Vanita Ben Dhagat [7] have given the definition of fuzzy probabilistic metric space and proved fixed point theorem for such space.

## 2. PRELIMINARY

**Definition 2.1:** Let (E, τ) be a topological vector space and P a subset of E, P is called a cone if

1. P is non-empty and closed, $P \neq \{0\}$,

2. For $x, y \in P$ and $a, b \in R \Rightarrow ax + by \in P$ where $a, b \geq 0$

3. If $x \in P$ and $-x \in P \Rightarrow x = 0$

For a given cone $P \subseteq E$, a partial ordering $\geq$ with respect to P is defined by $x \geq y$ if and only if $x - y \in P$,

$x > y$ if $x \geq y$ and $x \neq y$,

while $x \gg y$ will stand for $x - y \in \text{int } P$, int P denotes the interior of P.

**Definition 2.2:** A fuzzy probabilistic metric space (FPM space) is an ordered pair $(X, F_\alpha)$ consisting of a nonempty set X and a mapping $F_\alpha$ from XxX into the collections of all fuzzy distribution functions $F_\alpha \in R$ for all $\alpha \in [0,1]$. For x, y $\in$ X we denote the fuzzy distribution function $F_\alpha$ (x, y) by $F_{\alpha(x,y)}$ and $F_{\alpha(x,y)}(u)$ is he value of $F_{\alpha(x,y)}$ at u in R.
The functions $F_{\alpha(x,y)}$ for all $\alpha \in [0,1]$ assumed to satisfy the following conditions:
(a) $F_{\alpha(x,y)}(u) = 1 \ \forall\, u > 0$ iff x = y,
(b) $F_{\alpha(x,y)}(0) = 0 \ \forall$ x , y in X,
(c) $F_{\alpha(x,y)} = F_{\alpha(y,x)} \ \forall$ x , y in X,
(d) If $F_{\alpha(x,y)}(u) = 1$ and $F_{\alpha(y,z)}(v) = 1 \Rightarrow F_{\alpha(x,z)}(u+v) = 1$ $\forall$ x, y, z $\in$ X and u, v > 0.

**Definition 2.3:** A commutative, associative and non-decreasing mapping t: $[0,1] \times [0,1] \to [0,1]$ is a t-norm if and only if
$t(a,1) = a \ \forall\, a \in [0,1]$, $t(0,0) = 0$
and $t(c,d) \geq t(a,b)$ for $c \geq a$, $d \geq b$. .

**Definition 2.4:** A Fuzzy Menger space is a triplet $(X,F_\alpha,t)$, where $(X, F_\alpha)$ is a FPM-space, t is a t-norm and the generalized triangle inequality
$F_{\alpha(x,z)}(u+v) \geq t\,(F_{\alpha(x,z)}(u), F_{\alpha(y,z)}(v))$
holds for all x, y, z in X u, v > 0 and $\alpha \in [0,1]$.

The concept of neighbourhoods in Fuzzy Menger space is introduced as

**Definition 2.5:** Let $(X,F_\alpha,t)$ be a Fuzzy Menger space. If $x \in X$, $\varepsilon > 0$ and $\lambda \in (0,1)$, then $(\varepsilon,\lambda)$ - neighbourhood of x, called $U_x(\varepsilon,\lambda)$, is defined by

$U_x(\varepsilon,\lambda) = \{y\in X: F_{\alpha(x,y)}(\varepsilon) > (1-\lambda)\}$. An $(\varepsilon,\lambda)$-topology in X is the topology induced by the family $\{U_x(\varepsilon,\lambda): x \in X, \varepsilon > 0, \alpha \in [0,1]$ and $\lambda \in (0,1)\}$ of neighbourhood.

**Remark:** If t is continuous, then Fuzzy Menger space $(X,F_\alpha,t)$ is a Housdorff space in $(\varepsilon, \lambda)$-topology.

Let $(X,F_\alpha,t)$ be a complete Fuzzy Menger space and $A\subset X$. Then A is called a bounded set if $\lim_{u\to\infty} \inf_{x,y\in A} F_{\alpha(x,y)}(u) = 1$

**Definition 2.6:** A sequence $\{x_n\}$ in $(X,F_\alpha,t)$ is said to be convergent to a point x in X if for every $\varepsilon > 0$ and $\lambda > 0$, there exists an integer $N = N(\varepsilon,\lambda)$ such that $x_n \in U_x(\varepsilon,\lambda)$ $\forall n \geq N$ or equivalently $F_\alpha(x_n, x; \varepsilon) > 1-\lambda$ for all $n \geq N$ and $\alpha\in[0,1]$.

**Definition 2.7:** A sequence $\{x_n\}$ in $(X,F_\alpha,t)$ is said to be Cauchy sequence if for every $\varepsilon > 0$ and $\lambda > 0$, there exists an integer $N = N(\varepsilon,\lambda)$ such that for all $\alpha\in[0,1]$ $F_\alpha(x_n,x_m; \varepsilon) > 1-\lambda\ \forall n, m \geq N$.

**Definition 2.8:** A Fuzzy Menger space $(X,F_\alpha,t)$ with the continuous t-norm is said to be complete if every Cauchy sequence in X converges to a point in X for all $\alpha\in[0,1]$.

**Lemma 2.1:** Let $\{x_n\}$ be a sequence in a Fuzzy Menger space $(X,F_\alpha,t)$ with continuous t-norm $*$ and $t * t \geq t$. If there exists a constant $k \in (0,1)$ such that

$F_{\alpha(x_n,x_{n+1})}(kt) \geq F_{\alpha(x_{n-1},x_n)}(t)$ for all $t > 0$, $\alpha\in[0,1]$. and $n = 1, 2, \ldots,$ then $\{x_n\}$ is a Cauchy sequence in X.

**Lemma 2.2:** Let $(X, F_\alpha, t)$ be a Fuzzy Menger space. If there exists $k \in (0, 1)$ such that

$F_{\alpha(x,y)}(kt) \geq F_{\alpha(x,y)}(t)$ for all $x, y \in X$, for all $\alpha\in[0,1]$ and $t > 0$, then $x = y$.

**Definition 2.9:** Let M be a nonempty set and the mapping $d: M \to X$ and $P \subset X$ be a cone, satisfies the following conditions:

$$2.9.1)\ F_{\alpha(x,y)}(u) > 1\ \forall\ x, y\in X \Leftrightarrow x = y$$

$$2.9.2)\ F_{\alpha x,y}(u) = F_{\alpha y,x}(u)\ \ \forall x, y\in X,$$

$$2.9.3)\ F_{\alpha x,y}(u+v) \geq t(F_{\alpha x,z}(u)\ ,\ F_{\alpha z,y}(v))\ \forall x, y\in X.$$

2.9.4) For any $x, y\in X, (x, y)$

is non-increasing and left continuous.

## 3. MAIN RESULTS

**Theorem 3.1**: Let (X, d) be a complete Menger Cone Metric space and P a normal cone with normal constant K. Suppose M be a nonempty separable closed subset of Menger cone metric space X and let T and S be commuting mapping defined on M satisfying the contraction

$\|F_{\alpha(Tx,Ty)}(qu)\| \geq t\,\|F_{\alpha(Sx,\ Sy)}(u)\|$ for all $x, y \in X$ . …………… (3.1.1)

u > 0 and 0 < q < 1. And range of S contains range of T and if S is continuous, then T have unique common fixed point in X.

**Proof:** For each $x_0 \in X$ and $x_1 \in X$ considered such that

$y_0 = Tx_0 = Sx_1$. Therefore in general, $y_n = Tx_n = Sx_{n+1}$

$\|F_{\alpha(y_n,\ y_{n-1})}(qu)\| = t\|F_{\alpha(Tx_n,\ Tx_{n-1})}(u)\|$

$\geq t\|F_{\alpha(Sx_n,\ Sx_{n-1})}(u)\| = \phi\ \|F_{\alpha(y_{n-1},\ y_{n-2})}(u)\|$

$\Rightarrow \|F_{\alpha(y_n,\ y_{n-1})}(qu)\| \geq t\ \|F_{\alpha(y_{n-1},\ y_{n-2})}(u)\|$

$\geq t\ \|F_{\alpha(y_{n-2},\ y_{n-3})}(u)\| \leq t\ \|F_{\alpha(y_{n-3},\ y_{n-4})}(u)\|$

……………………………………………………………

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$\geq t\ \|F_{\alpha(y_1,\ y_0)}(u)\|$

Now for n > m

$\|F_{\alpha(y_n,\ y_m)}(qu)\| \geq t(\|F_{\alpha(y_n,\ y_{n-1})}(u_1)\|)\ .\ \|F_{\alpha(y_{n-1},\ y_{n-2})}(u_2)\|.\|F_{\alpha(y_{n-2},y_{n-3})}(u_3)\|$……

……………. $\|F_{\alpha(y_{m+1},\ y_m)}(u_{n-m})\|)$

Since P is normal cone

$\|F_{\alpha(y_n,\ y_m)}(qu)\| \geq [\|t(F_{\alpha(y_n,\ y_{n-1})}(u_1))\ .\ (F_{\alpha(y_{n-1},\ y_{n-2})}(u_2)).(F_{\alpha(y_{n-2},\ y_{n-3})}(u_3))$……..

…….. $(F_{\alpha(y_{m+1},\ y_m)}(u_{n-m}))\|]$

$\|F_{\alpha(y_n,\ y_m)}(qu)\| \geq [t(\|F_{\alpha(y_n,\ y_{n-1})}(u_1)\|).t(\|F_{\alpha(y_{n-1},\ y_{n-2})}(u_2)\|).t(\|F_{\alpha(y_{n-2},\ y_{n-3})}(u_3)\|)$…… …….$t(\|F_{\alpha(y_{m+1},\ y_m)}(u_{n-m})\|)]$

$\|F_{\alpha(y_n,\ y_m)}(qu)\| \geq t(\|F_{\alpha(y_1,\ y_0)}(u)\|)$

$\|F_{\alpha(y_n,\ y_m)}(qu)\| \geq \|F_{\alpha(y_1,\ y_0)}(u)\|$

$\Rightarrow \|F_{\alpha(y_n,\ y_m)}(u)\| \to 0$ as $m \to \infty$

Therefore sequences $\{y_n\} = \{Tx_n\} = \{Sx_{n+1}\}$ is Cauchy sequence and X in complete therefore there exist p in X such that

$$\lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sx_{n+1} = p$$

Now S is continuous and T and S are commuting mappings, we get

$$Sp = S\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} S^2x_n$$

$$Sp = S\lim_{n\to\infty} Tx_n = \lim_{n\to\infty} STx_n = \lim_{n\to\infty} TSx_n$$

Now from (3.1.1) we have

$\|F_{\alpha(TSx_n,Sp)}(qu)\| \geq t(\|F_{\alpha(S^2x_n,Sp)}(u)\|)$

On taking $n\to\infty$, we get

$\|F_{\alpha(Sp,Tp)}(qu)\| \geq t(\|F_{\alpha(Sp,Sp)}(u)\|)$

Since 0< q<1,

$\|F_{\alpha(Sp,Tp)}(u)\| = 0 \Rightarrow Sp = Tp$

Again from (3.1.1) we have

$\|TF_{\alpha(x_n,Tp)}(qu)\| \geq t(\|F_{\alpha(Sx_n,Sp)}(u)\|)$

$\|F_{\alpha(p,Tp)}(qu)\| \geq t(\|F_{\alpha(p,Sp)}(u)\|) = \|F_{\alpha(p,Tp)}(u)\|$

$\Rightarrow Tp = p.$

$\Rightarrow Sp = Tp = p.$

For uniqueness let there exists another fixed point q in X such that from (3.1.1)

$\|F_{\alpha(p,\ q)}(qu)\| = \|F_{\alpha(Tp,\ Tq)}(u)\| \geq \mathbf{t}\|F_{\alpha(Sp,\ Sq)}(u)\| = \|F_{\alpha(p,\ q)}(u)\|$

Hence for all 0< **q**<1 we have p = q.

**Theorem 3.2:** Let (X, d) be a complete Fuzzy Menger Cone metric space and P a normal cone with normal constant K. Suppose M be a nonempty separable closed subset of cone metric space X and let T and S be commuting operators defined on M satisfying contraction

$\|F_{\alpha(Tx,Ty)}(qu)\| \geq t(\|F_{\alpha(Sx,Sy)}(u)\|)$
$\forall$ x, y $\in$ and $0 < q < ½$ …………… (3.2.1)
$\|F_{\alpha(Tx,Ty)}(qu)\| \geq t(\|F_{\alpha(Tx,Sx)}(u)\|).t(\|F_{\alpha(Ty,Sy)}(u)\|)$
for all x, y $\in$ X and $0 < q < ½$…………… (3.2.2)
$\|F_{\alpha(Tx,Ty)}(qu)\| \geq t(\|F_{\alpha(Tx,Sy)}(u))\|).t(\|F_{\alpha(Ty,Sx)}(u)\|)$
for all x, y $\in$ X and $0 < q < ½$…………… (3.2.3)
and range of S contains range of T and if SX is continuous, then T and S have unique point of coincidence. If T and S weakly compatible, S and T have unique common fixed point in X.

**Proof:-** For each $x_0 \in X$ and $x_1 \in X$ considered such that $y_0 = T x_0 = Sx_1$.
Therefore in general, $y_n = Tx_n = Sx_{n+1}$
As per theorem 3.1 and for all the cases (3.2.1), (3.2.2), (3.2.3) we have
$\Rightarrow \|F_{\alpha(yn,yn-1)}(qu)\| \geq t\,\|F_{\alpha(yn-1,yn-2)}(u)\|)$ …….. (3.2.4)
Indeed by (3.2.1) it follows that
$\|F_{\alpha(yn,yn-1)}(qu)\| = \|F_{\alpha(Sxn+1,Sxn)}(qu)\|$
$= \|(F_{\alpha(Txn,Txn-1)}(qu)\|$
$\geq t(\|F_{\alpha(Sxn,Sxn-1)}(u)\|)$
$= t(\|F_{\alpha(yn-1,yn-2)}(u)\|)$.
Indeed by (3.2.2) it follows that
$\|F_{\alpha(yn,yn-1)}(qu)\| = \|(F_{\alpha(Sxn+1,Sxn)}(qu)\| = \|F_{\alpha(Txn,Txn-1)(qu)}\|$
$\geq t[\|F_{\alpha(Txn,Sxn)}(u_1)\| . \|F_{\alpha(Txn-1,Syn-1)}(u_2)\|]$
$\geq t[\|F_{\alpha(y_n,y_{n-1})}(u_1)\| . F_{\alpha(y_{n-1},y_{n-2})}(u_2)]$
$\|F_{\alpha(yn,yn-1)}(qu)\| \geq t((F_{\alpha(yn-1,yn-2)}(u))$
Indeed by (3.2.3) it follows that
$\|F_{\alpha(yn,yn-1)}(qu)\| = \|(F_{\alpha(Txn,Txn-1)}(qu)\|$
$\geq t[\|F_{\alpha(Txn,Sxn-1)}(u_1)\| . \|F_{\alpha(Txn-1,Syn)}(u_2)\|]$
$\geq t[\|F_{\alpha(y_n,y_{n-2})}(u_1)\| . t(\|F_{\alpha(y_{n-1},y_{n-1})}(u_2)\|]$
$\geq t[\|F_{\alpha(y_n,y_{n-1})}(u_1)\| . t(\|F_{\alpha(y_{n-1},y_{n-2})}(u_2)\|)]$
$\|F_{\alpha(yn,yn-1)}(qu)\| \geq t(\|F_{\alpha(yn-1,yn-2)}(u)\|)|$
Now, by (3.2.4) for all cases we get
$\|F_{\alpha(yn,yn-1)}(qu)\| \geq t(\|F_{\alpha(yn-1,yn-2)}(u)\|)$
$\geq t(\|F_{\alpha(yn-2,yn-3)}(u)\|)$
$\geq t(\|F_{\alpha(yn-3,yn-4)}(u)\|)$
…………………………………………………………………
………………..
$\geq t(\|F_{\alpha(y1,y0)}(u)\|)$
Now for n > m
$\|F_{\alpha(yn,ym)}(qu)\| \geq t(F_{\alpha(yn,yn-1)}(u_1) . F_{\alpha(yn-1,yn-2)}(u_2) . F_{\alpha(y_{n-2},y_{n-3})}(u_3)$ ……….
…….. $\|F_{\alpha(y_{m+1},y_m)}(u_{n-m})\|$
Since P is normal cone
$\|F_{\alpha(yn,ym)}(qu)\| \geq t[\|F_{\alpha(y_n,y_{n-1})}(u_1) . F_{\alpha(y_{n-1},y_{n-2})}(u_2) . F_{\alpha(y_{n-2},y_{n-3})}(u_3)$ ….
$F_{\alpha(y_{m+1},y_m)}(u_{n-m})\|]$
$\|F_{\alpha(y_n,y_m)}(qu)\| \geq [t(\|F_{\alpha(y_n,y_{n-1})}(u_1)\|)$.
$t(\|F_{\alpha(y_{n-1},y_{n-2})}(u_2)\|) . t(\|F_{\alpha(y_{n-2},y_{n-3})}(u_3)\|)$
…….$t(\|F_{\alpha(y_{m+1},y_m)}(u_{n-m})\|)]$
$\|F_{\alpha(yn,ym)}(qu)\| \geq t(\|F_{\alpha(y1,y0)}(u)\|)$
$\|F_{\alpha(yn,ym)}(qu)\| \geq \|F_{\alpha(y1,y0)}(u)\|$

$$\Rightarrow \|F_{\alpha(yn,ym)}(u)\| \rightarrow 0 \text{ as } m \rightarrow \infty$$

Therefore sequences $\{y_n\} = \{Tx_n\} = \{Sx_{n+1}\}$ is Cauchy sequence and S(X) is complete therefore there exist p in X such that Sp = z. Now we will show that for all cases T(p) = z.
From 3.2.1
$\|F_{\alpha(Sxn,Tp)}(qu)\| = \|F_{\alpha(Txn-1,Tp)}(qu)\|$
$\geq t(\|F_{\alpha(Sxn-1,Sp)}(u)\|)$
By taking $n \rightarrow \infty$, we get
$\Rightarrow \|F_{\alpha(Sp,Tp)}(qu)\| \geq t(\|F_{\alpha(Sp,Sp)}(u)\|) = 0$.
$\Rightarrow \|F_{\alpha(Sp,Tp)}(u)\| = 0$. Hence Sp = Tp.
Now for unique coincidence let us consider another point of coincidence $p_1$ in X such that $Tp_1 = Sp_1 = z_1$. Now
$\|F_{\alpha(Sp1,Sp)}(qu)\| = \|F_{\alpha(Tp1,Tp)}(qu)\|$
$\geq t(\|F_{\alpha(Sp1,Sp)}(u)\|)$.
$\Rightarrow \|F_{\alpha(Sp1,Sp)}(u)\| = 0$.
Hence $Sp_1 = Sp = Tp = Tp_1$.
Now, from 3.2.2 it follows
$\|F_{\alpha(Sxn,Tp)}(qu)\| = \|F_{\alpha(Txn-1,Tp)}(qu)\|$
$\geq t[\|F_{\alpha(Txn-1,Sxn-1)}(u_1)\| . \|F_{\alpha(Sp,Tp)}(u_2))\|$.
$\Rightarrow \|F_{\alpha(Sp,Tp)}(qu)\| \geq t(\|F_{\alpha(Sp,Sp)}(u_1)\|).t(\|F_{\alpha(Tp,Sp)}(u_2)\|)$
$= \|F_{\alpha(Tp,Sp)}(u)\|$.
$\Rightarrow$ Tp = Sp.
Again for uniqueness let us consider another point of coincidence $p_1$ in X such that $Tp_1 = Sp_1 = z_1$. Now
$\|F_{\alpha(Sp1,Sp)}(qu)\| = \|F_{\alpha(Tp1,Tp)}(qu)\|$
$\geq t[\|F_{\alpha(Tp1,Sp1)}(u_1)\| . \|F_{\alpha(Tp,Sp)}(u_2)\|]$.
$\Rightarrow \|F_{\alpha(Sp1,Sp)}(u)\| = 0$.
Hence $Sp_1 = Sp = Tp = Tp_1$.
Again from (3.2.3)
$\|F_{\alpha(Sxn,Tp)}(qu)\| = \|F_{\alpha(Txn-1,Tp)}(qu)\|$
$\geq t[\|F_{\alpha(Txn-1,Sp)}(u_1)\| . \|F_{\alpha(Tp,Sxn-1)}(u_2)\|]$.
By taking $n \rightarrow \infty$, we get
$\|F_{\alpha(Sp,Tp)}(qu)\|$
$\geq t[\|F_{\alpha(Tp,Sp)}(u_1)\| . \|F_{\alpha(Tp,Sp)}(u_2)\|]$ .
$\|F_{\alpha(Sp,Tp)}(qu)\| \geq t(\|F_{\alpha(Tp,Sp)}(u)\|)$
Since $0 < q < 1/2$
Therefore $\|F_{\alpha(Sp,Tp)}(u)\| = 0$.
Hence Sp = Tp.

For uniqueness let us consider another point of coincidence $p_1$in X such that $Tp_1 = Sp_1 = q_1$. Now
$\|F_{\alpha(Sp1Sp)}(qu)\| = \|F_{\alpha(Tp1,Tp)}(qu)\|$
$\geq t[\|F_{\alpha(Tp1,Sp)}(u_1)\| . \|F_{\alpha(Tp,Sp1)}(u_2)\|]$
$= t[\|F_{\alpha(Sp1,Sp)}(u)\| . \|F_{\alpha(Sp,Sp1)}(u)\|]$
$\Rightarrow \|F_{\alpha(Sp1,Sp)}(qu)\| \geq t(\|F_{\alpha(Sp1,Sp)}(u)\|)$.
Since $0 < \lambda < 1/2$ therefore $\|FSp_1,Sp(u)\| = 0$. Hence $Sp_1 = Sp = Tp = Tp_1$.
By the use of weak compatibility of F and S and for $\alpha \in [0,1]$ in all above cases we can find that p is unique common fixed point of T and S.

## 4. EXAMPLE

Let M = R and P = $\{x \in M: x \geq 0\}$. Let X = $[0,\infty)$ and define mapping as d:XxX $\rightarrow$ M by $F_{\alpha(x,y)}(u) = \alpha|x-y|$. Then $(X,F_\alpha,t)$ is a fuzzy Menger cone metric space. Define operator T from X to X as T(x) = x/2. Also sequence of mapping $x_n$ : X$\rightarrow$X is defined by

$x_n = \{1+1/n\}$ for every n $\in$ N.

T Satisfies all condition of the theorem 3.1 and hence 1 is fixed point of the space.

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