Fixed Point Results in Non-Archimedean Fuzzy Menger spaces

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***Abstract*—In this paper we study another Fuzzy Probabilistic metric space known as non-Archimedean Fuzzy Probabilistic metric space. Our object in this paper is to study on fixed points in non-Archimedean Fuzzy Menger Space for quasi-contraction type pair and triplets of maps.**

***Keywords*—Non-Archimedean Fuzzy Probabilistic Metric Space (FPM Space), Common Fixed Points.**

1. INTRODUCTION

Istrătescu and Crivăt [19] first studied Non-Archimedean probablistic metric spaces and some topological preliminaries on them. Achari [1] studied the fixed points of quasi-contraction type mappings in non-Archimedean PM-spaces and generalized the results of Istrătescu [18]. The fundamental results of Sehgal-Bharucha-Reid[31], Sherwood[34], were generalized and extended by many authors out of which some prominent one's are Achari[1], Chang [5, 7, 8, 9], Chang, S S and Huang, NJ [10], Ciric [12], Hadzic [14, 15, 16, 17], Istrătescu and Sacuiu[20], Mishra-Singh-Talwar[25], Singh & Pant [36,37,38,39,40] etc.

2. PRELIMINARIES:

Now we recall some definitions:

Definition 2.1: A Fuzzy probabilistic metric space is an ordered pair $(X,F_\alpha)$where X is a nonempty set, L be set of all distribution function and $F: X \times X \to L$ (collection of all distribution functions). The value of $F_\alpha(x,y)$ at $u \in X \times X$ is represented by $F_{\alpha_{x,y}}(u)$ or $F_\alpha(x,y;u)$ satisfy the following conditions:

$[FPM-1] F_{\alpha_{x,y}}(u) = 1$ for all $u > 0$ if and only if $x = y$

$[FPM-2]. F_{\alpha_{x,y}}(0) = 0$ for every $x, y \in X$

$[FPM-3]. F_{\alpha_{x,y}}(u) = F_{\alpha_{y,x}}(u)$ for every $x, y \in X$

$$[FPM-4]. F_{\alpha_{x,y}}(u) = 1 \text{ and } F_{\alpha_{y,z}}(v) = 1 \text{ then } F_{\alpha_{x,z}}(u+v) = 1$$

for every $x, y, z \in X$.

A Fuzzy Probabilistic Metric Space (X, F) is called non-Archimedean FPM – space if it satisfies $[FPM-5]. F_{\alpha_{x,z}}(u) = 1$ and $F_{\alpha_{z,y}}(v) = 1$ then $F_{\alpha_{x,y}} \max\{u,v\} = 1$ for every $x, y, z \in X$ instead of $[FPM-4]$.

Definition 2.2: A mapping $T: [0,1] \times [0,1] \to [0,1]$ is called t-norm if

1. $T(a,1) = a, \forall a \in [0,1]$
2. $T(0,0) = 0,$
3. $T(a,b) = T(b,a),$
4. $T(c,d) \geq T(a,b)$ for $c \geq a, d \geq b$, i.e. T is non-decreasing in both co-ordinates
5. $T(T(a,b),c) == T(a,T(b,c))$

$$\forall a,b,c,d \in [0,1]$$

i.e. T is associative.

Definition 2.3: In addition of definition 4.1, if T is continuous on $[0,1] \times [0,1]$ and $T(a,a) < a, a \in [0,1]$, then T is called an Archimedean t – norm. A characterization of Archimedean t – norm is due to Ling [16]. He proved that a t – norm T is Archimedean if and only if it admits the representation,

$$T(a,b) = g^{-1}[g(a) + g(b)]$$

where g is continuous and decreasing function from $[0,1]$ to $[0,\infty]$ with $g(1) = 0$ and $g(0) = \infty$ and g^{-1} is the pseudo inverse of g, (c.f. Chang [32])

$(g \circ g^{-1})(a) = a$, for all a in the range of g.

The continuous decreasing function g appearing in this characterization is called an additive generator of the Archimedean t-norm T.

Definition 2.4: A non-Archimedean Fuzzy Menger space is an ordered triplet (X,F_α,T) where (X,F_α) is non- Archimedean FPM-space, T is a t-norm with the Menger non-Archimedean triangle inequality;

$$F_{\alpha_{x,y}} \max\{u,v\} \geq T\{F_{\alpha_{x,z}}(u), F_{\alpha_{z,y}}(v)\}$$

$$F_{\alpha_{x,z}}(u) = 1 \text{ and } F_{\alpha_{z,y}}(v) = 1 \text{ then} F_{\alpha_{x,y}} \max\{u,v\} = 1$$

Definition 2.5: (Achari[1]) Let (X,F_α,T) be a non-Archimedean Fuzzy Probabilistic Metric space and let g be an additive generator, and let α be a number such that $0 < \alpha < 1$. A mapping $T: X \to X$ is a quasi – contraction mapping on X with respect g and α if for every $x, y \in X$,

$$g\{F_{\alpha_{Tx,Ty}}(u)\} \leq \alpha g \max\left\{F_{\alpha_{x,y}}\left(\frac{u}{\alpha}\right), F_{\alpha_{x,Tx}}\left(\frac{u}{\alpha}\right), F_{\alpha_{y,Ty}}\left(\frac{u}{\alpha}\right)\right\}$$

Definition 2.6: (Achari[1]) A mapping T: X→ X is a quasi-contraction type map on a non-Archimedean FPM-space (X, F_α) if and only if there exists a constant $\alpha \in (0,1)$ such that

$$F_{\alpha_{Tx,Ty}}(u) \leq \max\left\{F_{\alpha_{x,y}}\left(\frac{u}{\alpha}\right), F_{\alpha_{x,Tx}}\left(\frac{u}{\alpha}\right), F_{\alpha_{y,Ty}}\left(\frac{u}{\alpha}\right)\right\}$$

for all $x, y \in X$ and $u > 0, 0 < \alpha < 1$.

This can be interpreted as the probability that the distance between the image points Tx, Ty is less than u is at least equal to the probability that the maximum distances between x, y, x, Tx and y, Ty is less than u.

Definition 2.7: Let (X, F_α, T) be a non-Archimedean FPM-Space and let g be an additive generator, and let α be a number such that $0 < \alpha < 1$. The mappings $G, T, Q: X \to X$ is called a quasi-contraction type pair of mappings on X with respect g and α if for every $x, y \in X$ and $u > 0$,

$$g\left\{F_{\alpha_{Gx,Ty}}(u)\right\} \leq \alpha g\, \varphi\left\{F_{\alpha_{x,y}}\left(\frac{u}{\alpha}\right), F_{\alpha_{x,Gx}}\left(\frac{u}{\alpha}\right), F_{\alpha_{y,Ty}}\left(\frac{u}{\alpha}\right)\right\}$$

Definition 2.8: Let (X, F_α, T) be a non-Archimedean FPM-Space and let g be an additive generator, and let α be a number such that $0 < \alpha < 1$. The mappings $G, T, Q: X \to X$ is called a quasi-contraction type triplet of mappings on X with respect g and α if for every $x, y \in X$ and $u > 0$,

$$g\left\{F_{\alpha_{GQx,TQy}}(u)\right\} \leq \alpha g\, \varphi\left\{F_{\alpha_{x,y}}\left(\frac{u}{\alpha}\right), F_{\alpha_{x,GQx}}\left(\frac{u}{\alpha}\right), F_{\alpha_{y,TQy}}\left(\frac{u}{\alpha}\right)\right\}.$$

Definition 2.9: Mappings $G, T: X \to X$ is a quasi-contraction type A pair of maps on a non-Archimedean FPM-space (X, F_α) if and only if there exists a constant $\alpha \in (0,1)$ such that

$$F_{\alpha_{Gx,Ty}}(u) \leq \varphi\left\{\begin{matrix} F_{\alpha_{x,y}}\left(\frac{u}{\alpha}\right), F_{\alpha_{x,Gx}}\left(\frac{u}{\alpha}\right), F_{\alpha_{Gx,y}}\left(\frac{u}{\alpha}\right) \\ F_{\alpha_{y,Ty}}\left(\frac{u}{\alpha}\right) \end{matrix}\right\}$$

for all $x, y \in X$ and $u > 0, 0 < \alpha < 1$.

Definition 2.10: Mappings $G, T, Q: X \to X$ is a quasi-contraction type A triplet of maps on a non-Archimedean FPM-space (X, F_α) if and only if there exists a constant $\alpha \in (0,1)$ such that for all $x, y \in X$ and $u > 0, 0 < \alpha < 1$

$$F_{\alpha_{GQx,TQy}}(u) \leq \varphi\left\{\begin{matrix} F_{\alpha_{x,y}}\left(\frac{u}{\alpha}\right), F_{\alpha_{x,GQx}}\left(\frac{u}{\alpha}\right), F_{\alpha_{y,TQy}}\left(\frac{u}{\alpha}\right), \\ F_{\alpha_{GQx,y}}\left(\frac{u}{\alpha}\right) \end{matrix}\right\}.$$

3. MAIN RESULT:

We establish fixed point theorems for a quasi-contraction type pair A and a triplet of maps on complete non-Archimedean Fuzzy Menger space.

Theorem 3.1: Let (X, F_α, T) be a non Archimedean Fuzzy Menger space under the Archimedean t-norm T, with the additive generator g. Let G and T be two self mappings of X into itself satisfying;

$$(3.1(a))\; g\left\{F_{\alpha_{Gx,Ty}}(u)\right\} \leq \alpha g\, \varphi\left\{\begin{matrix} F_{\alpha_{x,y}}\left(\frac{u}{\alpha}\right), F_{\alpha_{x,Gx}}\left(\frac{u}{\alpha}\right), \\ F_{\alpha_{y,Ty}}\left(\frac{u}{\alpha}\right), F_{\alpha_{y,Gx}}\left(\frac{u}{\alpha}\right) \end{matrix}\right\}$$

for all $x, y \in X$ and $u > 0, 0 < \alpha < 1$.

(3.1(b)) G and T are continuous on X.

Then G and T have a unique common fixed point in X.

Proof: Let $x_0 \in X$ be a arbitrary element and $\{x_n\}$ be a sequence in X

such that $x_{2n+1} = Gx_{2n}, x_{2n+2} = Tx_{2n+1}, n = 0, 1, 2, 3 \dots\dots$ be the sequence of iterates under the pair {G, T} at x_0.

Now from (3.1(a))

$$g\left\{F_{\alpha_{x_1,x_2}}(u)\right\} = g\left\{F_{\alpha_{Gx_0,Tx_1}}(u)\right\}$$

$$\leq \alpha g\, \varphi\left\{\begin{matrix} F_{\alpha_{x_0,x_1}}\left(\frac{u}{\alpha}\right), F_{\alpha_{x_0,Gx_0}}\left(\frac{u}{\alpha}\right), F_{\alpha_{x_1,Gx_0}}\left(\frac{u}{\alpha}\right), \\ F_{\alpha_{x_1,Tx_1}}\left(\frac{u}{\alpha}\right) \end{matrix}\right\}$$

$$\leq \alpha g\, \varphi\left\{\begin{matrix} F_{\alpha_{x_0,x_1}}\left(\frac{u}{\alpha}\right), F_{\alpha_{x_0,x_1}}\left(\frac{u}{\alpha}\right), F_{\alpha_{x_1,x_1}}\left(\frac{u}{\alpha}\right), \\ F_{\alpha_{x_1,x_2}}\left(\frac{u}{\alpha}\right) \end{matrix}\right\}$$

$$\leq \alpha g\left\{F_{\alpha_{x_0,x_1}}\left(\frac{u}{\alpha}\right)\right\}$$

$$g\left\{F_{\alpha_{x_1,x_2}}(u)\right\} \leq \alpha g\left\{F_{\alpha_{x_0,x_1}}\left(\frac{u}{\alpha}\right)\right\}$$

Again, $g\left\{F_{\alpha_{x_2,x_3}}(u)\right\} \leq g\left\{F_{\alpha_{Gx_1,Tx_2}}(u)\right\}$

$$\leq \alpha g\left\{F_{\alpha_{x_1,x_2}}\left(\frac{u}{\alpha}\right)\right\}$$

$$\leq \alpha^2 g\left\{F_{\alpha_{x_0,x_1}}\left(\frac{u}{\alpha^2}\right)\right\}$$

Therefore, $g\left\{F_{\alpha_{x_2,x_3}}(u)\right\} \leq \alpha^2 g\left\{F_{\alpha_{x_0,x_1}}\left(\frac{u}{\alpha^2}\right)\right\}$

Hence it follows by induction that for every positive integer n,

$$g\left\{F_{\alpha_{x_n,x_{n+1}}}(u)\right\} \leq \alpha^n g\left\{F_{\alpha_{x_0,x_1}}\left(\frac{u}{\alpha^n}\right)\right\} \dots\dots (3.1.1)$$

Now for $m > n > 0$ and $u > 0$ we have,

$$F_{\alpha_{x_{2n+1},x_{2n+2m}}}(u) \geq T\{F_{\alpha_{x_{2n+1},x_{2n+2}}}(u), F_{\alpha_{x_{2n+2},x_{2n+2m}}}(u)\}$$

$$\geq T\{F_{\alpha_{x_{2n+1},x_{2n+2}}}(u), F_{\alpha_{x_{2n+2},x_{2n+2m}}}(\alpha\, u)\}$$

Since α < 1 and T is non decreasing and (FPM-5)

$$F_{\alpha_{x_{2n+1},x_{2n+2m}}}(u)$$

$$\geq T\left\{F_{\alpha_{x_{2n+1},x_{2n+2}}}(u), T(F_{\alpha_{x_{2n+2},x_{2n+3}}}(\alpha\, u), F_{\alpha_{x_{2n+3},x_{2n+2m}}}(\alpha^2\, u))\right\}$$

$$\geq T\left\{\begin{matrix} T(F_{\alpha_{x_{2n+1},x_{2n+2}}}(u), \\ F_{\alpha_{x_{2n+2},x_{2n+3}}}(\alpha\, u), F_{\alpha_{x_{2n+3},x_{2n+2m}}}(\alpha^2\, u)) \end{matrix}\right\}$$

$$= \quad g^{-1}\left\{g[T(F_{\alpha_{x_{2n+1},x_{2n+2}}}(u), F_{\alpha_{x_{2n+2},x_{2n+3}}}(\alpha\, u))] + g[F_{\alpha_{x_{2n+3},x_{2n+2m}}}(\alpha^2\, u)]\right\}$$

$$= g^{-1}\left\{g\left[g^{-1}\left\{g\left[(F_{\alpha_{x_{2n+1},x_{2n+2}}}(u)\right] + g\left[F_{\alpha_{x_{2n+2},x_{2n+3}}}(\alpha\, u)\right]\right\}\right] + g[F_{\alpha_{x_{2n+3},x_{2n+2m}}}(\alpha^2\, u)]\right\}$$

$$\geq g^{-1}\left\{g\left[g^{-1}\left\{\begin{matrix}\alpha^{2n+1} g\left[(F_{\alpha_{x_0,x_1}}\left(\frac{u}{\alpha^{2n+1}}\right)\right] \\ +\alpha^{2n+2} g\left[(F_{\alpha_{x_0,x_1}}\left(\frac{u}{\alpha^{2n+1}}\right)\right] \\ \dots\dots + \alpha^{2n+2m+2} g[F_{\alpha_{x_0,x_1}}\left(\frac{u}{\alpha^{2n+1}}\right)]\end{matrix}\right\}\right]\right\}$$

Hence we conclude $\{x_n\}$ is a Cauchy sequence, since g^{-1} and g are continuous, $\alpha \to 0$, as $n \to \infty, F_{x,y}(u) \to 1$ as $u \to \infty$ and $g^{-1}(0) = 1$.

Since (X, F_α, T) is complete there is point $z \in X$ such that $x_n \to z$.

According to Istrătescu and Sacuiu [20], the subsequences $\{x_n\}, \{x_{n+1}\}$ converges to z i.e. $x_n \to z, x_{n+1} \to z$ continuity of G and T implies $Gx_n \to Gz, Tx_n \to Tz$.

We shall now show that z is common fixed point of G and T.

However we have,

$$F_{\alpha_{z,Gz}}(u) \geq T\left\{F_{\alpha_{z,x_{2n}}}(u), F_{\alpha_{x_{2n},Gz}}(u)\right\}$$

$$= g^{-1}\left\{g\left[F_{\alpha_{z,x_{2n}}}(u)\right] + g\left[F_{\alpha_{x_{2n},Gz}}(u)\right]\right\}$$

$$= g^{-1}\left\{g\left[F_{\alpha_{z,x_{2n}}}(u)\right] + g\left[F_{\alpha_{Tx_{2n-1},Gz}}(u)\right]\right\}$$

$$= g^{-1}\left\{g\left[F_{\alpha_{z,x_{2n}}}(u)\right] + \alpha g\left[F_{\alpha_{x_{2n-1},z}}(u/\alpha)\right]\right\}$$

$$\geq \lim_{n\to\infty} g^{-1}\left\{g\left[F_{\alpha_{z,x_{2n}}}(u)\right] + \alpha g\left[F_{\alpha_{x_{2n-1},z}}(u/\alpha)\right]\right\} = 1$$

Using (3.1(a)) and (3.1(b)) we get $Gz = z$.

Again,

$$F_{\alpha_{z,Tz}}(u) \geq T\left\{F_{\alpha_{z,x_{2n+1}}}(u), F_{\alpha_{x_{2n+1},Tz}}(u)\right\}$$

$$= g^{-1}\left\{g\left[F_{\alpha_{z,x_{2n+1}}}(u)\right] + gF_{\alpha_{x_{2n+1},Tz}}(u)\right\}$$

$$= g^{-1}\left\{g\left[F_{\alpha_{z,x_{2n+1}}}(u)\right] + g\left[F_{\alpha_{Gx_{2n},Tz}}(u)\right]\right\}$$

$$= g^{-1}\left\{g\left[F_{\alpha_{z,x_{2n+1}}}(u)\right] + \alpha g\left[F_{\alpha_{x_{2n},z}}(u/\alpha)\right]\right\}$$

$$\geq \lim_{n\to\infty} g^{-1}\left\{g\left[F_{\alpha_{z,x_{2n+1}}}(u)\right] + \alpha g\left[F_{\alpha_{x_{2n},z}}(u/\alpha)\right]\right\} = 1$$

Thus z is common fixed point of G and T.

In order to show that z is the only common fixed point of G and T, if possible let w be any other common fixed point of G and T

We have from (3.1(a))

$$F_{\alpha_{z,w}}(u) = F_{\alpha_{Gz,Tw}}(u)$$

$$g\left\{F_{\alpha_{z,w}}(u)\right\} = g\left\{F_{\alpha_{Gz,Tw}}(u)\right\}$$

$$\leq \alpha g\, \varphi\left\{\begin{matrix} F_{\alpha_{z,w}}\left(\frac{u}{\alpha}\right), F_{\alpha_{z,Gz}}\left(\frac{u}{\alpha}\right), F_{\alpha_{w,Tw}}\left(\frac{u}{\alpha}\right), \\ F_{\alpha_{w,Gz}}\left(\frac{u}{\alpha}\right) \end{matrix}\right\}$$

$$\leq \alpha g\left\{F_{\alpha_{z,w}}\left(\frac{u}{\alpha}\right)\right\}$$

Therefore $g\left\{F_{\alpha_{z,w}}(u)\right\} \leq \alpha g\left\{F_{\alpha_{z,w}}\left(\frac{u}{\alpha}\right)\right\} < g\left\{F_{\alpha_{z,w}}\left(\frac{u}{\alpha}\right)\right\}$ since $\alpha < 1$.

This implies $F_{\alpha_{z,w}}(u) \geq F_{\alpha_{z,w}}\left(\frac{u}{\alpha}\right)$ since g is decreasing function.

This gives a contradiction, as $\frac{u}{\alpha} > u$ as $\alpha < 1$ and $F_{\alpha_{x,y}}(u)$ is non decreasing function

This implies $z = w$.

This completes the proof.

In the next theorem we further extend the results of theorem 3.1 for three self mappings.

Theorem 3.2: Let (X, F_α, T) be a non-Archimedean Fuzzy Menger space under the Archimedean t-norm T, with the additive generator g. Let G,T and Q be three self mappings of X into itself satisfying;

(3.2(a)) $g\left\{F_{\alpha_{GQx,TQy}}(u)\right\}$

$$\leq \alpha g\, \varphi\left\{\begin{matrix} F_{\alpha_{x,y}}\left(\frac{u}{\alpha}\right), F_{\alpha_{x,GQx}}\left(\frac{u}{\alpha}\right), \\ F_{\alpha_{y,TQy}}\left(\frac{u}{\alpha}\right), F_{\alpha_{y,GQx}}\left(\frac{u}{\alpha}\right) \end{matrix}\right\}$$

for all $x, y \in X$ and $u > 0, 0 < \alpha < 1$.

(3.2(b)) Q commutes with G and T, that is, $GQ = QG$ and $TQ = QT$

(3.2(c)) G, Q and T are continuous on X.

Then G, T and Q have a unique common fixed point in X.

Proof: Suppose $GQ = U$ and $TQ = V$, then U and V satisfy all conditions of theorem 3.1 and therefore U and V have unique common fixed point say z.

$$Uz = Vz = z$$

$$\text{i.e. } GQz = z, TQz = z.$$

Now we shall show z is a common fixed point of G, T and Q.

It will be sufficient to prove $Tz = z$.

We have, $F_{\alpha_{z,Qz}}(u) = F_{\alpha_{GQz,TQz}}(u)$

$$g\left\{F_{\alpha_{z,Qz}}(u)\right\} = g\left\{F_{\alpha_{GQz,TQQz}}(u)\right\}$$

From (3.2(a)) we have

$$g\left\{F_{\alpha_{GQz,TQQz}}(u)\right\} \leq \alpha g\,\varphi \left\{\begin{matrix} F_{\alpha_{z,Qz}}\left(\frac{u}{\alpha}\right), F_{\alpha_{z,GQQz}}\left(\frac{u}{\alpha}\right), \\ F_{\alpha_{Qz,TQQz}}\left(\frac{u}{\alpha}\right), F_{\alpha_{Qz,GQQz}}\left(\frac{u}{\alpha}\right) \end{matrix}\right\}$$

$$= \alpha g\left\{F_{\alpha_{z,Qz}}\left(\frac{u}{\alpha}\right)\right\}$$

Therefore, $g\left\{F_{\alpha_{z,Qz}}(u)\right\} \leq \alpha g\left\{F_{\alpha_{z,Qz}}\left(\frac{u}{\alpha}\right)\right\} < g\left\{F_{\alpha_{z,Qz}}\left(\frac{u}{\alpha}\right)\right\}$ since $\alpha < 1$

This implies

$F_{\alpha_{z,Qz}}(u) \geq F_{\alpha_{z,Qz}}\left(\frac{u}{\alpha}\right)$, for all $u > 0$ since g is decreasing function.

This gives a contradiction, as $\frac{u}{\alpha} > u$ as $\alpha < 1$ and $F_{\alpha_{x,y}}(u)$ is non decreasing function. This implies $Qz = z$.

Now, $z = Uz = GQz = Gz$ and $z = Vz = TQz = Tz$.

Thus $z = Gz = Tz = Qz$.

The uniqueness of z as a common fixed point of G, Q and T.

Follows from the fact that z is a unique common fixed point of GQ and TQ.

This completes the proof.

Remark : Set **Q**=I in above theorem, we get theorem 3.1.

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