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Application Of Munich Chain Ladder For An Albanian DMTPL Portfolio

Kleida HAXHI
Department of Mathematics, Mathematical and Physical Engineering Faculty, Polytechnic University of Tirana
Tirana, Albania
kleida_haxhi@yahoo.com

Bledar BAHOLLI
Department of Mathematics, Mathematical and Physical Engineering Faculty, Polytechnic University of Tirana
Tirana, Albania

***Abstract*—The most used method for the estimation of claims reserve is the Chain Ladder method. The actuaries apply the chain ladder method independently to the paid claims and to the incurred claims triangles. The Munich Chain Ladder method combine both triangles, by taking the paid-incurred ratios into account in projections.**

Keywords—Munich Chain Ladder; Standard Chain Ladder; Run-off triangle; Claims reserving; Incurred Claims, Paid Claims; P/I ratios

I. INTRODUCTION

Using the Standard Chain Ladder (SCL) method the claims reserve is calculated based on the run-off triangles of paid claims and on the run-off triangles of the incurred claims. Many times the projections based on the paid claims are different that the projections based on the incurred claims. The solution for this problem is the Munich Chain Ladder (MCL) method. We apply MCL to an Albanian DMTPL portfolio. The paid (P) claims triangle and the incurred (I) claims triangle cover ten accident years and also ten development years. The values are in Albanian currency.

II. MUNICH CHAIN LADDER METHOD

A. *Mack Chain Ladder Model*

We denote n∈N the number of accident years and T={1,2,…m} the development years, m∈N and generally m=n;$P_i = (P_{i,t})_{t\in T}$ and $I_i = (I_{i,t})_{t\in T}$ denote respectively the paid claims process and the incurred claims process. The processes P_i and I_i describe the development of the paid and the incurred claims.

$P_i(s)=\{P_{i,1},\dots,P_{i,s}\}$ represent the condition that the paid development of accident *i* is given until the end of development year *s* and $I_i(s)=\{I_{i,1},\dots,I_{i,s}\}$ for the condition that the incurred development of accident *i* is given up to and including *s*. [2]

The assumptions for the paid processes are:

- Expectation assumption PE
- Variance assumption PV
- Independence assumption PU, that means the accident years are stochastically independent

The assumptions for the incurred processes are:

- Expectation assumption IE
- Variance assumption IV
- Independence assumption IU, that means the accident years are stochastically independent

These assumptions does not say anything about the relationships between paid and incurred processes. If we knows the paid run-off triangle and the incurred run-off triangle, we can make projections based on the conditional expectations [2]

$$E\left(\frac{P_{i,t}}{P_{i,s}}|\mathcal{B}_i(s)\right) \text{ and } E\left(\frac{I_{i,t}}{I_{i,s}}|\mathcal{B}_i(s)\right)$$

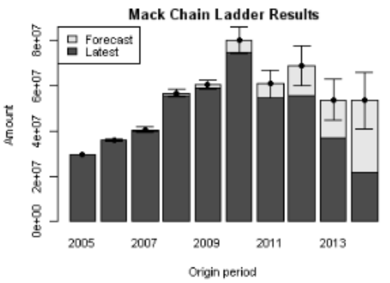
where $\mathcal{B}_i(s)=\{P_{i,1},\dots,P_{i,s}; I_{i,1},\dots,I_{i,s}\}$ stand for the fact of the development of both processes up to the end of development year *s*.

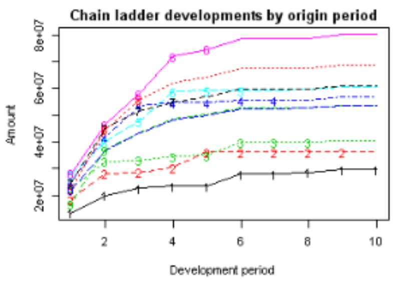
Cumulative Paid

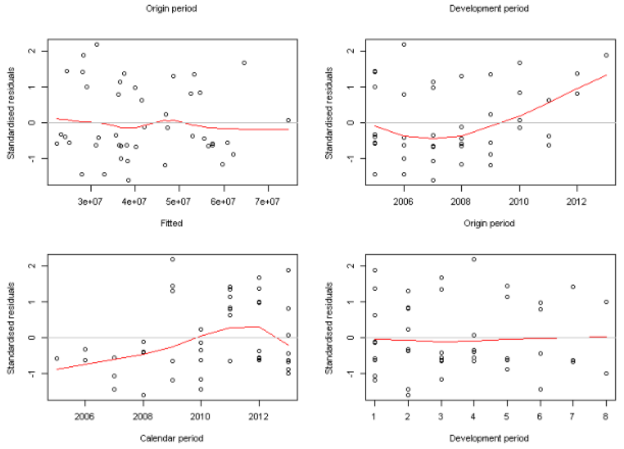
	0	1	2	3	4	5	6	7	8	9
2005	13,247,635	19,871,794	22,514,448	23,425,701	23,425,701	28,034,921	28,034,921	28,534,921	29,627,617	29,627,617
2006	18,545,283	28,096,622	28,505,022	30,384,866	36,210,297	36,210,297	36,210,297	36,210,297	36,210,297	
2007	22,805,548	32,677,287	33,056,967	34,544,398	34,901,048	39,901,048	39,901,048	39,901,048		
2008	25,061,272	41,450,112	53,532,128	54,632,128	54,632,128	55,482,000	55,482,000			
2009	27,800,292	39,885,124	47,736,642	58,826,959	59,226,959	59,226,959				
2010	28,171,203	46,485,088	57,877,529	72,033,666	74,682,850					
2011	24,747,420	45,051,323	51,535,064	54,831,028						
2012	22,311,896	44,679,545	55,642,105							
2013	16,811,300	36,981,386								
2014	21,903,227									

Cumulative Incurred

	0	1	2	3	4	5	6	7	8	9
2005	43,855,985	48,981,761	49,781,761	49,781,761	49,781,761	49,781,761	49,781,761	49,781,761	49,781,761	49,781,761
2006	35,549,677	39,261,019	47,261,019	47,261,019	47,638,070	47,638,070	47,638,070	47,638,070	47,638,070	
2007	34,563,960	61,123,045	61,123,045	62,748,791	62,748,791	62,748,791	62,748,791	62,748,791		
2008	39,563,188	61,731,322	67,677,605	84,949,636	91,255,164	91,255,164	91,421,764			
2009	39,467,409	58,680,818	66,055,173	67,412,109	67,412,109	67,412,109				
2010	43,035,761	69,523,122	83,069,602	83,319,602	88,216,570					
2011	43,195,291	67,457,014	76,275,810	76,325,810						
2012	41,495,136	54,703,915	58,893,154							
2013	40,571,725	40,571,725								
2014	53,596,544									







B. *Munich Chain Ladder Model*

In the MCL model we use PIU [1] instead of PU and IU. We denote the P/I process as

$$Q_i = \frac{P_i}{I_i} = \left(\frac{P_{i,t}}{I_{i,t}}\right)_{t\epsilon T}$$

If X is a random variable, C a condition and the conditional standard deviation of X given C is σ(X|C), we call the conditional residual of X given C

$$Res(X|C) = \frac{X - E(X|C)}{\sigma(X|C)}$$

The standardization of the conditional residual is

E(Res(X|C)|C)=0 and Var(Res(X|C)|C)=1

The assumptions for the MCL model:

- The conditional expectations for the paid development factors and their residuals are

$$Res\left(\frac{P_{i,t}}{P_{i,s}}|\mathcal{P}_i(s)\right)$$

- The conditional expectations for the incurred development factors and their residuals are

$$Res\left(\frac{I_{i,t}}{I_{i,s}}|\mathbb{I}_i(s)\right)$$

Hence we have

$$E\left(Res\left(\frac{P_{i,t}}{P_{i,s}}|\mathcal{P}_i(s)\right)|\mathcal{B}_i(s)\right)$$

and

$$E\left(Res\left(\frac{I_{i,t}}{I_{i,s}}|\mathbb{I}_i(s)\right)|\mathcal{B}_i(s)\right)$$

- Linear dependence of the expectations on the residuals of (P/I) or (I/P) ratios into an mathematical equation

$$Res(Q_{i,s}|\mathbb{I}_i(s)) \quad \text{or} \quad Res(Q_{i,s}^{-1}|\mathcal{P}_i(s))$$

- PQ – there exist a constant θ^P such that for all s, t ϵT with t=s+1 and i=1,2,….,n

$$E\left(Res\left(\frac{P_{i,t}}{P_{i,s}}|\mathcal{P}_i(s)\right)|\mathcal{B}_i(s)\right) = \theta^P\ Res(Q_{i,s}^{-1}|\mathcal{P}_i(s))$$

- IQ - there exist a constant θ^I such that for all s, t ϵT with t=s+1 and i=1,2,….,n

$$E\left(Res\left(\frac{I_{i,t}}{I_{i,s}}|\mathbb{I}_i(s)\right)|\mathcal{B}_i(s)\right) = \theta^I\ Res(Q_{i,s}|\mathbb{I}_i(s))$$

The parameters θ^P and θ^I represent the slopes of the regression lines in the residuals plots and are not independent on development year *s*.

III. ANALYSIS OF THE MCL MODEL

The factor θ^I is the coefficient of correlation of the residuals of the development factors and the residuals of the P/I ratios. The factor θ^P is the coefficient of correlation of the residuals of the development factors and the residuals of the I/P ratios. The values of factors θ^I and θ^P are between 0 and 1. The correlation parameters θ^I and θ^P represent the link between the incurred and the paid triangles.

$$\theta^P = corr\left(Q_{i,s}^{-1}, \frac{P_{i,t}}{P_{i,s}}|\mathcal{P}_i(s)\right)$$

$$\theta^I = corr\left(Q_{i,s}, \frac{I_{i,t}}{I_{i,s}}|\mathbb{I}_i(s)\right)$$

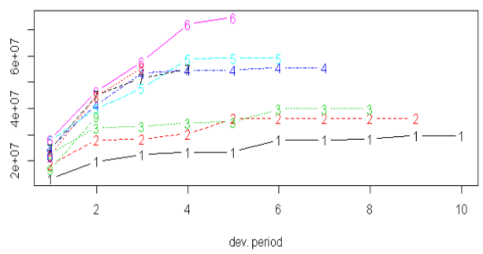
For the conditional correlation coefficients

$$\theta^P = corr\left(Res\ (Q_{i,s}^{-1}|\mathcal{P}_i(s)), Res\ \left(\frac{P_{i,t}}{P_{i,s}}|\mathcal{P}_i(s)\right)\right)$$

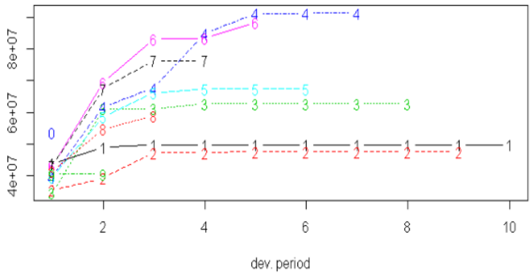
$$\theta^I = corr\left(Res(Q_{i,s}|\mathbb{I}_i(s)), Res\ \left(\frac{I_{i,t}}{I_{i,s}}|\mathbb{I}_i(s)\right)\right)$$

A. *Estimation of claims reserve using MCL*

The appearance of the paid claims before the application of MCL



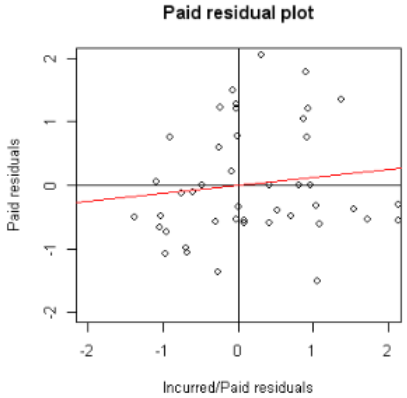
The appearance of the incurred claims before the application of MCL

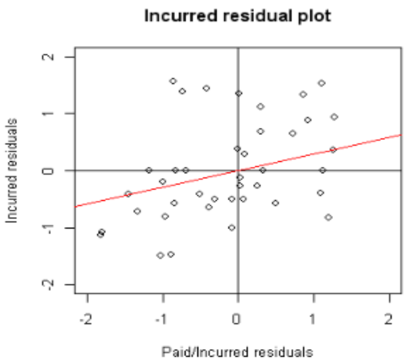


Using MCL method we can project the total claims reserve and also the paid and incurred triangles.

Totals	**Claims**		
	Paid	***Incurred***	***P/I Ratio***
Latest	464488517	636606298	0.7296323
Ultimate	534960659	687394853	0.7782436

The incurred residual plot shows a correlation of 58% and the paid residuals shows a correlation of 26%. The regression lines for the two plots are flat.





B. Results of projections

The results of the projection are the paid and the incurred quadrangle.

The projections for paid claims

> MCL Paid

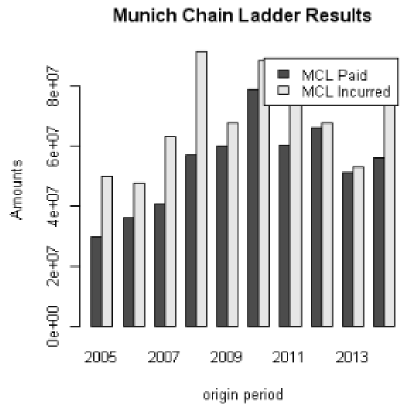
	0	1	2	3	4	5	6	7	8	9
2005	13247635	19871794	22514448	23425701	23425701	28034921	28034921	28534921	29627617	29627617
2006	18545283	28096622	28505022	30384866	36210297	36210297	36210297	36210297	36210297	36182431
2007	22805548	32677287	33056967	34544398	34901048	39901048	39901048	39901048	40599576	40612116
2008	25061272	41450112	53532128	54632128	54632128	55482000	55482000	55772566	56804566	56839121
2009	27800292	39885124	47736642	58826959	59226959	59226959	59226959	59353914	60005744	59905837
2010	28171203	46485088	57877529	72033666	74682850	77926334	77926334	78090458	78940934	78807308
2011	24747420	45051323	51535064	54831028	56622516	59392062	59392062	59586714	60405696	60355788
2012	22311896	44679545	55642105	61198085	62704245	65255587	65255587	65354501	65972112	65831451
2013	16811300	36981386	42904531	47226714	48409706	50401215	50401215	50482508	50971549	50866563
2014	21903227	37537900	44646838	50230164	52077653	54840558	54840558	55068368	55942615	55932426

The projections for incurred claims

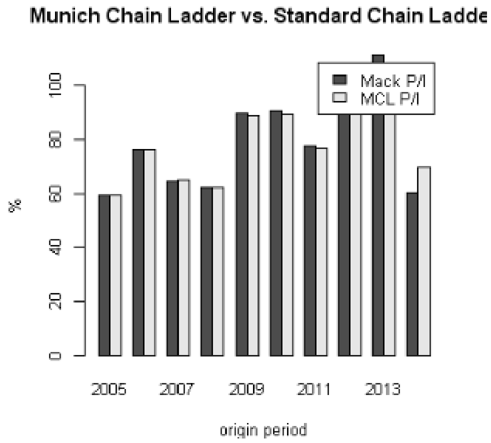
> MCL Incurred

	0	1	2	3	4	5	6	7	8	9
2005	43855985	48981761	49781761	49781761	49781761	49781761	49781761	49781761	49781761	49781761
2006	35549677	39261019	47261019	47261019	47638070	47638070	47638070	47638070	47638070	47638070
2007	34563960	61123045	61123045	62748791	62748791	62748791	62748791	62748791	62748791	62748791
2008	39563188	61731322	67677605	84949636	91255164	91255164	91421764	91421764	91421764	91421764
2009	39467409	58680818	66055173	67412109	67412109	67412109	67485979	67485979	67485979	67485979
2010	43035761	69523122	83069602	83319602	88216570	88216570	88314186	88314186	88314186	88314186
2011	43195291	67457014	76275810	76325810	78675472	78675472	78739749	78739749	78739749	78739749
2012	41495136	54703915	58893154	64522884	67536952	67536952	67624301	67624301	67624301	67624301
2013	40571725	40571725	46405079	50658890	52968438	52968438	53035191	53035191	53035191	53035191
2014	53596544	70682877	76622566	78563956	80552705	80552705	80605061	80605061	80605061	80605061

The Munich Chain Ladder results



The comparison of level of concurrence between paid and incurred projections by comparing the ultimate P/I ratios calculated using the Standard Chain Ladder and the Munich Chain Ladder.



IV. CONCLUSIONS

The standards chain ladder method don't consider the correlation between paid claims and incurred claims. The Munich chain ladder seeks to resolve the differences that arise between the standard paid claims and the incurred chain ladder indications. MCL provides separate estimations for paid and incurred, but they are closer to one another. In the cases where the correlations are not significant the MCL method provides the same results as the SCL method.

The implementation of MCL method is more complex that the other reserving methods. It may be not respond very well to the small data and sometimes the parameters may need smoothing or extrapolation.

References

[1] Quarg, G.; Mack, T. [2008] Munich Chain Ladder: A Reserving Method that Reduces the Gap between IBNR Projections Based on Paid Losses and IBNR Projections Based on Incurred Losses. In: CAS, Volume 2, Issue 2, pp. 266 -299

[2] Mack, T. [1993] Distribution–free calculation of the standard error of chain–ladder reserve estimates. ASTIN Bull. 23, 213–225.

[3] Mack, T. [1997] Measuring the variability of chain–ladder reserve estimates. In: Claims Reserving Manual, vol. 2. London: Institute of Actuaries.