Critical Velocity Of Fluid-Conveying Pipes Resting On Winkler Foundation With General Boundary Conditions

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Abstract— **The problem of critical velocity of fluid-conveying pipes resting on a Winkler foundation with elastically supported of the pipe is studied in this paper. The aim of this work is deriving a new analytical model to perform a general study to investigate the dynamic behavior of a pipe under general boundary conditions by considering the supports as compliant material with linear and rotational springs, and study the effect of foundation and stiffness values on the critical velocity.** ***This model describes both the classical (simply support, free, built, guide) and the restrained boundary condition and it is not required to derive a new critical velocity equation if the boundary conditions is changed ,also the result will be near to reality by knowing the physical parameters for the compliant material and the pipe. The cases studied in this work are flexible support of the ends pipe with or without intermediate restrain support, and pipe resting on Winkler foundation. . The general theoretical conclusions are that the supports and foundation values have significant effects on the dynamic characteristics of the pipe.***

Keywords— Pipe conveying fluid, Critical velocity, Elastic support, Winkler foundation.

I. Introduction .

The study of the dynamic behavior of a fluid conveying pipe, started in 1950, despite the great importance of this subject in pumps, heat exchanger, discharge lines, marines risers, etc.., the first observation of this phenomena was made by Ashley and Havilland [1], when examining the above ground Trans-Arabian oil pipe line. They considered the problem as a simply supported pipe. Housner[2] used an approximate power series for solving the governing equation but neglected the effect of internal pressure. Noirdson[3] studied the stability for straight pipe simply supported with steady flow and he reached to the same conclusions which were made by the above authors, that the natural frequency for a system reduces with the increase in the fluid flow velocity, and the system losses stability by buckling. Long [4] obtained the solution of the equation of motion using power series approximation. He studied the problem of fixed-free ended pipes as well for simply supported pipe. The vibration of simply and clamped ends infinitely pipe, also including the effect pressure was studied by Change and Shia [5]. The effect of fluid friction on the dynamics of the pipe was considered for the first time by Benjamin [6], he also studied the instability of a cantilever pipe both theoretically and experimentally. A theoretical and experimental investigation in to the instability of cantilever pipe was performed by Gregory and Paidoussis[7] and concluded, as did Benjamin, that the cantilever pipe loses its stability at a certain flow velocity by flutter rather than buckling. Doare and Langre [8] studied instability of fluid conveying pipes on Winkler type foundation. The focus in their paper was on instability of infinitely long fluid conveying pipes using wave propagation approach, wherein results are interpreted in terms of static neutrality as criteria for pinned–pinned, clamped– clamped ends and dynamic neutrality for clamped–free ends. Lilkova-Markova and Lolov [9] investigated the influence of the transverse force at the free end of the dynamic stability of a cantilevered pipe placed on a Winkler elastic foundation. Lottati and Kornecki [10] found that the critical flow velocity of a fluid conveying pipe on Winkler foundation is higher than the critical flow velocity of that pipe without foundation. In this manner, the Winkler foundation is proved to have a stabilizing effect on the pipe. Hauger and Vetter [11] studied the dynamic stability of axially compressed rods on variable elastic foundations. They found that some kinds of foundations have stabilizing effect on the rod (the critical load is increased), while the other ones can destabilize the rod (the critical load is decreased). Chen [12] investigated the steady flow and pulsating flow for simply supported pipe conveying fluid, and used Hsu's Bolotin's approximate method to find the stability zones for pulsating flow. Formatter will need to create these components, incorporating the applicable criteria that follow.

II. Theoretical approach

A. Equation of motion.

Consider a straight uniform pipe conveying fluid of length L. The following assumptions are considered in the analysis of the system under consideration [13]:

1. Neglecting the effect of gravity.

2. The pipe considered to be horizontal.

3. Neglecting the material damping.

4. The pipe is inextensible.

5. Neglecting the shear deformation and rotary inertia.

6. All motion considered small.

7. Neglecting the velocity distribution through the cross- section of the pipe.

Derivation of the equation of motion for straight pipe with steady flow are available in the literature Ref.[14].For a single-span pipe conveying fluid, the equation based on beam theory is given by,

$$EI\frac{\partial^4 y}{\partial x^4}+\left(M_f U^2+pA\right)\frac{\partial^2 y}{\partial x^2}+2M_f U\frac{\partial^2 y}{\partial x\partial t}+\left(M_f+M_p\right)\frac{\partial^2 y}{\partial t^2}=0 \quad (1)$$

where $F(x,t)$: is the external harmonic force being applied normally to the pipe axis in the y- direction.

$EI\frac{\partial^4 y}{\partial x^4}$: Stiffness term

$\left(M_f U^2+pA\right)\frac{\partial^2 y}{\partial x^2}$: Curvature term

$2M_f U\frac{\partial^2 y}{\partial x\partial t}$: Coriolis force term

$\left(M_f+M_p\right)\frac{\partial^2 y}{\partial t^2}$: Inertia force term

The Coriolis force is a result of the rotation of the system element due to the system lateral motion, since each point in the span rotates with angular velocity **[15]**.

The equation of motion Eq.(1) can be written in the following non-dimensional form:

$$\boldsymbol{Y^{iv}+(u^2+\gamma)Y''+2\beta U\dot{Y}'+\ddot{Y}=0} \quad (2)$$

Where,

$X=x/L$

$Y=y/L$

$Y^{iv}=\frac{\partial^4 y}{\partial x^4}$, $Y''=\frac{\partial^2 y}{\partial x^2}$, $\dot{Y}'=\frac{\partial^2 y}{\partial x\partial t}$, $\ddot{Y}=\frac{\partial^2 y}{\partial t^2}$,

$\tau=\sqrt{EI/(M_f+M_p)}(\frac{t}{L^2})$.

$\beta=\sqrt{M_f/(M_f+M_p)}$, $u=UL\sqrt{(M_f/EI)}$

$\gamma=(\frac{pA_p}{EI})L^2$

β: Non-dimensional mass ratio.

γ: Non-dimensional fluid pressure.

The natural frequencies for steady flow decrease with increasing the fluid flow velocity. If the natural frequencies of the pipe reach to zero, the flow velocity in this case is called critical flow velocity. When the flow velocity is equal to the critical velocity the pipe bows out and buckles, because the forces required to make the fluid deform to the pipe curvature are greater than the stiffness of the pipe.

Therefore the mechanism underlying instability may be illustrated by static method **[6]**, so deleting time dependent term from equation (2) yields:

$$Y^{iv}+(u^2+\gamma)Y''=0 \quad (3)$$

Whose its solution takes the form:

$$Y(X)=C_1+C_2X+C_3\sin(\alpha X)+\ C_4\cos(\alpha X)$$

Where $\alpha=\sqrt{\gamma+u_{cr}{}^2}$ and $C_1, C_2, C_3, C_4,$ are constant and can be evaluated by using the boundary conditions.

In order to investigate the influence of the Winkler foundation on the free vibration characteristics of pipe conveying fluid, we assumed that the pipe is divided to twenty spans, and intermediate linear springs are at ends of the spans as shown in Figure(1). The method of continuity of boundary condition (equilibrium) is used in order to account for different support locations.

To solve the problem of free vibration for a structure, first its boundary conditions must be known. The method of solution consists of formulating the support condition of ends pipe in terms of the compliant boundary material. The parameter of this material will be represented by linear and rotational spring. To describe the classical boundary conditions impedance values are taken to be zero or infinity values.

B. Evaluating Critical Velocity.

Assumed that the pipe is divided to twenty spans, and intermediate linear springs are at ends of the spans as shown in Figure (1), where t_i and r_i are the translational and rotational spring constant, it will be considered that the pipe is supported at the two end points, and c represent the distance between springs at along the pipe.

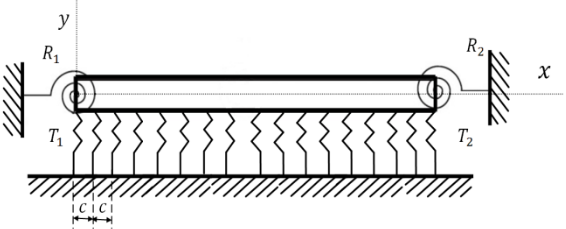


Figure (1) Pipe conveying fluid resting on Winkler foundation.

The method of continuity of boundary condition (equilibrium) is used in order to account for different support locations as following:

1- *at x=0,* $r_1 l y'(0,\tau)=EIy''(0,\tau)$

$r_1 l[C_2+C_3\alpha]=-EIC_4\alpha^2$

$R_1C_2+R_1\alpha C_3+C_4\alpha^2=0$

Where $R_1=(r_1 l/EI)$: dimensionless rotational stiffness at *x*=0.

2-*at x=0,* $t_1 y(0,\tau)=-EIy'''(0,\tau)$

$t_1 l^3[C_1+C_4]=-EI[-C_3\alpha^3]$

$T_1C_{1-}\alpha^3C_3+T_1C_4=0.$

Where $T_1=(t_1 l^3/EI)$: dimensionless longitudinal stiffness at *x*=0

3-*at* *x=c,* $r_c\frac{\partial y(c,t)}{\partial x}=EI\left(-\frac{\partial^2 y(c^-,t)}{\partial x^2}+\frac{\partial^2 y(c^+,t)}{\partial x^2}\right)$, $r_c=0$

$$-\alpha^2\sin(\alpha c)\,C_3-\alpha^2\cos(\alpha c)\,C_4+\alpha^2\sin(\alpha c)\,C_7+\ \alpha^2\cos(\alpha c)\,C_8\ =0$$

4-*at x=c,* $t_c l^3 y(c,t)=\ EI\left(\frac{\partial^3 y(c^-,t)}{\partial x^3}-\frac{\partial^3 y(c^+,t)}{\partial x^3}\right)$.

$$t_1 l^3[C_1+cC_2+\sin(\alpha c)\,C_3+\cos(\alpha c)\,C_4]=\mathrm{EI}[-\alpha^3\cos(\alpha c)\,C_3+\alpha^3\sin(\alpha c)\,C_4+\alpha^3\cos(\alpha c)\,C_7-\alpha^3\sin(\alpha c)\,C_8=0.$$

$T_c C_1 + cT_c C_2 + [T_c \sin(\alpha c) + \alpha^3 \cos(\alpha c)]C_3 + [T_c \cos(\alpha c) - \alpha^3 \sin(\alpha c)]C_4 - \alpha^3 \cos(\alpha c)\, C_7 + \alpha^3 \sin(\alpha c)\, C_8$=0.

Where $T_c = (t_c l^3/EI)$: dimensionless longitudinal stiffness at *x*=c

5-at *x*=c, $y(c^-,t) = y(c^+,t)$.

$$C_1 + cC_2 + \sin(\alpha c)\, C_3 + \cos(\alpha c)\; C_4 - C_5 - cC_6 - \sin(\alpha c)\, C_7 - \cos(\alpha c)\, C_8 = 0.$$

6- at *x*=c, $\frac{\partial y(c^-,t)}{\partial x} = \frac{\partial y(c^+,t)}{\partial x}$.

$C_2 + \alpha \cos(\alpha c)\, C_3 - \alpha \sin(\alpha c)\, C_4 - C_6 - \alpha \cos(\alpha c)\, C_7 + \alpha \sin(\alpha c)\, C_8 = 0$

7-at *x*=2c,$r_{2c} \frac{\partial y(2c,t)}{\partial x} = EI\left(-\frac{\partial^2 y(2c^-,t)}{\partial x^2} + \frac{\partial^2 y(2c^+,t)}{\partial x^2}\right)$,$r_{2c} = 0$

$-\alpha^2 \sin(\alpha c)\, C_7 - \alpha^2 \cos(\alpha c)\, C_8 + \alpha^2 \sin(\alpha c)\, C_{11} + \alpha^2 \cos(\alpha c)\, C_{12}$

8-at *x*=2c, $t_{2c} l^3 y(c,t) = EI\left(\frac{\partial^3 y(2c^-,t)}{\partial x^3} - \frac{\partial^3 y(2c^+,t)}{\partial x^3}\right)$.

$T_{2c} C_5 + cT_c C_6 + [T_{2c} \sin(\alpha c) + \alpha^3 \cos(\alpha c)]C_7 + [T_{2c} \cos(\alpha c) - \alpha^3 \sin(\alpha c)]C_8 - \alpha^3 \cos(\alpha c)\, C_{11} + \alpha^3 \sin(\alpha c)\, C_{12}$=0.

Where $T_{2c} = (t_{2c} l^3/EI)$: dimensionless longitudinal stiffness at *x*=2c.

9-at *x*=2c, $y(2c^-,t) = y(2c^+,t)$.

$C_5 + 2cC_6 + \sin(2\alpha c)\, C_7 + \cos(2\alpha c)\, C_8 - C_9 - 2cC_{10} - \sin(2\alpha c)\, C_{11} - \cos(2\alpha c)\, C_{12} = 0.$

10- at *x*=2c, $\frac{\partial y(2c^-,t)}{\partial x} = \frac{\partial y(2c^+,t)}{\partial x}$.

$C_6 + \alpha \cos(\alpha c)\, C_7 - \alpha \sin(\alpha c)\, C_8 - C_{10} - \alpha \cos(\alpha c)\, C_{11} + \alpha \sin(\alpha c)\, C_{12} = 0$

11-at *x*=3c,$r_{3c} \frac{\partial y(3c,t)}{\partial x} = EI\left(-\frac{\partial^2 y(3c^-,t)}{\partial x^2} + \frac{\partial^2 y(3c^+,t)}{\partial x^2}\right)$,$r_{3c} = 0$

$-\alpha^2 \sin(3\alpha c)\, C_{11} - \alpha^2 \cos(3\alpha c)\, C_{12} + \alpha^2 \sin(3\alpha c)\, C_{15} + \alpha^2 \cos(3\alpha c)\, C_{16}$

12-at *x*=3c, $t_{3c} l^3 y(3c,t) = EI\left(\frac{\partial^3 y(3c^-,t)}{\partial x^3} - \frac{\partial^3 y(3c^+,t)}{\partial x^3}\right)$.

$T_{3c} C_9 + 3cT_{3c} C_{10} + [T_{3c} \sin(3\alpha c) + \alpha^3 \cos(3\alpha c)]C_{11} + [T_{3c} \cos(3\alpha c) - \alpha^3 \sin(\alpha c)]C_{12} - \alpha^3 \cos(3\alpha c)\, C_{15} + \alpha^3 \sin(3\alpha c)\, C_{16}$=0.

Where $T_{3c} = (t_{3c} l^3/EI)$: dimensionless longitudinal stiffness at *x*=3c

13-at *x*=3c, $y(3c^-,t) = y(3c^+,t)$.

$C_9 + 3cC_{10} + \sin(3\alpha c)\, C_{11} + \cos(3\alpha c)\, C_{12} - C_{13} - 3cC_{14} - \sin(3\alpha c)\, C_{15} - \cos(3\alpha c)\, C_{16} = 0.$

14- at *x*=3c, $\frac{\partial y(3c^-,t)}{\partial x} = \frac{\partial y(3c^+,t)}{\partial x}$.

$C_{10} + \alpha \cos(3\alpha c)\, C_{11} - \alpha \sin(3\alpha c)\, C_{12} - C_{14} - \alpha \cos(3\alpha c)\; C_{15} + \alpha \sin(3\alpha c)\, C_{16} = 0$

15-at *x*=4c, $r_{4c} \frac{\partial y(4c,t)}{\partial x} = EI\left(-\frac{\partial^2 y(4c^-,t)}{\partial x^2} + \frac{\partial^2 y(4c^+,t)}{\partial x^2}\right)$, $r_{4c} = 0$

$-\alpha^2 \sin(4\alpha c)\, C_{15} - \alpha^2 \cos(4\alpha c)\, C_{16} + \alpha^2 \sin(4\alpha c)\, C_{19} + \alpha^2 \cos(4\alpha c)\, C_{20}$

16-at *x*=4c, $t_{4c} l^3 y(4c,t) = EI\left(\frac{\partial^3 y(4c^-,t)}{\partial x^3} - \frac{\partial^3 y(4c^+,t)}{\partial x^3}\right)$.

$T_{4c} C_{13} + 4cT_{4c} C_{14} + [T_{4c} \sin(4\alpha c) + \alpha^3 \cos(4\alpha c)]C_{15} + [T_{4c} \cos(4\alpha c) - \alpha^3 \sin(4\alpha c)]C_{16} - \alpha^3 \cos(4\alpha c)\, C_{19} + \alpha^3 \sin(4\alpha c)\, C_{20}$=0.

Where $T_{4c} = (t_{4c} l^3/EI)$: dimensionless longitudinal stiffness at *x*=4c

17-at *x*=4c, $y(3c^-,t) = y(4c^+,t)$.

$C_{13} + 4cC_{14} + \sin(4\alpha c)\, C_{15} + \cos(4\alpha c)\, C_{16} - C_{17} - cC_{18} - \sin(4\alpha c)\, C_{19} - \cos(4\alpha c)\, C_{20} = 0.$

18- at *x*=4c, $\frac{\partial y(4c^-,t)}{\partial x} = \frac{\partial y(4c^+,t)}{\partial x}$.

$C_{14} + \alpha \cos(4\alpha c)\, C_{15} - \alpha \sin(4\alpha c)\, C_{16} - C_{18} - \alpha \cos(4\alpha c)\, C_{19} + \alpha \sin(4\alpha c)\, C_{20} = 0.$

19-at *x*=5c,$r_{5c} \frac{\partial y(5c,t)}{\partial x} = EI\left(-\frac{\partial^2 y(5c^-,t)}{\partial x^2} + \frac{\partial^2 y(5c^+,t)}{\partial x^2}\right)$,$r_{5c} = 0$

$-\alpha^2 \sin(5\alpha c)\, C_{19} - \alpha^2 \cos(5\alpha c)\, C_{20} + \alpha^2 \sin(5\alpha c)\, C_{23} + \alpha^2 \cos(5\alpha c)\, C_{24}$

20-at *x*=5c, $t_{5c} l^3 y(5c,t) = EI\left(\frac{\partial^3 y(5c^-,t)}{\partial x^3} - \frac{\partial^3 y(5c^+,t)}{\partial x^3}\right)$.

$T_{5c} C_{17} + 5cT_{5c} C_{18} + [T_{5c} \sin(5\alpha c) + \alpha^3 \cos(5\alpha c)]C_{19} + [T_{5c} \cos(5\alpha c) - \alpha^3 \sin(5\alpha c)]C_{20} - \alpha^3 \cos(5\alpha c)\, C_{23} + \alpha^3 \sin(5\alpha c)\, C_{24}$=0.

Where $T_{5c} = (t_{5c} l^3/EI)$: dimensionless longitudinal stiffness at *x*=5c

21-at *x*=5c, $y(5c^-,t) = y(5c^+,t)$.

$C_{17} + 5cC_{18} + \sin(5\alpha c)\, C_{19} + \cos(5\alpha c)\, C_{20} - C_{21} - 5cC_{22} - \sin(5\alpha c)\, C_{23} - \cos(5\alpha c)\, C_{24} = 0.$

22- at *x*=5c, $\frac{\partial y(5c^-,t)}{\partial x} = \frac{\partial y(5c^+,t)}{\partial x}$.

$$C_{18} + \alpha \cos(5\alpha c)\, C_{19} - \alpha \sin(5\alpha c)\;\; C_{20} - C_{22} - \alpha \cos(5\alpha c)\, C_{23} + \alpha \sin(5\alpha c)\, C_{24} = 0.$$

23-at *x*=6c,$r_{6c} \frac{\partial y(6c,t)}{\partial x} = EI\left(-\frac{\partial^2 y(6c^-,t)}{\partial x^2} + \frac{\partial^2 y(6c^+,t)}{\partial x^2}\right)$,$r_{6c} = 0$

$-\alpha^2 \sin(6\alpha c)\, C_{23} - \alpha^2 \cos(6\alpha c)\, C_{24} + \alpha^2 \sin(6\alpha c)\, C_{27} + \alpha^2 \cos(6\alpha c)\, C_{28}$

24-at *x*=6c, $t_{6c} l^3 y(6c,t) = EI\left(\frac{\partial^3 y(6c^-,t)}{\partial x^3} - \frac{\partial^3 y(6c^+,t)}{\partial x^3}\right)$.

$T_{6c} C_{21} + 6cT_{6c} C_{22} + [T_{6c} \sin(6\alpha c) + \alpha^3 \cos(6\alpha c)]C_{23} + [T_{6c} \cos(6\alpha c) - \alpha^3 \sin(6\alpha c)]C_{24} - \alpha^3 \cos(6\alpha c)\, C_{27} + \alpha^3 \sin(6\alpha c)\, C_{28}$=0.

Where $T_{6c} = (t_{6c} l^3/EI)$: dimensionless longitudinal stiffness at *x*=6c

25-at *x*=6c, $y(6c^-,t) = y(6c^+,t)$.

$C_{21} + 6cC_{22} + \sin(6\alpha c)\, C_{23} + \cos(6\alpha c)\, C_{24} - C_{25} - 6cC_{26} - \sin(6\alpha c)\, C_{27} - \cos(6\alpha c)\, C_{28} = 0.$

26- at *x*=6c, $\frac{\partial y(6c^-,t)}{\partial x} = \frac{\partial y(6c^+,t)}{\partial x}$.

$C_{22} + \alpha \cos(6\alpha c)\, C_{23} - \alpha \sin(6\alpha c)\, C_{24} - C_{26} - \alpha \cos(6\alpha c)\, C_{27} + \alpha \sin(6\alpha c)\, C_{28} = 0.$

27-at *x*=7c,$r_{7c} \frac{\partial y(7c,t)}{\partial x} = EI\left(-\frac{\partial^2 y(7c^-,t)}{\partial x^2} + \frac{\partial^2 y(7c^+,t)}{\partial x^2}\right)$,$r_{7c} = 0$

$-\alpha^2 \sin(7\alpha c)\, C_{27} - \alpha^2 \cos(7\alpha c)\, C_{28} + \alpha^2 \sin(7\alpha c)\, C_{31} + \alpha^2 \cos(7\alpha c)\, C_{32}$

28-at *x*=7c, $t_{7c} l^3 y(7c,t) = EI\left(\frac{\partial^3 y(7c^-,t)}{\partial x^3} - \frac{\partial^3 y(7c^+,t)}{\partial x^3}\right)$.

$T_{7c} C_{25} + 7cT_{6c} C_{26} + [T_{7c} \sin(7\alpha c) + \alpha^3 \cos(7\alpha c)]C_{27} + [T_{7c} \cos(7\alpha c) - \alpha^3 \sin(7\alpha c)]C_{28} - \alpha^3 \cos(7\alpha c)\, C_{31} + \alpha^3 \sin(7\alpha c)\, C_{32}$=0.

Where $T_{7c} = (t_{7c} l^3/EI)$: dimensionless longitudinal stiffness at *x*=7c

29-at *x*=7c, $y(7c^-,t) = y(7c^+,t)$.

$C_{25} + 7cC_{26} + \sin(7\alpha c)\, C_{27} + \cos(7\alpha c)\, C_{28} - C_{29} - 7cC_{30} - \sin(7\alpha c)\, C_{31} - \cos(7\alpha c)\, C_{32} = 0.$

30- *at x=7c*, $\frac{\partial y(7c^-,t)}{\partial x}=\frac{\partial y(7c^+,t)}{\partial x}$.

$$C_{26}+\alpha\cos(7\alpha c)\,C_{27}-\alpha\sin(7\alpha c)\,C_{28}-C_{30}-\alpha\cos(7\alpha c)\,C_{31}+\alpha\sin(7\alpha c)\,C_{32}=0$$

.

31-*at* *x=8c*, $r_{8c}\frac{\partial y(8c,t)}{\partial x}=EI\left(-\frac{\partial^2 y(8c^-,t)}{\partial x^2}+\frac{\partial^2 y(8c^+,t)}{\partial x^2}\right)$, $r_{8c}=0$

$-\alpha^2\sin(8\alpha c)\,C_{31}-\alpha^2\cos(8\alpha c)\,C_{32}+\alpha^2\sin(8\alpha c)\,C_{35}+\alpha^2\cos(8\alpha c)\,C_{36}$

32-*at x=8c*, $t_{8c}l^3y(8c,t)=EI\left(\frac{\partial^3 y(8c^-,t)}{\partial x^3}-\frac{\partial^3 y(8c^+,t)}{\partial x^3}\right)$.

$T_{8c}C_{29}+8cT_{8c}C_{30}+[T_{8c}\sin(8\alpha c)+\alpha^3\cos(8\alpha c)]C_{31}+[T_{7c}\cos(8\alpha c)-\alpha^3\sin(8\alpha c)]C_{32}-\alpha^3\cos(8\alpha c)\,C_{35}+\alpha^3\sin(8\alpha c)\,C_{36}$=0.

Where $T_{8c}=(t_{8c}l^3/EI)$: dimensionless longitudinal stiffness at *x*=8c

33-*at x=8c*, $y(8c^-,t)=y(8c^+,t)$.

$C_{29}+8cC_{30}+\sin(8\alpha c)\,C_{31}+\cos(8\alpha c)\,C_{32}-C_{33}-8cC_{34}-\sin(8\alpha c)\,C_{35}-\cos(8\alpha c)\,C_{36}=0$.

34- *at x=8c*, $\frac{\partial y(8c^-,t)}{\partial x}=\frac{\partial y(8c^+,t)}{\partial x}$.

$C_{30}+\alpha\cos(8\alpha c)\,C_{31}-\alpha\sin(8\alpha c)\,C_{32}-C_{34}-\alpha\cos(8\alpha c)\,C_{35}+\alpha\sin(8\alpha c)\,C_{36}=0$.

35-*at* *x=9c*, $r_{9c}\frac{\partial y(9c,t)}{\partial x}=EI\left(-\frac{\partial^2 y(9c^-,t)}{\partial x^2}+\frac{\partial^2 y(9c^+,t)}{\partial x^2}\right)$, $r_{9c}=0$

$-\alpha^2\sin(9\alpha c)\,C_{35}-\alpha^2\cos(9\alpha c)\,C_{36}+\alpha^2\sin(9\alpha c)\,C_{39}+\alpha^2\cos(9\alpha c)\,C_{40}$

36-*at x=9c*, $t_{8c}l^3y(9c,t)=EI\left(\frac{\partial^3 y(9c^-,t)}{\partial x^3}-\frac{\partial^3 y(9c^+,t)}{\partial x^3}\right)$.

$T_{9c}C_{33}+9cT_{9c}C_{34}+[T_{9c}\sin(9\alpha c)+\alpha^3\cos(9\alpha c)]C_{35}+[T_{9c}\cos(9\alpha c)-\alpha^3\sin(9\alpha c)]C_{36}-\alpha^3\cos(9\alpha c)\,C_{39}+\alpha^3\sin(9\alpha c)\,C_{40}$=0.

Where $T_{9c}=(t_{9c}l^3/EI)$: dimensionless longitudinal stiffness at *x*=9c

37-*at x=9c*, $y(9c^-,t)=y(9c^+,t)$.

$C_{33}+9cC_{34}+\sin(9\alpha c)\,C_{35}+\cos(9\alpha c)\,C_{36}-C_{37}-9cC_{38}-\sin(9\alpha c)\,C_{39}-\cos(9\alpha c)\,C_{40}=0$.

38- *at x=9c*, $\frac{\partial y(9c^-,t)}{\partial x}=\frac{\partial y(9c^+,t)}{\partial x}$.

$C_{34}+\alpha\cos(9\alpha c)\,C_{35}-\alpha\sin(9\alpha c)\,C_{36}-C_{38}-\alpha\cos(9\alpha c)\,C_{39}+\alpha\sin(9\alpha c)\,C_{40}=0$.

39-*at* *x=10c*, $r_{10c}\frac{\partial y(10c,t)}{\partial x}=EI\left(-\frac{\partial^2 y(10c^-,t)}{\partial x^2}+\frac{\partial^2 y(10c^+,t)}{\partial x^2}\right)$, $r_{10c}=0$

$-\alpha^2\sin(10\alpha c)\,C_{39}-\alpha^2\cos(10\alpha c)\,C_{40}+\alpha^2\sin(10\alpha c)\,C_{43}+\alpha^2\cos(10\alpha c)\,C_{44}$

40-*at* *x=10c*, $t_{8c}l^3y(10c,t)=EI\left(\frac{\partial^3 y(10c^-,t)}{\partial x^3}-\frac{\partial^3 y(10c^+,t)}{\partial x^3}\right)$.

$T_{10c}C_{37}+10cT_{9c}C_{38}+[T_{10c}\sin(10\alpha c)+\alpha^3\cos(10\alpha c)]C_{39}+[T_{10c}\cos(10\alpha c)-\alpha^3\sin(10\alpha c)]C_{40}-\alpha^3\cos(10\alpha c)\,C_{41}+\alpha^3\sin(10\alpha c)\,C_{44}$=0.

where $T_{10c}=(t_{10c}l^3/EI)$: dimensionless longitudinal stiffness at *x*=10c

41-*at x=10c*, $y(10c^-,t)=y(10c^+,t)$.

$C_{37}+10cC_{38}+\sin(10\alpha c)\,C_{39}+\cos(10\alpha c)\,C_{40}-C_{41}-10cC_{42}-\sin(10\alpha c)\,C_{43}-\cos(10\alpha c)\,C_{44}=0$.

42- *at x=10c*, $\frac{\partial y(10c^-,t)}{\partial x}=\frac{\partial y(10c^+,t)}{\partial x}$.

$C_{38}+\alpha\cos(10\alpha c)\,C_{39}-\alpha\sin(10\alpha c)\,C_{40}-C_{42}-\alpha\cos(10\alpha c)\,C_{43}+\alpha\sin(10\alpha c)\,C_{44}=0$.

43-*at* *x=11c*, $r_{11c}\frac{\partial y(11c,t)}{\partial x}=EI\left(-\frac{\partial^2 y(11c^-,t)}{\partial x^2}+\frac{\partial^2 y(11c^+,t)}{\partial x^2}\right)$, $r_{11c}=0$

$-\alpha^2\sin(11\alpha c)\,C_{43}-\alpha^2\cos(11\alpha c)\,C_{44}+\alpha^2\sin(11\alpha c)\,C_{47}+\alpha^2\cos(11\alpha c)\,C_{48}$

44-*at* *x=11c*, $t_{11c}l^3y(11c,t)=EI\left(\frac{\partial^3 y(11c^-,t)}{\partial x^3}-\frac{\partial^3 y(11c^+,t)}{\partial x^3}\right)$.

$T_{11c}C_{41}+11cT_{9c}C_{42}+[T_{11c}\sin(11\alpha c)+\alpha^3\cos(11\alpha c)]C_{43}+[T_{11c}\cos(11\alpha c)-\alpha^3\sin(11\alpha c)]C_{44}-\alpha^3\cos(11\alpha c)\,C_{45}+\alpha^3\sin(11\alpha c)\,C_{48}$=0.

Where $T_{11c}=(t_{11c}l^3/EI)$: dimensionless longitudinal stiffness at *x*=11c

45-*at x=11c*, $y(11c^-,t)=y(11c^+,t)$.

$C_{41}+11cC_{42}+\sin(11\alpha c)\,C_{43}+\cos(11\alpha c)\,C_{44}-C_{45}-11cC_{46}-\sin(11\alpha c)\,C_{47}-\cos(11\alpha c)\,C_{48}=0$.

46- *at x=11c*, $\frac{\partial y(11c^-,t)}{\partial x}=\frac{\partial y(11c^+,t)}{\partial x}$.

$C_{42}+\alpha\cos(11\alpha c)\,C_{43}-\alpha\sin(11\alpha c)\,C_{44}-C_{46}-\alpha\cos(11\alpha c)\,C_{47}+\alpha\sin(11\alpha c)\,C_{48}=0$.

47-*at* *x=12c*, $r_{12c}\frac{\partial y(12c,t)}{\partial x}=EI\left(-\frac{\partial^2 y(12c^-,t)}{\partial x^2}+\frac{\partial^2 y(12c^+,t)}{\partial x^2}\right)$, $r_{12c}=0$

$-\alpha^2\sin(12\alpha c)\,C_{47}-\alpha^2\cos(12\alpha c)\,C_{48}+\alpha^2\sin(12\alpha c)\,C_{51}+\alpha^2\cos(12\alpha c)\,C_{52}$

48-*at* *x=12c*, $t_{12c}l^3y(12c,t)=EI\left(\frac{\partial^3 y(12c^-,t)}{\partial x^3}-\frac{\partial^3 y(12c^+,t)}{\partial x^3}\right)$.

$T_{12c}C_{45}+12cT_{12c}C_{46}+[T_{12c}\sin(12\alpha c)+\alpha^3\cos(12\alpha c)]C_{47}+[T_{12c}\cos(12\alpha c)-\alpha^3\sin(12\alpha c)]C_{48}-\alpha^3\cos(12\alpha c)\,C_{49}+\alpha^3\sin(12\alpha c)\,C_{52}$=0.

Where $T_{12c}=(t_{12c}l^3/EI)$: dimensionless longitudinal stiffness at *x*=12c

49-*at x=12c*, $y(12c^-,t)=y(12c^+,t)$.

$C_{45}+12cC_{46}+\sin(12\alpha c)\,C_{47}+\cos(12\alpha c)\,C_{48}-C_{49}-12cC_{50}-\sin(12\alpha c)\,C_{51}-\cos(12\alpha c)\,C_{52}=0$.

50- *at x=12c*, $\frac{\partial y(12c^-,t)}{\partial x}=\frac{\partial y(12c^+,t)}{\partial x}$.

$C_{46}+\alpha\cos(12\alpha c)\,C_{47}-\alpha\sin(12\alpha c)\,C_{48}-C_{50}-\alpha\cos(12\alpha c)\,C_{51}+\alpha\sin(12\alpha c)\,C_{52}=0$.

51-*at* *x=13c*, $r_{13c}\frac{\partial y(13c,t)}{\partial x}=EI\left(-\frac{\partial^2 y(13c^-,t)}{\partial x^2}+\frac{\partial^2 y(13c^+,t)}{\partial x^2}\right)$, $r_{13c}=0$

$-\alpha^2\sin(13\alpha c)\,C_{51}-\alpha^2\cos(13\alpha c)\,C_{52}+\alpha^2\sin(13\alpha c)\,C_{55}+\alpha^2\cos(13\alpha c)\,C_{56}$

52-*at* *x=13c*, $t_{13c}l^3y(13c,t)=EI\left(\frac{\partial^3 y(13c^-,t)}{\partial x^3}-\frac{\partial^3 y(13c^+,t)}{\partial x^3}\right)$.

$T_{13c}C_{49}+13cT_{13c}C_{50}+[T_{13c}\sin(13\alpha c)+\alpha^3\cos(13\alpha c)]C_{51}+[T_{13c}\cos(13\alpha c)-\alpha^3\sin(13\alpha c)]C_{52}-\alpha^3\cos(13\alpha c)\,C_{53}+\alpha^3\sin(13\alpha c)\,C_{56}$=0.

Where $T_{13c}=(t_{13c}l^3/EI)$: dimensionless longitudinal stiffness at *x*=13c

53-*at x=13c*, $y(13c^-,t)=y(13c^+,t)$.

$C_{49} + 13cC_{50} + \sin(13\alpha c)\, C_{51} + \cos(13\alpha c)\, C_{52} - C_{53} - 13cC_{54} - \sin(13\alpha c)\, C_{55} - \cos(13\alpha c)\, C_{56} = 0.$

54- *at x=13c,* $\frac{\partial y(13c^-,t)}{\partial x} = \frac{\partial y(13c^+,t)}{\partial x}$.

$C_{50} + \alpha\cos(13\alpha c)\, C_{51} - \alpha\sin(13\alpha c)\, C_{52} - C_{54} - \alpha\cos(13\alpha c)\, C_{55} + \alpha\sin(13\alpha c)\, C_{56} = 0.$

55-*at x=14c,* $r_{14c}\frac{\partial y(14c,t)}{\partial x} = EI\left(-\frac{\partial^2 y(14c^-,t)}{\partial x^2} + \frac{\partial^2 y(14c^+,t)}{\partial x^2}\right)$, $r_{14c} = 0$

$-\alpha^2\sin(14\alpha c)\, C_{55} - \alpha^2\cos(14\alpha c)\, C_{56} + \alpha^2\sin(14\alpha c)\, C_{59} + \alpha^2\cos(14\alpha c)\, C_{60}$

56-*at x=14c,* $t_{14c}l^3 y(14c,t) = EI\left(\frac{\partial^3 y(14c^-,t)}{\partial x^3} - \frac{\partial^3 y(14c^+,t)}{\partial x^3}\right)$.

$T_{14c}C_{53} + 14cT_{14c}C_{54} + [T_{14c}\sin(14\alpha c) + \alpha^3\cos(14\alpha c)]C_{55} + [T_{14c}\cos(14\alpha c) - \alpha^3\sin(14\alpha c)]C_{56} - \alpha^3\cos(14\alpha c)\, C_{57} + \alpha^3\sin(14\alpha c)\, C_{60}$=0.

Where $T_{14c} = (t_{14c}l^3/EI)$: dimensionless longitudinal stiffness at *x*=13c

57-*at x=14c,* $y(14c^-,t) = y(14c^+,t)$.

$C_{53} + 14cC_{54} + \sin(14\alpha c)\, C_{55} + \cos(14\alpha c)\, C_{56} - C_{57} - 14cC_{58} - \sin(14\alpha c)\, C_{59} - \cos(14\alpha c)\, C_{60} = 0.$

58- *at x=14c,* $\frac{\partial y(14c^-,t)}{\partial x} = \frac{\partial y(14c^+,t)}{\partial x}$.

$C_{54} + \alpha\cos(14\alpha c)\, C_{55} - \alpha\sin(14\alpha c)\, C_{56} - C_{58} - \alpha\cos(14\alpha c)\, C_{59} + \alpha\sin(14\alpha c)\, C_{60} = 0.$

59-*at x=15c,* $r_{15c}\frac{\partial y(15c,t)}{\partial x} = EI\left(-\frac{\partial^2 y(15c^-,t)}{\partial x^2} + \frac{\partial^2 y(15c^+,t)}{\partial x^2}\right)$, $r_{15c} = 0$

$-\alpha^2\sin(15\alpha c)\, C_{59} - \alpha^2\cos(15\alpha c)\, C_{60} + \alpha^2\sin(15\alpha c)\, C_{61} + \alpha^2\cos(15\alpha c)\, C_{64}$

60-*at x=15c,* $t_{15c}l^3 y(15c,t) = EI\left(\frac{\partial^3 y(15c^-,t)}{\partial x^3} - \frac{\partial^3 y(15c^+,t)}{\partial x^3}\right)$.

$T_{15c}C_{57} + 15cT_{15c}C_{58} + [T_{15c}\sin(15\alpha c) + \alpha^3\cos(15\alpha c)]C_{59} + [T_{15c}\cos(15\alpha c) - \alpha^3\sin(15\alpha c)]C_{60} - \alpha^3\cos(15\alpha c)\, C_{61} + \alpha^3\sin(15\alpha c)\, C_{64}$=0.

Where $T_{15c} = (t_{15c}l^3/EI)$: dimensionless longitudinal stiffness at *x*=15c

61-*at x=15c,* $y(15c^-,t) = y(15c^+,t)$.

$C_{57} + 15cC_{58} + \sin(15\alpha c)\, C_{59} + \cos(15\alpha c)\, C_{60} - C_{61} - 15cC_{62} - \sin(15\alpha c)\, C_{63} - \cos(15\alpha c)\, C_{64} = 0.$

62- *at x=15c,* $\frac{\partial y(15c^-,t)}{\partial x} = \frac{\partial y(15c^+,t)}{\partial x}$.

$C_{58} + \alpha\cos(15\alpha c)\, C_{59} - \alpha\sin(15\alpha c)\, C_{60} - C_{62} - \alpha\cos(15\alpha c)\, C_{63} + \alpha\sin(15\alpha c)\, C_{64} = 0.$

63-*at x=16c,* $r_{16c}\frac{\partial y(16c,t)}{\partial x} = EI\left(-\frac{\partial^2 y(16c^-,t)}{\partial x^2} + \frac{\partial^2 y(16c^+,t)}{\partial x^2}\right)$, $r_{16c} = 0$

$-\alpha^2\sin(16\alpha c)\, C_{63} - \alpha^2\cos(16\alpha c)\, C_{64} + \alpha^2\sin(16\alpha c)\, C_{65} + \alpha^2\cos(16\alpha c)\, C_{68}$

64-*at x=16c,* $t_{16c}l^3 y(16c,t) = EI\left(\frac{\partial^3 y(16c^-,t)}{\partial x^3} - \frac{\partial^3 y(16c^+,t)}{\partial x^3}\right)$.

$T_{16c}C_{61} + 16cT_{16c}C_{62} + [T_{16c}\sin(16\alpha c) + \alpha^3\cos(16\alpha c)]C_{63} + [T_{16c}\cos(16\alpha c) - \alpha^3\sin(16\alpha c)]C_{64} - \alpha^3\cos(16\alpha c)\, C_{65} + \alpha^3\sin(16\alpha c)\, C_{68}$=0.

Where $T_{16c} = (t_{16c}l^3/EI)$: dimensionless longitudinal stiffness at *x*=16c

65-*at x=16c,* $y(16c^-,t) = y(16c^+,t)$.

$C_{61} + 16cC_{62} + \sin(16\alpha c)\, C_{63} + \cos(16\alpha c)\, C_{64} - C_{65} - 16cC_{66} - \sin(16\alpha c)\, C_{67} - \cos(16\alpha c)\, C_{68} = 0.$

66- *at x=16c,* $\frac{\partial y(16c^-,t)}{\partial x} = \frac{\partial y(16c^+,t)}{\partial x}$.

$C_{62} + \alpha\cos(16\alpha c)\, C_{63} - \alpha\sin(16\alpha c)\, C_{64} - C_{66} - \alpha\cos(16\alpha c)\, C_{67} + \alpha\sin(16\alpha c)\, C_{68} = 0.$

67-*at x=17c,* $r_{17c}\frac{\partial y(17c,t)}{\partial x} = EI\left(-\frac{\partial^2 y(17c^-,t)}{\partial x^2} + \frac{\partial^2 y(17c^+,t)}{\partial x^2}\right)$, $r_{17c} = 0$

$-\alpha^2\sin(17\alpha c)\, C_{67} - \alpha^2\cos(17\alpha c)\, C_{68} + \alpha^2\sin(17\alpha c)\, C_{69} + \alpha^2\cos(17\alpha c)\, C_{72}$

68-at x=17c, $t_{17c}l^3 y(17c,t) = EI\left(\frac{\partial^3 y(17c^-,t)}{\partial x^3} - \frac{\partial^3 y(17c^+,t)}{\partial x^3}\right)$.

$T_{17c}C_{65} + 17cT_{17c}C_{66} + [T_{17c}\sin(17\alpha c) + \alpha^3\cos(17\alpha c)]C_{67} + [T_{17c}\cos(17\alpha c) - \alpha^3\sin(17\alpha c)]C_{68} - \alpha^3\cos(17\alpha c)\, C_{69} + \alpha^3\sin(17\alpha c)\, C_{72}$=0.

Where $T_{17c} = (t_{17c}l^3/EI)$: dimensionless longitudinal stiffness at *x*=17c

69-*at x=17c,* $y(17c^-,t) = y(17c^+,t)$.

$C_{65} + 17cC_{66} + \sin(17\alpha c)\, C_{67} + \cos(17\alpha c)\, C_{68} - C_{69} - 17cC_{70} - \sin(17\alpha c)\, C_{71} - \cos(17\alpha c)\, C_{72} = 0.$

70- *at x=17c,* $\frac{\partial y(17c^-,t)}{\partial x} = \frac{\partial y(17c^+,t)}{\partial x}$.

$C_{66} + \alpha\cos(17\alpha c)\, C_{67} - \alpha\sin(17\alpha c)\, C_{68} - C_{70} - \alpha\cos(17\alpha c)\, C_{71} + \alpha\sin(17\alpha c)\, C_{72} = 0.$

71-*at x=18c,* $r_{18c}\frac{\partial y(18c,t)}{\partial x} = EI\left(-\frac{\partial^2 y(18c^-,t)}{\partial x^2} + \frac{\partial^2 y(18c^+,t)}{\partial x^2}\right)$, $r_{18c} = 0$

$-\alpha^2\sin(18\alpha c)\, C_{71} - \alpha^2\cos(18\alpha c)\, C_{72} + \alpha^2\sin(18\alpha c)\, C_{73} + \alpha^2\cos(18\alpha c)\, C_{76}$

72-*at x=18c,* $t_{18c}l^3 y(18c,t) = EI\left(\frac{\partial^3 y(18c^-,t)}{\partial x^3} - \frac{\partial^3 y(18c^+,t)}{\partial x^3}\right)$.

$T_{18c}C_{69} + 18cT_{18c}C_{70} + [T_{18c}\sin(18\alpha c) + \alpha^3\cos(18\alpha c)]C_{71} + [T_{18c}\cos(18\alpha c) - \alpha^3\sin(18\alpha c)]C_{72} - \alpha^3\cos(18\alpha c)\, C_{73} + \alpha^3\sin(18\alpha c)\, C_{76}$=0.

Where $T_{18c} = (t_{18c}l^3/EI)$: dimensionless longitudinal stiffness at *x*=18c

73-*at x=18c,* $y(18c^-,t) = y(18c^+,t)$.

$C_{69} + 18cC_{70} + \sin(18\alpha c)\, C_{71} + \cos(18\alpha c)\, C_{72} - C_{73} - 18cC_{74} - \sin(18\alpha c)\, C_{75} - \cos(18\alpha c)\, C_{76} = 0.$

74- *at x=18c,* $\frac{\partial y(18c^-,t)}{\partial x} = \frac{\partial y(18c^+,t)}{\partial x}$.

$C_{70} + \alpha\cos(18\alpha c)\, C_{71} - \alpha\sin(18\alpha c)\, C_{72} - C_{74} - \alpha\cos(18\alpha c)\, C_{75} + \alpha\sin(18\alpha c)\, C_{76} = 0.$

75-*at x=19c,* $r_{19c}\frac{\partial y(19c,t)}{\partial x} = EI\left(-\frac{\partial^2 y(19c^-,t)}{\partial x^2} + \frac{\partial^2 y(19c^+,t)}{\partial x^2}\right)$, $r_{19c} = 0$

$-\alpha^2\sin(19\alpha c)\, C_{75} - \alpha^2\cos(19\alpha c)\, C_{76} + \alpha^2\sin(19\alpha c)\, C_{77} + \alpha^2\cos(19\alpha c)\, C_{80}$

76-*at x=19c,* $t_{19c}l^3 y(19c,t) = EI\left(\frac{\partial^3 y(19c^-,t)}{\partial x^3} - \frac{\partial^3 y(19c^+,t)}{\partial x^3}\right)$.

$T_{19c}C_{73} + 19cT_{19c}C_{74} + [T_{19c}\sin(19\alpha c) + \alpha^3\cos(19\alpha c)]C_{75} + [T_{19c}\cos(19\alpha c) -$

$\alpha^3 \sin(19\alpha c)]C_{76} - \alpha^3 \cos(19\alpha c)\, C_{77} +$
$\alpha^3 \sin(19\alpha c)\, C_{80}$=0.
Where $T_{19c} = (t_{19c} l^3/EI)$: dimensionless longitudinal stiffness at *x*=19c

77*-at x=18c,* $y(18c^-, t) = y(18c^+, t)$.
$C_{73} + 19cC_{74} + \sin(19\alpha c)\, C_{75} + \cos(19\alpha c)\, C_{76} - C_{77} - 19cC_{78} - \sin(19\alpha c)\, C_{79} - \cos(19\alpha c)\, C_{80} = 0.$

78*- at x=19c,* $\frac{\partial y(19c^-,t)}{\partial x} = \frac{\partial y(19c^+,t)}{\partial x}$.
$C_{74} + \alpha\cos(19\alpha c)\, C_{75} - \alpha\sin(19\alpha c)\, C_{76} - C_{78} - \alpha\cos(19\alpha c)\, C_{79} + \alpha\sin(19\alpha c)\, C_{80} = 0.$

79*-at x=l,* $r_2 l \frac{\partial y(l,t)}{\partial x} = -EI \frac{\partial^2 y(l,t)}{\partial x^2}$,
$R_2 C_{78} + (\alpha R_2 \cos(\alpha) - \alpha^2 \sin(\alpha))C_{79} + (-\alpha R_2 \sin(\alpha) - \alpha^2 \cos(\alpha))C_{80} = 0$
Where $R_2 = (r_2 l/EI)$: dimensionless rotational stiffness at *x*= l

80*-atx=l,* $t_2 l^3 y(l,t) = EI \frac{\partial^3 y(l,t)}{\partial x^3}$,
$t_2 l^3 [C_{77} + C_{78} + C_{79}\sin(\alpha) + C_{80}\cos(\alpha) = EI[-C_{79}\,\alpha^3 \cos(\alpha) + C_{80}\alpha^3 \sin(\alpha)]$
$T_2 C_{77} + T_2 C_{78} + (T_2 \sin(\alpha) + \alpha^3 \cos(\alpha))C_{79} + (T_2 \cos(\alpha) - \alpha^3 \sin(\alpha))C_{80} = 0$
Where $T_2 = (t_2 l^3/EI)$: dimensionless longitudinal stiffness at *x*= l.

These (80) equations can be written in matrix form as follows:

$$[Wi, j]\{Cj\} = 0$$
$$\Delta = |Wi, j|$$

Where i, j=1, 2, 3, 4 …etc.

Trial and error procedure is used to find the value of (α) that makes the determinant |Wi, j| Vanish. From $(\alpha = \sqrt{\gamma + u_{cr}^2})$ the non-dimensional critical velocity $(u_{cr})$is obtained.

III.Results and Discussions.

The natural frequencies for steady flow decreases with increasing the fluid flow velocity. If the natural frequencies of the pipe reach to zero, the flow velocity in this case is called critical flow velocity. When the flow velocity equals the critical velocity the pipe bows out and buckles, because the forces required to make the fluid deform to the pipe curvature are greater than the stiffness of the pipe. The effects of the transverse parameter related to the Winkler model on the critical flow velocity are studied based on the numerical results obtained for various pipe end conditions. From the results obtained, it is observed that the instability caused by the fluid flow velocity is effectively countered by the foundation and the fluid conveying pipe is stabilized by an appropriate choice of the stiffness parameters of the Winkler foundation. A detailed study is made on the influence of Winkler foundation on the critical flow velocity and interesting conclusions are drawn from the numerical results presented for pipes under different boundary conditions.

Results have been obtained for critical flow velocity condition, where $\Omega_1 = 0$. This condition constitutes a pipe on elastic foundation.

In Figure (2), the critical flow velocity parameter u_{cr} is plotted against Winkler foundation parameter γ_1, for the pinned-pinned case. The results show that the critical flow velocity increases appreciably for values of γ_1 greater than 1000. Figures (3) and (4) show the results for the clamped-pinned and the clamped-clamped boundary conditions respectively. Here, too, the trend is similar.

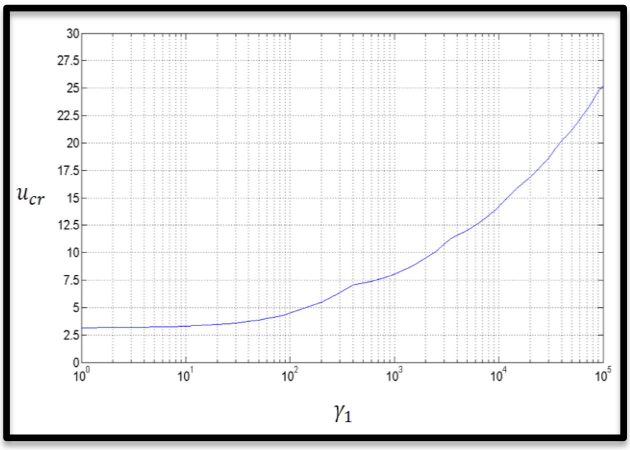


Figure (2) Influence of $\gamma_1$on u_{cr} for Pinned-Pinned pipe.

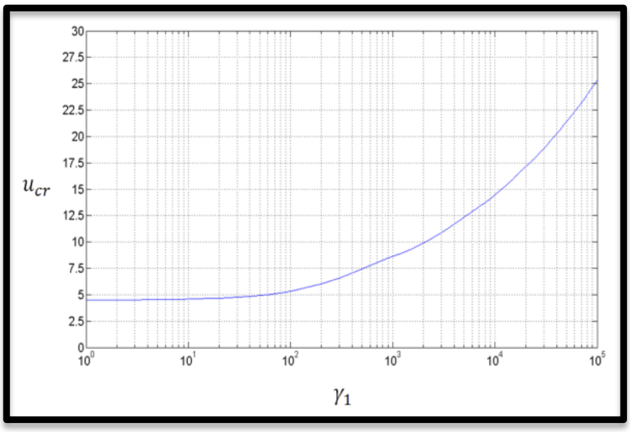


Figure (3) Influence of $\gamma_1$on u_{cr} for Clamped-Pinned pipe.

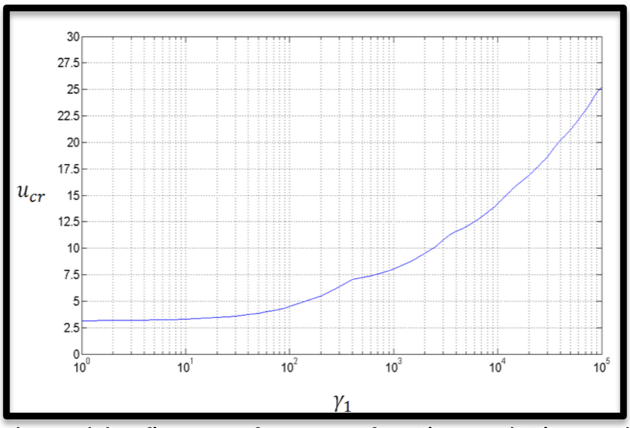
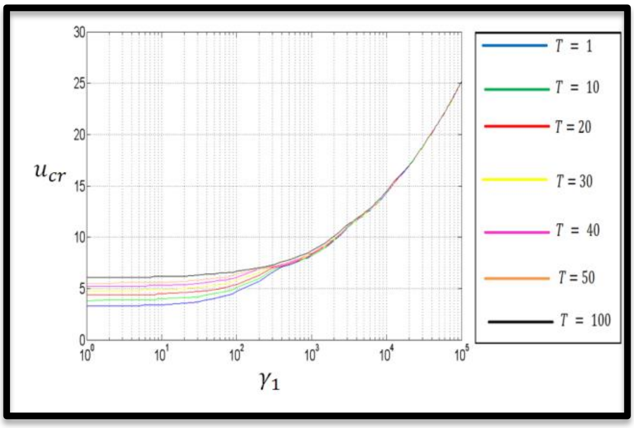


Figure (4) Influence of $\gamma_1$on u_{cr} for Clamped-Clamped pipe.

New results for pipe conveying fluid on Winkler parameter elastic foundation under flexible boundary condition have been presented. Figure (5) show the effect of changing the linear impedance value on the critical velocity with different foundation parameter, and same rotational impedance value ($R_1 = R_2 = R = 50$). The results show that the critical flow velocity increases for values of linear stiffness increasing. This increasing vanishes with arrived the foundation value to 10000 because of the effect of foundation parameter on the critical velocity greater than the effect of linear stiffness in this point.



Figure(5) Effect of linear impedance ($T_1 = T_1 = T$) and foundation parameter(γ_1) values on the critical velocity

.

Figers(6,7,8) shows some representative values of u_{cr} for different values of the Winkler foundation parameter γ_1 for all the three flexible boundary conditions. Figure. (6) shows the plot of u_{cr} versus γ_1 for type ($T_1 = 100$, $R_1 = 10$, $T_2 = 10$, $R_2 = 100$) boundary condition. The Winkler foundation has a stabilizing effect in the pipe and increasing values of γ_1 tend to increase the critical flow velocity. Figures (7) and (8) shows the plots for type ($T_1 = 110, R_1 = 25, T_2 = 130, R_2 = 15$) and the type ($T_1 = 750, R_1 = 180, T_2 = 450, R_2 = 250$) cases respectively. A similar trend is noticed in these cases also. This new results are very accurate and may be useful to other researchers so as compare their result.

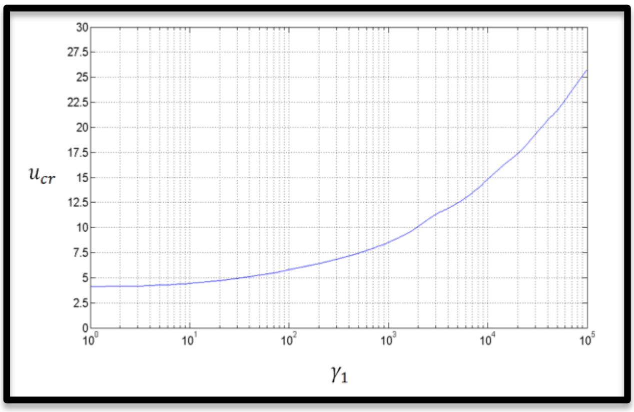


Figure (6) Influence of γ_1 on u_{cr} for type ($T_1 = 100, R_1 = 10, T_2 = 10, R_2 = 100$.) pipe conveying fluid.

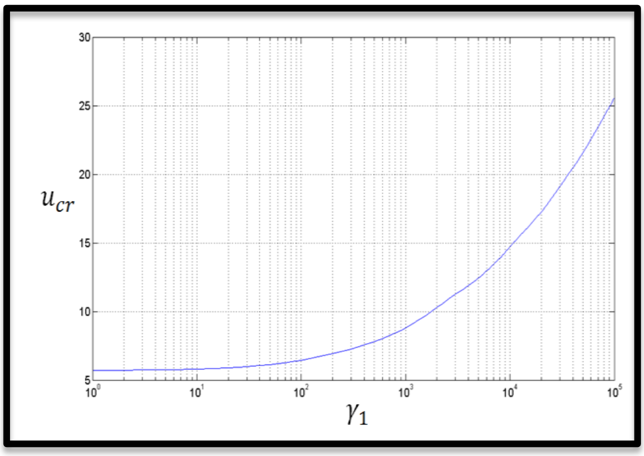


Figure (7) Influence of γ_1 on u_{cr} for type ($T_1 = 110, R_1 = 25, T_2 = 130, R_2 = 15$.) pipe conveying fluid.

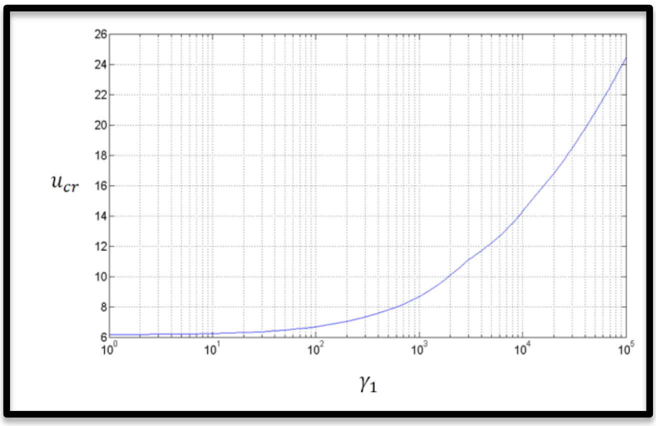


Figure (8) Influence of γ_1 on u_{cr} for type ($T_1 = 750, R_1 = 180, T_2 = 450, R_2 = 250$.) pipe conveying fluid.

V. Conclusions.

Following the main summarized conclusions raised by this research:

1-The technique used for modeling the compliant boundary material in terms of linear and rotational impedance allows the designer to describe both the classical and restrained boundary conditions.

4- The critical velocity increase with increasing the value of the Winkler foundation.

3-The instability caused by the fluid flow velocity is effectively countered by the foundation and the fluid conveying pipe is stabilized by an appropriate choice of the stiffness parameters of the Winkler foundation.

5-The mass ratio has no effect on the critical velocity.

6- The critical flow velocity increases for values of linear impedance increasing. This increasing vanishes with arrived the foundation value to 10000 because of the effect of foundation parameter on the critical velocity greater than the effect of linear stiffness in this point.

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