Linear of Viscoelastic Oscillations of Mechanical Systems with a Finite Number of Degrees of Freedom

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***Abstract*—In work describes the statements of problems on Eigen vibrations of the system with a finite number of degrees of freedom. Are considered oscillations of mechanical of systems consisting of absolutely rigid bodies, of interconnected, without massive viscoelastic elements?**

Keywords: Vibrations, viscoelastic system, the damping factor.

Introduction. Questions of use of dynamical of viscoelastic of damping of systems with two degrees of freedom, with all the scrutiny of the problem [1,2], is rarely discussed in the scientific literature. However, the protection tasks an object as solid in two elastic supports have implications for engineering practice; because of the "beam" of the type widely enough are used in the transportation dynamics [3, 4, 5, 6].

Mechanical problem. Are considered the natural oscillations of mechanical of systems consisting of S Tel (SKK - hard, Se - of viscoelastic ;). System of bodies connected to each other and the base without a mass (or by massive) viscoelastic elements. The viscoelastic properties of the materials described by the integral ratios of the Boltzmann-Voltaire [1,4]. Some of the deformable elements may be resilient, in this case the kernel of heredity describing the rheological properties of the elements are identically zero. System in which the rheological properties of deformable elements are identical (core elements of heredity are equal) will be called dissipative homogeneous, and the system with different rheological characteristics of deformable elements - dissipative inhomogeneous [2].

The main objective of work - the study of dissipative (damping) properties of the system as a whole. With free vibration damping rate quantifies the dissipative properties of the system: the higher the speed, the higher dissipation of. To quantify the dissipative properties of the system offers two quantities: the minimum rate of decay of the natural oscillations and maximum amplitude of the resonant. Dissipative properties of the system are determined primarily damping properties of elements [2]. This is true for dissipative homogeneous systems, completely inapplicable to dissipative inhomogeneous system. Displays the notion of global damping coefficient. Global dissipative damping characteristics of the inhomogeneous system as a whole are determined not only (and not so much), the viscoelastic properties of elements of the system, how the interaction of different natural modes of oscillations, which is substantially determined by the structure, design, geometry, dimensions, by elastic links, the mutual arrangement of elements [4, 5, 6]. In this case the real part of the complex natural frequency is the frequency of damped oscillations, the imaginary - the coefficient a damping of the natural oscillations of the system.

Statement of the problem and solution methods. In formulating the problem on their own and forced vibrations of the system uses the principle of virtual displacements, according to which the sum of all the works acting on the system of active forces, including the forces of inertia is zero [3]. Analyze of the dynamical coefficients for a dissipative inhomogeneous mechanical design CEA shown in Figure 1. Figure. 1 $\tilde{C}_j$ - The operator of the spring stiffness, which has the form (j=1, 2, 3) [2]

$$\tilde{C}_j\varphi(t) = C_{01j}\left[\varphi(t) - \int_0^t R_{cj}(t-\tau)\varphi(t)d\tau\right]; \qquad (1)$$

$\varphi(t)$- Arbitrary function of time; $R_{cj}(t-\tau)$- The core relaxation. Further, applying the procedure of freezing [2], we replace the relation (5) is approximately of the form

$$\bar{C}_j\varphi = C_j\left[1-\Gamma_j^C(\omega_R) - i\Gamma_j^S(\omega_R)\right]\varphi = \bar{C}_j\varphi,$$

Where $\Gamma_j^C(\omega_R) = \int_0^\infty R_j(\tau)\cos\omega_R\tau\, d\tau$,

$\Gamma_j^S(\omega_R) = \int_0^\infty R_j(\tau)\sin\omega_R\tau\, d\tau$, respectively, the cosine and sine - Fourier transforms relaxation kernel material. As an example, assume a three-parameter the viscoelastic material relaxation kernel $R_j(t) = A_j e^{-\beta_j t}/t^{1-\alpha_j}$, has a weak the singularity [2]. The technical challenge is to varying within a physically realizable stiffness of the deformable element, its size and weight, to achieve the maximum reduction of the amplitude of resonant vibrations. For a system with a finite number of degrees of freedom the variational problem reduces to a system of linear equations of Lagrange type II with complex generalized rigidity:

$$\sum_{k=1}^{6N}(a_{jk}q_k + \overline{C}_{jk}q_k) = 0,$$

$$j = 1,2,3,...6N$$

where a_{jk} - components of the real of a symmetric matrix of generalized masses; $\overline{C}_{jk} = C_{R_{jk}} + C_{I_{jk}}$ - The components of a complex of a symmetric matrix of generalized stiffness; q_k -are complex generalized coordinates (components of displacement of the center of mass and the angles of rotation of the rigid bodies). A solution is sought in the form of;

$q_j = A_j \exp(-i\omega t)$, Where $\omega = \omega_R + i\omega_I$ the complex natural frequency; A_j - complex natural modes. The problem reduces to of a complex algebraic eigenvalue problem of the system, of equations species

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = \{0\},$$

Reduces to the solution of the characteristic equation

$$det\big[[M]\lambda^2 + [C]\lambda + [K]\big] = 0, \qquad (2)$$

With nonlinearly within a complex parameter. The characteristic equation of the system (2) is solved by Muller, as an initial approximation decided to close (2) conservative goals. Kernel relaxation taken in the form of

$$R(t-\tau) = \frac{Ae^{-\beta(t-\tau)}}{(t-\tau)^{(1-\alpha)}}.$$

Cosine sine s - the images of this nucleus expressed by the formulas

$$U_n^c = \frac{A\Gamma(\alpha)}{(p^2n^2+\beta^2)^{\frac{\alpha}{2}}} cos\left(\alpha \cdot arctg\left(\frac{p_n}{\beta}\right)\right),$$

$$U_n^s = \frac{A\Gamma(\alpha)}{(p^2n^2+\beta^2)^{\alpha/2}} sin\left(\alpha \cdot arctg\left(\frac{p_n}{\beta}\right)\right),$$

Somewhere gamma functions. Consider the natural oscillations of the system with two degrees of freedom (Fig. 1). The following parameters [2]: $A = 0{,}048;\ \beta = 0{,}05;\ \alpha = 0{,}1;\ C_1 = 1;\ M = 1,$

The instantaneous rigidity varies. We consider two mechanical systems. In a first embodiment all elements of viscoelastic

$$R_1 = R_2 = R_3 = \frac{Aexp(-\beta t)}{t^{1-a}}, A = 0.048,$$
$$\beta = 0.05, \qquad \alpha = 0.1.$$

The calculation results are shown in Fig. 2a. The dependence of the natural frequencies of C_2

is the same as in the case of a homogeneous system, the corresponding curves are identical up to 5%. As for the damping coefficients, their behavior changes radically: the dependence ω_1 what C_2 becomes no monotonic. Particular interest is the minimum value of the damping factor at fixed stiffness C_2:

$$\delta = \min_k\{-\omega_{Ik}\}.$$

Magnitude δ determines the damping properties of the system. In the case of of a homogeneous system value C_2 (we call it the global damping factor) is entirely determined by the imaginary part of the complex modulus lowest natural frequency. In case of a heterogeneous of the system as a global damping factor depending on the magnitude C_2 act as the imaginary parts of the first and second natural frequencies. "Turn the Tables" occurs when the value of the characteristic value C_2, when real parts of the first and second natural frequencies are most similar. Global damping factor at the specified characteristic value C_2 has a pronounced maximum. This circumstance is, in our opinion, the new mechanical effect, which can be formulated as: fluctuations in their own forms of inhomogeneous viscoelastic of the system with close frequencies mutually cancel each other. The instantaneous rigidity C_2 is the geometric parameter determines the size, rather than the physical properties of the material. The main feature of the observed effect is the qualitative dependence of the dissipative properties of the system from its geometric parameters. Thus, the results obtained for the dissipative inhomogeneous viscoelastic structures are fully consistent with the decisions of the problem of free damped oscillations and confirm the sharp increase in the intensity of the dissipative processes in the approximation of the fundamental frequencies in inhomogeneous viscoelastic systems. The role of theology reduces both vibration damping, and to mutually enhance the interaction of oscillations of different modes, which significantly increases the dissipative properties of the system as a whole. This effect of the interaction of different forms of continuous motion of bodies is of fundamental outlook for the synthesis of optimal on dissipative properties and material engineering inhomogeneous dissipative structures, building products, damping materials and composites of various vibration isolation systems and devices. To elucidate the physical nature of the observed effect, we write the equation of motion of a system with n degrees of freedom in the normal coordinates of the elastic system. In the case of a homogeneous system all the relaxation kernel R_{ij}

same: $R_{ij} = R$, so that the matrix of generalized complex stiffness is positive definite real matrix multiplication by a complex scalar:

$$\overline{C}_{ij} = C_{ij}[1 - \Gamma^c(\omega_R) - i\Gamma^s(\omega_R)]$$

In the normal coordinates of the elastic problem system (1) takes the form

$$\theta_n + \Omega_n^2\theta_n\left(1 - \Gamma^c - i\Gamma^s\right) = \Psi_n \qquad (3)$$

Where Ω - complex natural frequency of the elastic system; Ψ_n - Generalized force corresponding to n - normal coordinate. The system (3) disintegrated into n individual equations. This means that the motion of a mechanical of a viscoelastic system represents a superposition independent normal oscillation damping, and forced to have finite resonance amplitude. The basic properties of conservative systems - the ability to drive oscillations of a normal coordinate without exciting the other - completely preserved in the case of a homogeneous viscoelastic system.

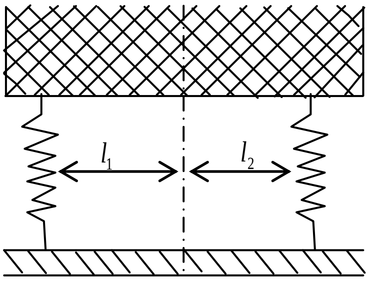

Fig.1. Design scheme.

The situation changes radically in the case of dissipative of an inhomogeneous system. Here generalized complex stiffness represents the sum of two matrices - real and complex that generally speaking, are not similar.

Three symmetrical not like generalized matrices (matrix of generalized mass, the real and imaginary parts of the matrix of generalized stiffness) not cause the EQSs C_k to diagonal form of a non-singular transformation. Therefore in the case of an inhomogeneous system of Lagrange's equation under normal kordinitah the elastic system has the form

$$\theta_n + \Omega_n^2\theta_n - \Omega_n^2\sum_{j=1}^{N}(\theta_{nj}^c + \theta_{nj}^s)\theta_j = \Psi_k \qquad (4)$$

Where $\theta_{nj}^c, \theta_{nj}^s$ -non-negative definite real matrix.

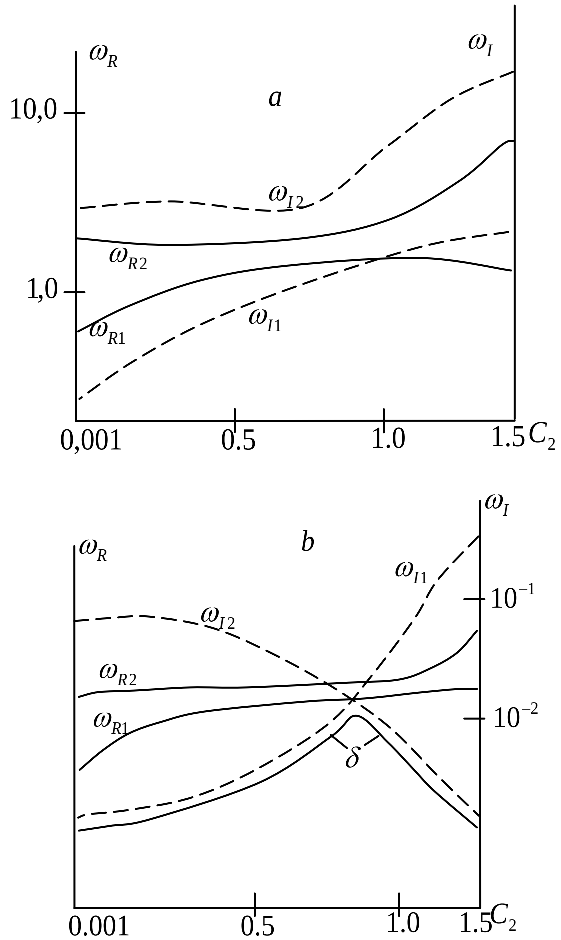

Fig.2. Dependence on of complex frequencies

The system (4) consists of N interconnected equations. Mechanical coupling, this means you cannot swing a separate excitation of normal coordinates.

Every movement of the inhomogeneous system is a superposition of several interacting vibrations normal coordinates, and the interaction of various normal coordinates, the most intense at close Eigen frequencies leads to an intensification of the dissipative processes in the system.

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