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Difference Equations and Time Series Analysis in Approximating the Index Prices (For Albania)

Eglantina Kalluci
Department of Applied Mathematics
Faculty of Natural Sciences, University of Tirana
Albania
eglantina.xhaja@fshn.edu.al

Eralda |Dhamo (Gjika)
Department of Applied Mathematics
Faculty of Natural Sciences, University of Tirana
Albania
eralda.dhamo@fshn.edu.al

***Abstract*—A lot of real life problems can be modelled as difference equations, which are solved analytically and numerically. The aim of this paper it to study the stability conditions of a real problem, the index prices (for consummators) and if it is gained a numerical prediction is made. This method has the priority compared with other extrapolation methods, that before making any prediction it studies the stability of the existing values and if this values suit the stability conditions than we consider the model as a suitable one to be used in extrapolation. To gain a full insight of the problem we are using the time series approximation. We have considered the SARIMA (*Seasonal Autoregressive Integrated Moving Average*) and exponential smoothing techniques of ETS (*Error-Trend-Seasonality*) models which also examine and provide important indicators of trend, seasonality and irregular component of the time series.We have calculated the MAE and RMSE errors to compare the results provided by the nonlinear difference equation and time series models (SARIMA, ETS).**

Keywords—stability, difference equation, equilibrium, time series, prediction.

I. INTRODUCTION

The concepts of stability and chaos are very important in Mathematical Economics. Many of economic theory is based on the comparative statics of equilibrium states. The definition of a point of equilibrium is different according to many authors. In fact the *equilibrium is a situation that is characterized by the lack of tendency to change.*

Modeling financial data is difficult because we demand accountability at predictions. Models built upon the techniques of time series should be good to increase confidence in predictions, especially when financial indicators that are modeled are important for decision-making.

In paragraph 2 we are going to study the stability of price index (for Albanian consummators) and we have done some predictions using a nonlinear difference equation. In paragraph 3 we have presented a time series analysis and we have chosen SARIMA and ETS modes to approximate the data.

II. STUDING THE STABILITY OF INDEX PRICES USING THE DIFFERENCE EQUATIONS

To understand an economic process it is necessary to look for a method which will give forecasts or overview of what might happen in a certain sector of economy. Forecasts are necessary for planning, decision-making, understanding and implementing prospective choices. In relation to this, economics has to use both quantitative and qualitative methods in explaining the economic situation of the market. A lot of models involve differential equations in the representation of the qualitative methods in explaining the economic situation of the market. A lot of models involve differential equations in the representation of the relationships among variables which concern changes over time.

One of the many great results of ordinary differential equations is the derivation of the logistic equation $P(t)=\dfrac{MCe^{rt}}{1+Ce^{rt}}$.

Several studies have revealed the usefulness of the logistic equation in forecasting events.

In this section we are using the above theoretical result applied in a real data base and more exactly studying the stability of the index prices (for the consummators) in the time period 1994-2014. The data are taken from the website of Bank of Albania.

The logistic equation is a model of the growth population published for the first time by Pierre – Francois Verhulst (1845,1847) [Weisstein, 2009]. The model is a continuous function with respect to time, but a modification of the continuous equation in a recurrent discreet quadratic equation, is known as the logistic map.

A continuous version of the logistic model is the differential equation:

$$\frac{dP}{dt}=rP\left(1-\frac{P}{M}\right) \tag{1}$$

where r – is the Malthusian coefficient (the rate of population growth) and M – is maximal capacity. Dividing both sides with M and denoting $X=P/M$ obtaining the differential equation:

$$\frac{dX}{dt}=rX(1-X) \tag{2}$$

The discrete version of the logistic model is described as below:

$$X_{n+1} = rX_n(1 - X_n) \qquad (3)$$

We shall calculate the growth rate for a real data base (index price) and check if we are in the boundaries of convergence.

The concept of stability (or equilibrium) is related with the absence of small changes in the system. In the concept of difference equation the the stable state X_{SS} is defined as below:

$$X_{n+1} = X_n = X_{SS} \qquad (4)$$

Following the analysis given by [Gonze, 2013] it is justified that if $|r| < 1$, the stage is state is stable (the perturbations x_n goes to zeroas n grows), and if $|r| > 1$, the state is unstable (the perturbations grow as n is increased).

Firstly we study the stability of the index price for the period 1994-2004. To have a better outlook of the situation we use different time intervals, $t_1 = 2, t_1 = 3, t_1 = 4$, $t_1 = 5$ and we obtain the respectively values of the rate *r* and *M* as presented in the Table 1 and Table 3 and using the growth rates calculated and the difference equation (3) we generate the approximated values shown in Table 2 and Table 4.

TABLE 1. The growth rate and maximal value for different time intervals for 1994-2004.

t	2	3	4	5
r	-1.2581	-0.5614	-0.507	-0.065
M	1.2731	1.3927	1.4334	-0.2186

As we see from the values in the above table, except the result for time interval $t_1 = 2$, which does not ensure us convergence conditions (or stability), the other result satisfy the boundary values for which we have convergence and this leads us to the next step of making approximations and predictions. In the following table we are presenting the exact and approximated values using the growth rates calculated above.

TABLE 2. The exact and approximated values for different time intervals for 1994-2004.

Exact val. 94-04	r=-1.2581, M=1.2731	r=-0.561, M=1.3927	r=-0.507, M=1.4334	r=-0.065, M=-0.2186
1.2581	1.2581	1.2581	1.2581	1.2581
0.5083	1.2405	1.1768	1.1701	0.8692
1.3583	1.1849	1.0577	1.0480	0.6511
3.033	1.0303	0.8992	0.8930	0.5136
0.7167	0.7167	0.7129	0.7167	0.4191
-0.083	0.3547	0.5239	0.5394	0.3503
0.3583	0.1317	0.3583	0.3821	0.2981
0.3083	0.0424	0.2311	0.2573	0.2572
0.1667	0.0129	0.1427	0.1667	0.2243
0.275	0.0039	0.0857	0.1051	0.1974
0.175	0.0012	0.0505	0.0651	0.1750

In the following table we have calculated the growth rate and maximal values using the monthly average index prices for the period 2004-2014.

TABLE 3 The growth rate and maximal value for different time intervals for 2004-2014.

t	2	3	4	5
r	0.1095	*inf*	-0.1551	-0.3745
M	0.1996	0.267	0.1665	0.1839

From the values of the above table we see that except of the case of time interval T=3 (which has this value because are used as target values to equal ones 0.267 and 0.267) the other values guarantee stability.

TABLE 4. The exact and approximated values for different time intervals for 2004-2014.

Exact values	Approximated values T=2	Approximated values T=3	Approximated values T=4	Approximated values T=5
0.175	0.175	0.175	0.175	0.175
0.1667	0.1773	0.2670	0.1765	0.1712
0.225	0.1793	0.2670	0.1783	0.1660
0.267	0.1813	0.2670	0.1804	0.1590
0.183	**0.1830**	0.2670	**0.1830**	0.1498
0.275	0.1846	0.2670	0.1861	0.1382
0.267	0.1860	0.2670	0.1898	0.1241
0.1417	0.1874	0.2670	0.1944	0.1081
0.2	0.1886	0.2670	**0.2000**	0.0911
0.1583	0.1896	0.2670	0.2070	0.0741
0.0583	0.1906	0.2670	0.2158	**0.0583**
Predicted vl.	0.1915		0.2270	0.0445
Predicted vl.	0.1923		0.2418	0.0331

For this part we are saving space from the figures and conclude that even for this year in the three cases where the time intervals are taken two, three and four months the growth rate is smaller than 1, which yields that we are in conditions of stability. After analysing these data using the times series methods we will present in a common table the error of all the methods used.

III. TIME SERIES APPROXIMATION OF INDEX PRICES

Time series of consumer price index (referred as C.P.I) is an interesting time series to be studied. It contains monthly data that besides the presence of trend also exhibit a clear seasonal pattern. Excluding the special case of observation in July and August 2013 (months in which the consumer price index shows an apparent discount) time series can be modeled by several popular models of time series.

Time series techniques [Ngailo et al, 2014] such as Autoregressive models with parameter *p* and Moving average models with parameter *q,* if used separately, are not suitable for time series that present clear seasonality and trend. A combination of them together would be interesting and could better model the time series.

We have considered the SARIMA (*Seasonal Autoregressive Integrated Moving Average*) and exponential smoothing techniques of ETS (*Error-Trend-Seasonality*) models [Junior et al, 2014] which also examine and provide important indicators of trend, seasonality and irregular component of the time series.

The **Seasonal Autoregressive Integrated Moving Average** model of *Box and Jenkins (1970)* is given by:

$$\Phi_P(B^s)\phi(B)\nabla_s^D\nabla^d X_t = \alpha + \Theta_Q(B^s)\theta(B)w_t \quad (5)$$

where w_t is a Gausian white noise [Leng, 2014]. The model is denoted as an ***ARIMA(p,d,q)x(P,D,Q)$_s$***. the autoregressive and moving average components are represented by the polynomials: $\phi(B)$ and $\theta(B)$ of degree *p* and *q* respectively, the seasonal autoregressive and moving average components are represented by the polynomial $\Phi_P(B^s)$ and $\Theta_Q(B^s)$ of degree *P* and *Q,* and the difference operator represented by $\nabla^d = (1-B)^d$ and $\nabla_s^D = (1-B^s)^D$.

Hyndman et al. (2002, 2005) propose 15 models based on the combination of *Error-Trend- Seasonality*. Further additive or multiplicative combination of components provides in detail nearly 30 models. The exponential smoothing method Holt- Winters is one of these models.

Consumer price index Growth 1994- 2015

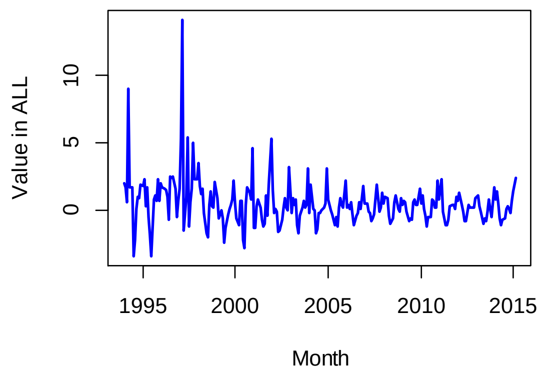

Figure 1 Time series of consumer growth rate (data from INSTAT)

Consumer price index growth is a time series which becomes stationary in the recent years. After a detailed analysis of a time series the proposed SARIMA model is:

Consumer price index growth (C.P.I growth):
ARIMA(0,1,3)(2,0,1)[12]

	ma1	ma2	ma3	sar1	sar2	sma1
Coefficients:	-0.7834	-0.2352	0.0483	1.1432	-0.1567	-0.9215
s.e.	0.0640	0.0899	0.0670	0.1058	0.0868	0.1025

sigma^2 estimated as 2.046: log likelihood=-454.45

Forecasted values for the next month using the SARIMA model with seasonality 12 are show graphically in Fig. 2. Fig. 2 is zoomed to give a clear information to real values and forecasts.

Forecast C.P.I growth from SARIMA 1994- 2015

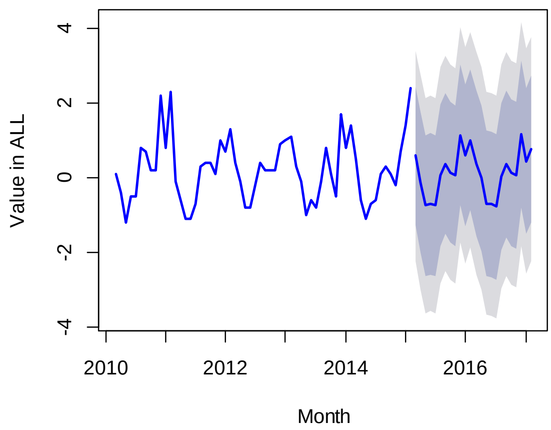

Figure 2 The forecasted values from SARIMA model

The proposed models from ETS are also nice models to use for prediction purposes of consumer price index (C.P.I). These are the models :

Consumer index price growth model

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ETS(A,N,A)
Smoothing parameters: alpha = 0.0849 ; gamma = 1e-
04
Initial states: l = 1.3923 ;
s=1.6402 0.1241 0.0203 0.3262 -0.1228 -1.3668 -
1.068 -1.0979 0.0961 0.4418 0.613 0.3938
sigma:  1.3894
```

Forecast C.P.I growth from ETS 1994- 2015

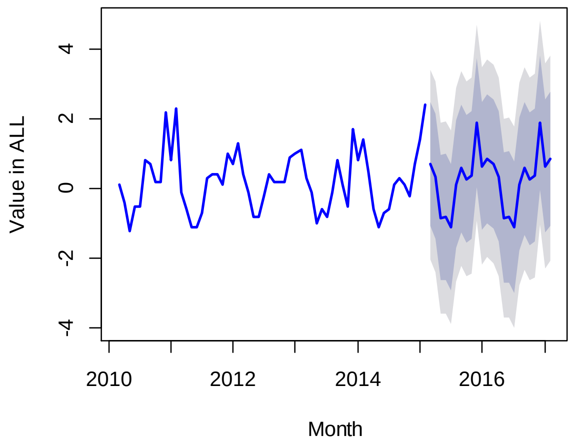

Figure 3 Forecasted values of C.P.I growth from ETS models

The predictions of these two models appear to be reliable and with good confidence intervals constructed. Thinking about the priorities that SARIMA models have in modeling the financial data and also indicators of goodness such as: MSE, AIC, AICc, BIC etc. we have calculated these indicators for each of the models proposed. TABLE5 shows the values of each indicator in the two proposed models. We selected as the best model the model with smaller values of theindicators (especially AIC values).

TABLE 5 Accuracy of SARIMA and ETS models for *C.P.I* and *C.P.I growth* time series

Models for C.P.I growth

```
 ME       RMSE  MAE  MASE     AIC     AICc        BIC
SARIMA -0.047 1.427 0.811 0.707   922.9      923.36
947.63
ETS-0.052   1.389   0.797   0.695   1601.56   1603.31
1651.08
```

Based on all the indicators above the proposed SARIMA model for the original time series of C.P.I is better than the proposed ETS model, we see that the values of all indicators are smaller for the SARIMA model compared to the ETS values. So, we decide to use as a better model for the C.P.I the SARIMA. In the other part observing carefully the indices of the SARIMA and ETS model for the C.P.I growth values only the ME and the information criteria's (AIC, AICc, BIC) have smaller values in SARIMA model all the other indices are smaller in ETS model.

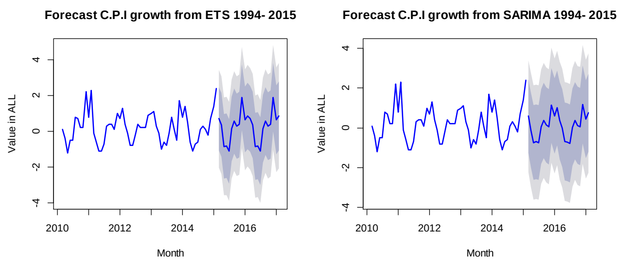

Figure 4 Comparison of prediction from SARIMA and ETS for C.P.I growth

Therefore here we believe that the SARIMA model is suitable for modeling time series of consumer price index (C.P.I). And can be used to predict the performance of this indicator in the coming months helping public and private institutions who have an interest either directly or indirectly to the performance of this indicator.

In the following table we are presenting the two errors (MAE and RMSE) to compare the accuracy of all the methods presented in this papers; and more precisely the nonlinear difference equation (3) focusing in observations done in different time intervals (leading to different values of the growth rate and capacity building) and times series methods, ARIMA and ETS.

Table 6. The error analysis of ARIMA, ETS and difference equations (for T=2, 3, 4, 5)

Model	MAE	RMSE
ARIMA	0.3578	0.5679
ETS	0.3898	0.6328
Diff. eq. T=2	**0.2284**	**0.2396**
Diff. eq.T=3	0.2261	0.26645
Diff. eq.T=4	0.2164	0.26604
Diff. eq.T=5	0.2513	0.2467

We have bolded the smallest error and it corresponds to the approximation made using the nonlinear difference equation with time interval equal to 2.

CONCLUSIONS

The aim of this study was the approximation of the index prices for Albania using two powerful tools of applied mathematics; numerical analysis methods and statistical ones. As we are dealing with real life values we have decided to use a nonlinear difference equation, because they represent more precisely the reality. The evaluation of the growth rate and the capacity building is a made using different time interval (T=2, 3, 4, 5) and from the error analysis it has come out that approximations made for T=2 are more accurate. In the third paragraph we have introduced the ARIMA and ETS models which give good results. From a final comparison of all these methods yields that the nonlinear difference equation gives a best result.

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