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Model of Strategic Decision Making in Mining Industry Based on Fuzzy Dynamic TOPSIS Method

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***Abstract*— Large capital intensive projects, such as those in the mineral resource industry, are often associated with diverse sources of uncertainties. These uncertainties can greatly influence the project success. Making optimal decision in such environment is high intensity exercise task. The value of managerial flexibility is assessed using data on input parameters (criteria), possible technical solutions (alternatives) and interaction between them.**

Keywords—mining, strategic decision making, uncertainty, fuzzy dynamic criteria, rank of alternatives, TOPSIS method

I. Introduction

A typical mining operation presents uncertainties and challenges in all its aspects, including evaluation, finance and construction. Mine engineers are often expected to make evaluation decisions at different stages of mine projects based on limited and uncertain data. For example, selection of surface mining technology belongs to the strategic decision making process. Selection of adequate mining equipment is also recognized as one of the most influencing decision. In underground mining, selection of suitable mining method can be treated almost as irreversible process which cannot be recouped without loss of significant amount of capital. Decisions pertaining to a capacity investment can have vital short and long-term consequences on the mine company's ability to compete, and even survive.

Decision making in mining industry in today's environment is much complex than it was just a few years ago. Having the ability to incorporate flexible alternatives and uncertainties of influencing parameters into decision making process is increasingly recognized as critical to long-term mining project success.

Many researchers in mining industry have applied different mathematical approaches to make optimal decision. Samis and Poulin (1996, 1998) provide a related decision-tree model where mineral price is the underlying source of uncertainty. In their model, management has the option to develop a large low-quality mineralized zone when a smaller high-quality zone, where operations are currently focused, is exhausted in nine years' time [10], [11]. Alpay and Yavuz (2009) applied Analytic Hierarchy Process (AHP) to select suitable underground mining method with respect to deposit characteristics [1]. Musingwini (2010) also applied AHP to make techno-economic optimization of level and raise spacing in inclined narrow reef mining [9]. Gligoric et al. (2010) developed a model for shaft location selection at deep multiple orebody deposit where the available alternative locations are defined by using network optimization and optimal solution is obtained by Fuzzy Technique for Order Preference by Similarity to Ideal Solution (FTOPSIS) [6].

The main aim of this paper is to develop an integrated dynamic model based on FTOPSIS method in order to help mining engineers and management of the company in the process of strategic decision making and mine design. In order to avoid subjective preference of decision makers we applied the concept of Shannon's entropy to calculate criteria weights. To decrease uncertainty related to influencing parameters, we have applied the concept of fuzzy sets theory. If there is only one criterion depending on time then we are faced with dynamic multiple criteria decision making problem. It means the rank of proposed alternatives is changed over defined time horizon. The model is developed on the basis of rank of proposed alternatives changing over defined time horizon and takes into account the variability of input parameters.

The rest of this paper is organized as follows. In section 2, a brief description of fuzzy theory is presented, including fuzzy sets, fuzzy numbers, linguistic variables and way of transformation of linguistic variables to triangular fuzzy numbers. Section 3 illustrates the process of decision making based on fuzzy dynamic TOPSIS method. Section 4 presents the testing of proposed model by hypothetical example related to selection of new mining technology at one operating surface clay mine. Finally, concluding remarks are discussed in section 5.

II. Basics of Fuzzy sets theory

In order to deal with vagueness of human thought, Zadeh [16] first introduced the fuzzy set theory. This theory was oriented to the rationality of uncertainty, owing to imprecision or vagueness. A fuzzy set is a class of objects with a continuum of grades of membership. The role of fuzzy sets is significant when applied to complex phenomena not easily described by traditional mathematical methods, especially when the goal is to find a good approximate solution [3]. Modeling using fuzzy sets has proved to be an effective way of formulating decision problems, where the information available is subjective and imprecise [20].

A fuzzy number $\widetilde{M}$ is a convex normalized fuzzy set $\widetilde{M}$ of the real line R [2]:

- it exists such that one $x_0 \in R$ with $\mu_{\widetilde{M}}(x_0) = 1$ (x_0 is called mean value of $\widetilde{M}$)

- $\mu_{\widetilde{M}}(x_0)$ is piecewise continuous.

There are many possibilities to use different fuzzy numbers according to the situation. Triangular fuzzy numbers (TFN) are very convenient to work with because of their computational simplicity and they are useful in promoting representation and information processing in fuzzy environment. In this paper, we use TFNs.

Triangular fuzzy numbers can be defined as a triplet (*a*,*b*,*c*). The parameters *a*, *b* and *c* respectively, indicate the smallest possible value, the most promising value and the largest possible value that describe a fuzzy event. The membership function is defined as [8], Fig.1:

$$\mu_{\widetilde{M}}(x_0) = \begin{cases} 0, x < a \\ \frac{x-a}{b-a}, a \le x \le b \\ \frac{c-x}{c-b}, b \le x \le c \\ 0, x > c \end{cases} \quad (1)$$

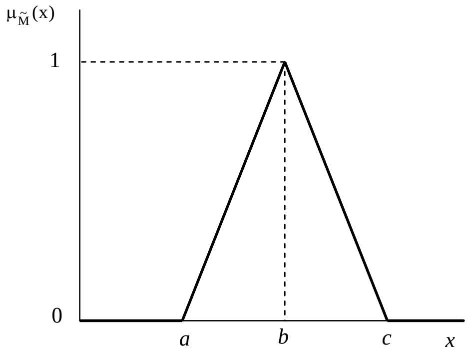

Fig. 1. *Triangular fuzzy number*

Triangular fuzzy numbers can be used to perform common mathematical operations. The basic fuzzy arithmetic operations on two triangular fuzzy memberships $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ are defined as follows:

inverse: $\tilde{A}^{-1} = (1/a_3, 1/a_2, 1/a_1)$;

addition: $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$;

subtraction: $\tilde{A} - \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$;

scalar multiplication: $\forall \varphi > 0, \varphi \in R, \varphi \cdot \tilde{A} = (\varphi \cdot a_1, \varphi \cdot a_2, \varphi \cdot a_3)$;

multiplication: $\tilde{A} \cdot \tilde{B} = (a_1 \cdot b_1, a_2 \cdot b_2, a_3 \cdot b_3)$;

division: $\tilde{A}/\tilde{B} = (a_1/b_3, a_2/b_2, a_3/b_1)$.

An important concept related to the applications of fuzzy numbers is defuzzification, which converts a fuzzy number into a crisp value. Such a transformation is not unique because different methods are possible. The most commonly used defuzzification method is the centroid defuzzification method, which is also known as center of gravity or center of area defuzzification. The centroid defuzzification method can be expressed as follows (Yager 1981) [14]:

$$\bar{x}_0(\tilde{A}) = \int_a^c x\mu_{\tilde{A}}(x)dx / \int_a^c \mu_{\tilde{A}}(x)dx \quad (2)$$

where $\bar{x}_0(\tilde{A})$ is the defuzzified value. The defuzzification formula of triangular fuzzy numbers (*a*,*b*,*c*) is

$$\bar{x}_0(\tilde{A}) = (a + b + c)/3 \quad (3)$$

and it will be used in this paper.

Input parameter can be expressed in quality way (a linguistic variable). A linguistic variable is a variable whose values are words or sentences in a natural or artificial language (Zadeh 1975) [17]. As an illustration, age is a linguistic variable if its values are assumed to be fuzzy variables labelled young, not young, very young, not very young, etc. rather than the numbers 0, 1, 2, 3. (Bellman 1977) [2]. The concept of a linguistic variable provides a means of approximate characterization of phenomena which are too complex to be amenable to description in conventional quantitative terms (Zadeh 1975) [17]. To overcome difficulties related to linguistic variable it is necessary to create fuzzy linguistic variable scale as follows: very low (VL), low (L), medium (M), high (H) and very high (VH). The next step concerns the transformation of the fuzzy linguistic variables to fuzzy triangular numbers. Such transformation is based on the knowledge of expert dealing with input parameter (for example, impact of mining activities on environment).

III. Decision Making Model Based on Fuzzy Dynamic TOPSIS Method

The problem of strategic decision making with uncertainty can be represented as Alternatives, Criteria, Evaluations model. We consider:

1. A finite set of alternatives: $= \{A_1, A_2, \ldots, A_m\}$.

2. A finite set of criteria: $\tilde{C} = \{\tilde{C}_1, \tilde{C}_2, \tilde{C}_3, \ldots, \tilde{C}_n\}$, where criteria are defined as fuzzy values. Some of criteria can be defined as crisp values and in that case we have a hybrid model.

3. A set of evaluations of alternatives with respect to defined criteria: $\tilde{D} = |\tilde{x}_{ij}|$.

Suppose we want to select the optimal surface mining technology according to deposit characteristics.

The proposed model is composed of the following main steps:

Step 1. Create a set of deposit properties (morphology of deposit, depth, inclination, reserves, quality, etc.)

Step 2. Identify feasible mining technologies (alternatives) with respect to deposit properties (continuous mining, discontinuous mining, semi-continuous mining, etc.)

Step 3. Create a set of criteria and define their uncertainties (fuzzy approach) and time depending nature (static or dynamic). It is necessary to assign the target value to each criterion (*max* or *min*). Usually applied criteria are capital costs, production costs, impact of mining activities on environment, reliability, etc.

Step 4. Form the time depending decision matrices for defined time horizon, $\tilde{D}(t) = \{\tilde{x}_{ij}(t_1), \tilde{x}_{ij}(t_2), \dots\}$ $i = 1,2,\dots,m;$ $j = 1,2,\dots,n;$ $t = 1,2,\dots,T$

Step 5. Calculate fuzzy aggregated overall relative closeness value of each alternative, $\tilde{Q}_i^{ag} = \sum_{t=1}^{T} \tilde{\lambda}(t) \cdot \tilde{Q}_i(t)$ $i = 1,2,\dots,m$.

Step 6. Create overall rank of alternatives according to the descending order of defuzzified aggregated overall relative closeness value.

To obtain the most efficiency alternative it is necessary to execute the fuzzy dynamic TOPSIS procedure.

TOPSIS method is a technique for order preference by similarity to ideal solution proposed by Hwang and Yoon (1981) [7]. The basic concept of this method is that the chosen alternative should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. Positive ideal solution is a solution that maximizes the benefit criteria and minimizes cost criteria, whereas the negative ideal solution maximizes the cost criteria and minimizes the benefit criteria (Wang 2006) [13]. In the classical TOPSIS method, the weights of the criteria and the ratings of alternatives are known precisely and crisp values are used in the evaluation process. However, under many conditions crisp data are inadequate to describe real-life decision problems. In such cases, the fuzzy TOPSIS method is proposed where the weights of criteria and ratings of alternatives are evaluated by linguistic variables represented by fuzzy numbers.

There are many applications of fuzzy TOPSIS in the literature. For instance, Chen (2000) [4] extended the TOPSIS to the fuzzy environment and gave numerical example of system analysis engineer selection for a software company. Chu (2002) [5] presented a fuzzy TOPSIS model under group decisions for solving the facility location selection problem. Yang and Hung (2007) [15] proposed to use TOPSIS and fuzzy TOPSIS methods for plant layout design problem. In general, a multiple criteria decision making problem can be concisely expressed in matrix format as:

$$\tilde{D} = |\tilde{x}_{ij}| = \begin{vmatrix} A/C & C_1 & C_2 & \dots & C_n \\ A_1 & \tilde{x}_{11} & \tilde{x}_{12} & \dots & \tilde{x}_{1n} \\ A_2 & \tilde{x}_{21} & \tilde{x}_{22} & \dots & \tilde{x}_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_m & \tilde{x}_{m1} & \tilde{x}_{m2} & \dots & \tilde{x}_{mn} \end{vmatrix} \tag{4}$$

where A_1, A_2,., A_m are possible alternatives, C_1, C_2,., C_n are criteria which measure the performance of alternatives and $\tilde{x}_{ij}$ is the rating of alternative A_i with respect to criteria C_j. In this paper, the rating of alternative A_i with respect to criteria is represented as triangular fuzzy numbers.

The fuzzy TOPSIS method is based on the following steps.

Step 1. Construct the normalized decision matrix $\tilde{R}$

The first step concerns the normalization of the judgment matrix $\tilde{D} = |\tilde{x}_{ij}|$. Each element $\tilde{x}_{ij}$ is transformed using the following equation

$$\tilde{r}_{ij} = \frac{\tilde{x}_{ij}}{\sum_{i=1}^{m} \tilde{x}_{ij}} = \frac{(a_{ij}, b_{ij}, c_{ij})}{\sum_{i=1}^{m} (a_{ij}, b_{ij}, c_{ij})} \quad j = 1,2,\dots,n \tag{5}$$

The normalized decision matrix is as follows:

$$\tilde{R} = |\tilde{r}_{ij}| = \begin{vmatrix} A/C & C_1 & C_2 & \dots & C_n \\ A_1 & \tilde{r}_{11} & \tilde{r}_{12} & \dots & \tilde{r}_{1n} \\ A_2 & \tilde{r}_{21} & \tilde{r}_{22} & \dots & \tilde{r}_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_m & \tilde{r}_{m1} & \tilde{r}_{m2} & \dots & \tilde{r}_{mn} \end{vmatrix} \tag{6}$$

Step 2. Construct the weighted normalized decision matrix $\tilde{V}$

Criteria importance is a reflection of the decision maker's subjective preference as well as the objective characteristics of the criteria themselves (Zeleny 1982) [18]. In order to determine criteria importance, we applied concept of the entropy method. Shannon and Weaver (1947) [12] proposed the entropy concept and this concept has been highlighted by Zeleny (1982) [18] for deciding the objective weights of criteria. Entropy is a measure of uncertainty in the information formulated using probability theory. To determine weights by the entropy measure, the normalized decision matrix $\tilde{R} = |\tilde{r}_{ij}|$ given by (6) is considered. The amount of decision information contained in (6) and associated with each criterion can be measured by the entropy value $\tilde{e}_j$ as:

$$\tilde{e}_j = -k \sum_{i=1}^{m} \tilde{r}_{ij} \cdot ln(\tilde{r}_{ij}) \tag{7}$$

where $k = 1/ln(m)$ is a constant that guarantees $0 \le \tilde{e}_j \le 1$. The degree of divergence (d_{ij}) of the average information contained by each criterion C_j (j = 1,2,…,n) can be calculated as:

$$\tilde{d}_j = 1 - \tilde{e}_j \tag{8}$$

The objective weight for each criterion C_j (j =1,2,…,n) is thus given by:

$$\tilde{w}_j = \tilde{d}_j / \sum_{j=1}^{n} \tilde{d}_j \tag{9}$$

Finally the weighted normalized decision matrix is as follows:

$$\tilde{V} = \left|\tilde{r}_{ij}\tilde{w}_{ij}\right| = \begin{vmatrix} A/C & C_1 & C_2 & \dots & C_n \\ A_1 & \tilde{r}_{11}\tilde{w}_1 & \tilde{r}_{12}\tilde{w}_2 & \dots & \tilde{r}_{1n}\tilde{w}_n \\ A_2 & \tilde{r}_{21}\tilde{w}_1 & \tilde{r}_{22}\tilde{w}_2 & \dots & \tilde{r}_{2n}\tilde{w}_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_m & \tilde{r}_{m1}\tilde{w}_1 & \tilde{r}_{m2}\tilde{w}_2 & \dots & \tilde{r}_{mn}\tilde{w}_n \end{vmatrix} \quad (10)$$

Step 3. Define the ideal and the negative-ideal solutions

Let us suppose that $\tilde{A}^{+}$ identifies the ideal solution and $\tilde{A}^{-}$ the negative one. They are defined as follows:

$$\tilde{A}^{+} = \left\{\left(\underbrace{max}_{i=1,2,.,m}\ \tilde{v}_{ij}|j \in J\right), \left(\underbrace{min}_{i=1,2,.,m}\ \tilde{v}_{ij}|j \in J'\right)\right\} = \{\tilde{v}_1^{+}, \tilde{v}_2^{+}, \dots, \tilde{v}_n^{+}\} \quad (11)$$

$$\tilde{A}^{-} = \left\{\left(\underbrace{min}_{i=1,2,.,m}\ \tilde{v}_{ij}|j \in J\right), \left(\underbrace{max}_{i=1,2,.,m}\ \tilde{v}_{ij}|j \in J'\right)\right\} = \{\tilde{v}_1^{-}, \tilde{v}_2^{-}, \dots, \tilde{v}_n^{-}\} \quad (12)$$

where

J = {*j* = 1,2,.,*n* | *j* associated with the benefit criteria}

J' = {*j* = 1,2,.,*n* | *j* associated with the cost criteria}

With benefit and cost attributes, we discriminate between criteria that the decision maker desires to maximize or minimize, respectively.

Step 4. Measure the distance between alternatives and ideal solutions

To calculate the *n*-Euclidean distance from each alternative to $\tilde{A}^{+}$ and $\tilde{A}^{-}$ the following equations can be easily adopted:

$$\tilde{S}_i^{+} = \sqrt{\sum_{j=1}^{n}\left(\tilde{v}_{ij} - \tilde{v}_j^{+}\right)^2} \quad i = 1,2,\dots,m \quad (13)$$

$$\tilde{S}_i^{-} = \sqrt{\sum_{j=1}^{n}\left(\tilde{v}_{ij} - \tilde{v}_j^{-}\right)^2} \quad i = 1,2,\dots,m \quad (14)$$

Step 5. Measure of the relative closeness to the ideal solution and final ranking

The final ranking of alternatives is obtained by referring to the value of the relative closeness to the ideal solution, defined as follows:

$$\tilde{Q}_i = \frac{\tilde{S}_i^{-}}{\tilde{S}_i^{+} + \tilde{S}_i^{-}} \quad i = 1,2,\dots,m \quad (15)$$

The best alternative is the one which has the shortest distance to the ideal solution. Ranking the defuzzified values is carried out in descending order.

If there is only one criterion depending on time then we are faced with dynamic multiple criteria decision making problem (DMCDMP). It means the rank of proposed alternatives is changed over defined time horizon. DMCDMP can be expressed as follows:

$$\tilde{D}(t) = \left|\tilde{x}_{ij}(t)\right| \ i = 1,2,\dots,m;\ j = 1,2,\dots,n;\ t = 1,2,\dots,T \quad (16)$$

where *T* is total project time.

***Definition* 1**: Let $\tilde{Q}_i(t), i = 1,2,\dots,m;\ t = 1,2,\dots,T$, be a set of the relative closeness to the ideal solution obtained for *T* different time periods, and $\tilde{\lambda}(t) = \left(\tilde{\lambda}(t_1), \tilde{\lambda}(t_2), \dots, \tilde{\lambda}(t_T)\right)$ be the weight vector of the *T* periods, then aggregated overall relative closeness value $\tilde{Q}_i^{ag}$ of the *i*-th alternative is defined as follows [19]:

$$\tilde{Q}_i^{ag} = \sum_{t=1}^{T}\tilde{\lambda}(t) \cdot \tilde{Q}_i(t) \ \ i = 1,2,\dots,m \quad (17)$$

where $\tilde{\lambda}(t) \geq 0, \sum_{t=1}^{T}\tilde{\lambda}(t) = 1$. In generally, $\tilde{\lambda}(t) = \left(\tilde{\lambda}(t_1), \tilde{\lambda}(t_2), \dots, \tilde{\lambda}(t_T)\right)$ can be given by the decision maker's subjective preference. To define $\tilde{\lambda}(t)$ we also apply the entropy method described above.

***Definition* 2**: Let $\tilde{W} = \left[\tilde{w}_{jt}\right]_{n\times T}, \tilde{w}_{jt} \geq 0, \sum_{j=1}^{n}\tilde{w}_{jt} = 1$, be a matrix of criteria weight over project time. The weight vector $\tilde{\lambda}(t) = \left(\tilde{\lambda}(t_1), \tilde{\lambda}(t_2), \dots, \tilde{\lambda}(t_T)\right)$ is defined as follows:

$$\tilde{\lambda}(t) = \tilde{d}_w(t) / \sum_{t=1}^{T}\tilde{d}_w(t) \quad (18)$$

where

$\tilde{d}_w(t) = 1 - \tilde{e}_w(t)$ – degree of divergence of the average weight information contained within each time interval,

$\tilde{e}_w(t) = -k_w\sum_{j=1}^{n}\tilde{w}_{jt} \cdot ln\left(\tilde{w}_{jt}\right)$ – the entropy value of weight information contained in the criteria weight matrix $\tilde{W}$,

$k_w = 1/ln(n)$ – a constant that guarantees $0 \leq \tilde{e}_w(t) \leq 1$.

Overall rank of alternatives is obtained according to the descending order of defuzzified $\tilde{Q}_i^{ag}$, that is, larger defuzzified $\tilde{Q}_i^{ag}$ means better alternative.

IV. NUMERICAL EXAMPLE

Management of surface clay mine is faced with need to develop new mining zone, because the zone where operations are currently focused, is exhausted in two years' time. Equipment that is currently employed to mining is depreciated and cannot be used for new increased production rate. Management of company must select the optimal mining technology with respect to deposit properties and defined production rate.

Note, the situation is hypothetical and the numbers used are in to permit calculation.

Mining technologies (alternatives) are given as follows:

Alternative A1: Stream-Cyclic (Semi-Continuous) mining technology

Excavation on the bench is done by continuous equipment. Transportation of mined clay from bench to dump site is performed by continuous equipment. Discontinuous equipment is used to load and transport the clay from dump site to processing facility. Technological system structure is composed of the following components: bucket excavator→belt conveyor→loader→truck.

Alternative A2: Cyclic (Continuous) mining technology

Excavation-loading and transport operations are done by continuous equipment. Technological system structure is composed of the following components: bucket excavator→belt conveyor→spreader.

Alternative A3: Stream-Cyclic (Semi-Continuous) mining technology

Equal to alternative A1 but with different technological system structure: bucket excavator→belt conveyor→loader→rail.

Criteria used in the process of decision making are as follows:

Criterion C1: Capital costs (mill USD). This criterion is defined by fuzzy triangular number and its value should be minimized.

Criterion C2: Production costs (USD/m3). This criterion is also defined by fuzzy triangular number and its value should be minimized. Although there is some intention to create correlation between mineral price and operating cost it is very hard to define it, since price and cost vary continuously and are different over time. At the project level, there will not be a perfect correlation between price and cost because of adjusting in variables such as labor, energy, fuel, as well as other material expenditures that are supplied by industries that are not directly linked to mineral price fluctuations. In order to protect themselves suppliers are offering short term contracts to mines that is opposite to traditional long term contracts. Some components of operating cost are usually purchased at market prices that fluctuate monthly, annually or even in shorter periods. Obviously production costs are of dynamic nature. In this paper we didn't analyze and predict the future states of costs by using special forecasting methods but we just assigned values varying over the time only in the purpose to verify the model.

Criterion C3: Environmental efficiency. Inputs used in the mining process can have an impact, either positive or negative, on the environment and environmental efficiency aims to take account of this impact in ranking mining technologies according to their level of efficiency. This criterion is defined as linguistic variable and and its value should be maximized. Transformation of the fuzzy linguistic variables to fuzzy triangular numbers is as follows: very low (VL) → TFN(0,1,1); low (L) → TFN(1,2,3); medium (M) → TFN(2,3,4); high (H) → TFN(3,4,5); very high (VH) → TFN(4,5,5)

Criterion C4: Applicability of mining technology with respect to given conditions. This criterion is also defined as linguistic variable and and its value should be maximized. Transformation of this criterion is equal to transformation of criterion C3.

The input parameters that are required for the evaluation are given in the Table I. Evaluations of criterion C2 are changed over five years.

TABLE I. FUZZY DYNAMIC DECISION MAKING MATRIX

A(t)		C1			C2(t)			C3			C4	
A1(t1)	16.01	17.79	20.46	52.38	55.14	57.90	2	3	4	3	4	5
A2(t1)	15.90	17.67	20.32	32.45	34.16	35.87	4	5	5	4	5	5
A3(t1)	9.51	10.57	12.16	47.28	49.77	52.26	3	4	5	4	5	5
A1(t2)	16.01	17.79	20.46	36.40	38.32	42.15	2	3	4	3	4	5
A2(t2)	15.90	17.67	20.32	43.09	45.36	49.90	4	5	5	3	4	5
A3(t2)	9.51	10.57	12.16	68.74	72.36	79.60	3	4	5	2	3	4
A1(t3)	16.01	17.79	20.46	45.93	48.35	53.19	2	3	4	3	4	5
A2(t3)	15.90	17.67	20.32	49.76	52.38	57.62	4	5	5	3	4	5
A3(t3)	9.51	10.57	12.16	62.05	65.32	71.85	3	4	5	2	3	4
A1(t4)	16.01	17.79	20.46	26.94	28.36	31.20	2	3	4	3	4	5
A2(t4)	15.90	17.67	20.32	59.19	62.31	68.54	4	5	5	3	4	5
A3(t4)	9.51	10.57	12.16	51.63	54.35	59.79	3	4	5	2	3	4
A1(t5)	16.01	17.79	20.46	42.97	45.23	49.75	2	3	4	3	4	5
A2(t5)	15.90	17.67	20.32	30.64	32.25	35.48	4	5	5	3	4	5
A3(t5)	9.51	10.57	12.16	59.45	62.58	68.84	3	4	5	2	3	4

The relative closeness of alternative to the ideal solution at each time episode is represented in the Table II.

TABLE II. RELATIVE CLOSENESS

A1(t1)	0.400602	0	0.430754
A2(t1)	0.670455	0.5344	0.361931
A3(t1)	0.434857	0.61745	0.578844
A1(t2)	0.306829	0.610231	0.720397
A2(t2)	0.293327	0.634629	0.775124
A3(t2)	0.5411	0.339869	0.460229
A1(t3)	0.300252	0.177609	0.715096
A2(t3)	0.29725	0.478281	0.76875
A3(t3)	0.547246	0.677324	0.466313
A1(t4)	0.317986	0.680851	0.709311
A2(t4)	0.293651	0.229389	0.744096
A3(t4)	0.53341	0.365961	0.463906
A1(t5)	0.304231	0.415277	0.712088
A2(t5)	0.293896	0.680669	0.778821
A3(t5)	0.542452	0.357714	0.460782

The weight vector $\tilde{\lambda}(t) = \left(\tilde{\lambda}(t_1), \tilde{\lambda}(t_2), \dots, \tilde{\lambda}(t_5)\right)$ obtained by using (18) is represented in the Table III.

TABLE III. WEIGHT VECTOR OF THE 5 PERIODS

$\tilde{\lambda}(t_1)$	0.109555	0.188596	0.988045
$\tilde{\lambda}(t_2)$	0.029419	0.185769	0.585748
$\tilde{\lambda}(t_3)$	0.082956	0.197089	0.951737
$\tilde{\lambda}(t_4)$	0.019092	0.261076	0.520225
$\tilde{\lambda}(t_5)$	0.032760	0.167470	0.606790

Aggregated overall relative closeness value $\tilde{Q}_i^{ag}, i = 1,2,3$ is represented in the Table IV.

TABLE IV. AGGREGATED OVERALL RELATIVE CLOSENESS

A1	0.0939	0.3957	2.3292
A2	0.1220	0.4868	2.4030
A3	0.1369	0.4685	1.8062

According to obtained defuzzified values of aggregated overall relative closeness, the final rank order of proposed alternatives is: A2(1.003), A1(0.939) and A3(0.803).

V. CONCLUSIONS AND FUTHER RESEARCH

Large capital intensive projects, such as those in the mineral resource industry require careful analysis with respect to given conditions, available technical solutions and desired objectives. Ability to incorporate uncertainty of input parameters and create a set of possible solutions into process of decision making increases the flexibility and reliability of such process. Developed model is a mathematical representation of reality and allows management (especially in small mining companies) to test different scenarios and select the best. It allows strategies to be very quickly and easily tested. The model brings forth an issue that has the dynamic nature of the assessment of available alternatives. The model is not closed and can be extended. Our intention is to incorporate different mathematical methods in model helping to forecast future states of dynamic criteria. Further explorations are related to application of stochastic differential equations in order to describe dynamic nature of some input parameters such as production costs.

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