Control Systems Optimal Multi-Stage Compensation

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***Abstract*—The contribution of this paper is in suggesting a useful technique for achieving optimal multi-stage compensation of process control systems. A multi-stage compensator, designed according to a number of proposed rules, is connected in series with the original control system. It eliminates some properly selected dominant poles of the system's transfer function. At the same time it introduces a specifically designed amplification and new dominant poles. This improves the quality of the system's performance in terms of its transient response, stability and accuracy. Rise and settling times, percent maximum overshoot, damping ratio, phase margin and steady-state error are optimized in the process.**

Keywords—marginal; multi-stage; phase-lag; phase-lead; dominant, poles, transient responses, compensation; stability;

I. Introduction

A useful technique is proposed to achieve a high quality performance of n-order process control systems. It emphasizes on development of a number of rules for design of a proper multi-stage compensation. The compensator, connected in series with the original control system, eliminates some properly selected dominant poles of the system's high order closed-loop transfer function. At the same time it introduces a specifically designed amplification and new dominant poles. This improves the quality of the system's performance in terms of its transient response, stability and accuracy. Rise time, percentage overshoot, settling time, damping ratio, phase margin and steady-state error are considerably optimized. The stability of the system is also improved. The suggested technique is originally designed for marginal control systems. Its application can be easily extended for systems different from marginal and can be used successfully for second, third or higher order process control systems of Type 0 and Type 1. The transfer function of the process is usually determined through known system identification methods. The compensation technique is applied, considering that the transfer function is presented as [1]:

$$G_p(s) = \frac{K\prod_{i=1}^{k}(s-z_i)}{s^n\prod_{j=1}^{m}(s-p_j)} \qquad (1)$$

It is well known that poles that close to the imaginary axis in the left-half s-plane are dominant and are used to design the dynamic performance of a system [2]. The insignificant poles should ensure that the applied compensator transfer function could be realized by physical components. In practice, the magnitude of the real part of an insignificant pole is considered at least 10 times larger than that of a dominant pole [3], [4]. This effect is going to be used in the suggested multi-stage compensation technique. To meet the ITAE criterion [5], [6] the following system objectives are set:

Damping ratio (DR) ζ = 0.707 Percent maximum overshoot (PMO) $\leq$ 4%

Settling / to max overshoot time $t_s / t_m \leq 1.4$ Steady-State error $e_{ss} \leq 1\%$ (type 0 systems)

These objectives will be used for establishing the rules and applying the suggested method.

II. Rules of the Compensation Technique

A. *Rules of the Compensation Technique*

The rules of the compensation technique are developed on the base on cascade compensation and a unity feedback. According to the suggested rules the compensator may consist of a multi-stage lead section and/or a lag section, depending on the system type. Additional attenuation and amplification, that is part of the compensator, with factors provided by the rules, are also applied to bring the system to the desired performance. The purpose of the rules is to set a design procedure of a compensator that eliminates some properly selected dominant poles, introduces new dominant poles and applies proper amplification.

Rule 1

To optimize ζ , t_s / t_m and the PMO of a Type 0 marginal closed-loop system, a cascade multi-stage lead compensation with factors of $\alpha_{1,2,3,...}$= 10 is applied for a zero-pole cancellation.

The number of the compensating stages N should be one less than the order of the open-loop system, i.e. N = n + m – 1.

The most dominant pole of the open-loop system should be left uncompensated. The current gain is maintained by attenuation equal to the product $\alpha_1\alpha_2\alpha_3\ldots$

Rule 2

To optimize a Type 0 marginal closed-loop system, a single-stage lag compensation with factor β = 10 is applied for a zero-pole cancellation of the uncompensated most dominant pole of the open-loop system. To optimize the steady-state error e_{ss} the current gain should be increased γ = 10 times.

Rule 3

If the most significant pole of the open-loop system has a real part close in value to that of an insignificant pole, following Rule 1, Rule 2 is modified. Then a successive two-stage lag compensation with factors $\beta_1 = \beta_2$ = 10 and a factor □ = 80 should be applied.

Rule 4

To optimize ζ, t_s/t_m and the PMO of a Type 1 marginal closed-loop system, a cascade multi-stage lead compensation with factors of $\alpha_{1,2,3,\ldots}$=10 should be applied for a zero-pole cancellation.

The number of the compensating stages N should be one less than the order of the open-loop system, i.e. N = n +m – 1.

The pure integration or the most significant pole of the open-loop system should be left uncompensated.

The existing system gain should be maintained by attenuation equal to the product $\varepsilon\alpha_1\alpha_2\alpha_3\ldots$, where the compensation attenuation factor is ε = (0.1 to 1.27).

B. Rules Design and Application

By testing different third order transfer functions the applicability of the suggested technique is proved in practice. It can be easily extended to any higher order system.

Case 1 Application of Rules 1 and 2 (Type 0 System)

Following the suggested rules, the compensation technique can be demonstrated for a case of a plant with a transfer function of Type 0 given in its Bode format:

$$G_P(s) = \frac{70}{(1+0.02s)(1+0.05s)(1+s)} \qquad (2)$$

There are two important steps, establishing the rules of the method. First, the values of the factors α_1 and α_2 are varied. Further, the value of the factor γ is varied, in this way searching for the optimum performance of the compensated system. The results are shown in Figure 1 and Figure 2.

From Figure 1, it can be seen that the set of the initial objectives to satisfy the ITAE criterion can be met if $\alpha_1 = \alpha_2$ = 10. The factor $\beta = 10$ is chosen as a realistic figure for the physical realization of the lag compensation stage [7], [8].

Figure 2 shows that the set of the objectives described in Equations (2), (3) and (4) can be met if γ= 10. In this case the steady-state error is measured as e_{ss} = 0.14%, which satisfies the initial objectives.

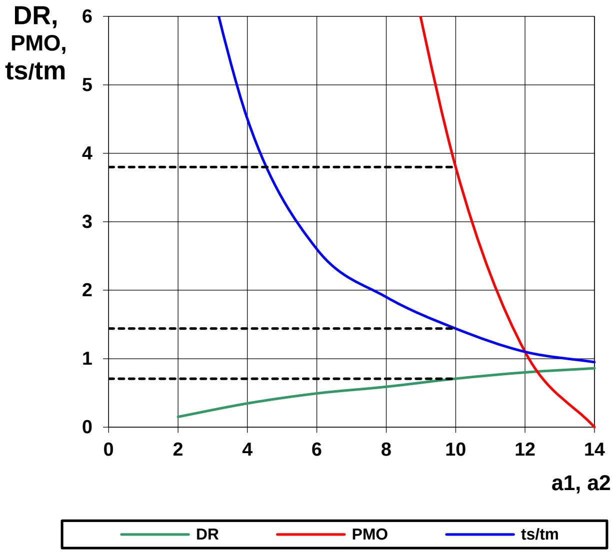

Fig. 1. *Results of the tracking procedure for determination of optimum values of α_1 and α_2*

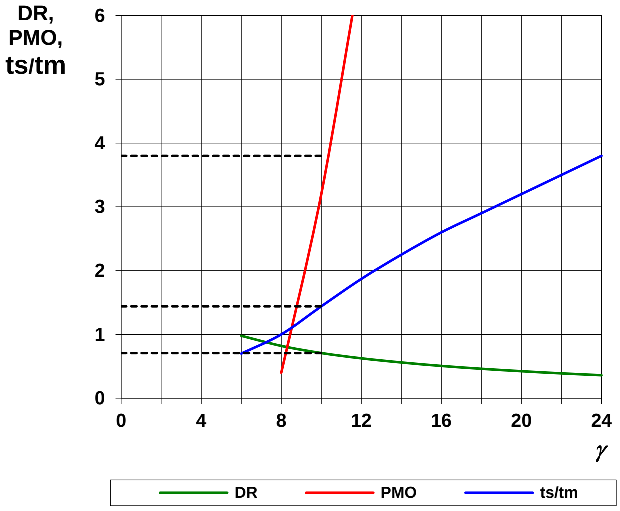

Fig. 2. *Results of the tracking procedure for determination of optimum value of γ*

Since the plant transfer function from Equation (6) is of a third order, two-stage lead plus one-stage lag compensation is applied. The two less significant poles in Equation (6) are p_1 = –1/0.02 = –50 and p_2 = –1/0.05 = –20. Then, according to Rule 1, the multi-stage lead section of the compensator should have a transfer function

$$G_c^{'}(s) = \frac{(1+\alpha_1 T_1 s)(1+\alpha_2 T_2 s)}{\alpha_1 \alpha_2 (1+T_1 s)(1+T_2 s)} = \qquad (3)$$
$$= \frac{(1+0.02s)(1+0.05s)}{100(1+0.002s)(1+0.005)}$$

Following Rule 1, additional attenuation should be also applied

$$G_c^{''}(s) = \alpha_1 \alpha_2 = 10 \times 10 = 100 \qquad (4)$$

The most significant pole in Equation (6) is $p_3 = -1$. Applying Rule 2, the section of the lag compensation and amplification is presented by:

$$G_c^{'''}(s) = \frac{\gamma(1+T_3 s)}{(1+\beta T_3 s)} = \frac{10(1+s)}{(1+10s)} \qquad (5)$$

Now the transfer function of the full compensator is:

$$G_c(s) = G_c^{'}(s) \times G_c^{''}(s) \times G_c^{'''}(s) \qquad (6)$$

Finally, after applying the full compensation, the transfer function of the open-loop system becomes:

$$G(s) = G_c(s) \times G_p(s) = \qquad (7)$$
$$= \frac{700}{(1+0.002s)(1+0.005s)(1+10s)}$$

The system's original phase margin is PM = $\Delta\varphi = 0$ and the original system is considered as marginal, or practically unstable [9]. By introducing the lead compensation and attenuation (Rule 1), the phase margin becomes PM = $\Delta\varphi$ = 70.7 and the damping ratio is $\zeta = 0.707$. The performance and the stability of the closed-loop system are improved.

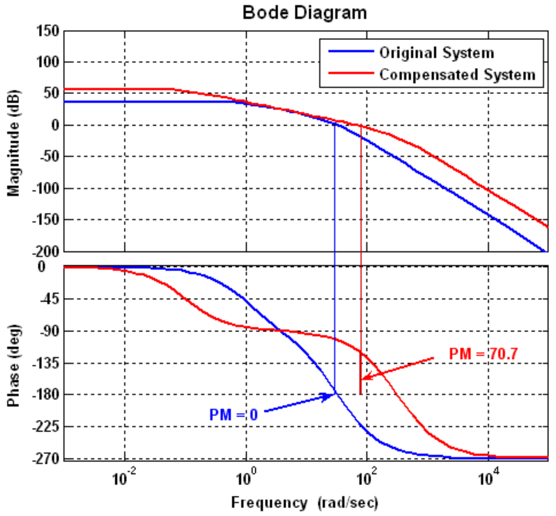

Fig. 3. *Case 1 Bode diagrams of the original system and of the system after applying Rules 1 and 2*

The main contribution of the lag compensation and amplification (Rule 2) is eliminating the influence of the most significant pole and reduction of the steady-state error of the closed-loop system. The Bode diagrams in Figure 3 represent the case of the original system and the case of the system after the implementation of Rule 1 and Rule 2.

The diagrams in Figure 3 are obtained by the code, as shown below:

```
>> z={0}
>> p={-50;-20;-1}
>> k=70000
>> s = zpk('s')
>> Gp=70000/(s+50)/(s+20)/(s+1)
Zero/pole/gain:
70000
------------------
 (s+50) (s+20) (s+1)
 >> z={0}
 >> p={-500;-200;-0.1}
 >> k=7000000
 >> s = zpk('s')
 >> G= 7000000/(s+500)/(s+200)/(s+0.1)
 Zero/pole/gain:
 7000000
 ----------------------
 (s+500) (s+200) (s+0.1)
 >> bode(Gp,G)
```

The system transient responses before and after the full compensation are achieved by the code as shown below and are presented in Figure 4:

```
>> Gpfb=feedback(Gp,1)
Zero/pole/gain:
 70000
--------------------------------
(s+70.17) (s^2 + 0.8292s + 1012)
>> Gfb=feedback(G,1)
Zero/pole/gain:
 7000000
------------------------------------
(s+538.4) (s^2 + 161.7s + 1.302e004)
>> step(Gpfb,Gfb)
```

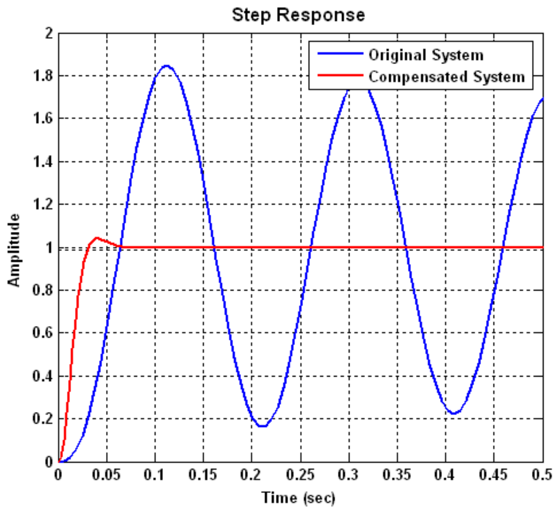

Fig. 4. *Case 1 Transient responses of the original and the fully compensated closed-loop system*

The performance specifications of the compensated system, determined from the transient response shown in Figure 4, are compared with the objectives for optimal performance.

TABLE I OBJECTIVES AND REAL RESULTS FOR CASE 1

Specifications	Objectives	Real	Consideration
ζ	= 0.707	= 0.707	Matching
PMO	≤ 4%	= 3.8%	Better
$t_{s(1\%)}$/tm	≤ 1.49	= 1.44	Better
$e_{ss}(t)$	< 1%	= 0.14%	Better

From the summary in Table I, it is seen that the transient response of the compensated system in terms of damping ratio, percent maximum overshoot, time ratio and steady-state error is either matching or is better than the one of the set objective.

Case 2 Application of Rules 1 and 3 (Type 0 System)

Rule 3 can be illustrated for a Type 0 marginal control system with a transfer function as shown:

$$G_P(s) = \frac{20}{(1+0.02s)(1+0.01s)(1+0.1s)} \tag{8}$$

In this case the real part of the most significant pole, p_3 = –10, is only 5 times smaller than the real part of the one of the insignificant pole p_1 = –50. This implies application of Rule 3.

According to Rule 1, two-stage lead compensation and attenuation with factors $\alpha_1 = \alpha_2 = 10$ are applied.

Then, following Rule 3, a two-stage lag compensation with factors $\beta_1 = \beta_2 = 10$ and a factor amplification δ= 80 is suggested.

The values of the factors β_1 and β_2 are chosen by the same considerations as in Rule 2. Using tracking procedures on the transient response, the value of the factor δ is varied, searching for the optimum performance of the compensated system. It can be seen from Figure 5 that the set of objectives can be met if δ = 80. The steady-state error is e_{ss} = 0.06%.

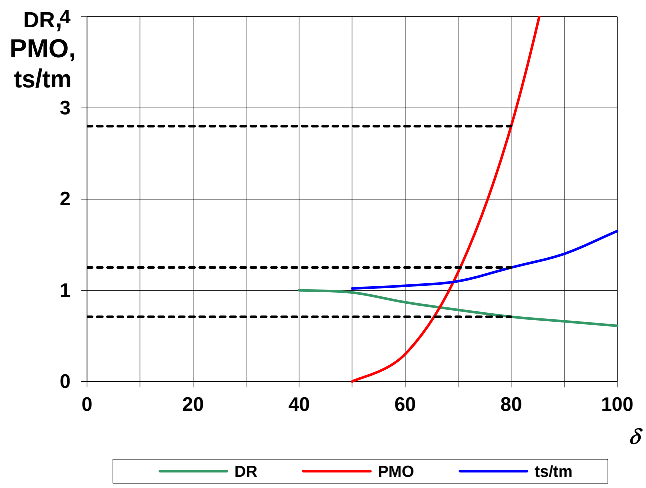

Fig. 5. *Results of the tracking procedure for determination of optimum value of δ*

Using a similar sequence as in Case 1, applying Equation (3), the two-stage lead stage is presented by

$$G_c^{'}(s) = \frac{(1+0.02s)(1+0.01s)}{100(1+0.002s)(1+0.001)} \tag{9}$$

The attenuation $G_c^{''}(s)$ is the same as in Equation (4). Then applying Rule 3, the two-stage lag compensation and amplification is presented by:

$$G_c^{'''}(s) = \frac{\delta(1+T_3 s)(1+T_4 s)}{(1+\beta_1 T_3 s)(1+\beta_2 T_4 s)} = \frac{80(1+0.1s)(1+s)}{(1+s)(1+10s)} \tag{10}$$

Now, applying the full compensation, considering Equations (11) and (12), the transfer function of the open-loop system becomes

$$G(s) = \frac{1600}{(1+0.002s)(1+0.001s)(1+10s)} \tag{11}$$

The effect of the application of the method can be seen from the Bode diagrams shown in Figure 6. The phase margin of the system and hence the damping ratio and stability are improved considerably and are close to the desirable objectives. The Bode diagrams are plotted with the aid of the following code:

```
>> z={0}
>> p={-10;-50;-100}
>> k=1000000
>> s = zpk('s');
>> Gp=1000000/(s+10)/(s+50)/(s+100)
Zero/pole/gain:
1000000
--------------------
(s+10) (s+50) (s+100)
>> z={0}
>> p={-0.1;-500;-1000}
>> k=80000000
>> s = zpk('s');
>> G=80000000/(s+0.1)/(s+500)/(s+1000)
Zero/pole/gain:
80000000
-----------------------
(s+0.1) (s+500) (s+1000)
>> bode(Gp,G)
```

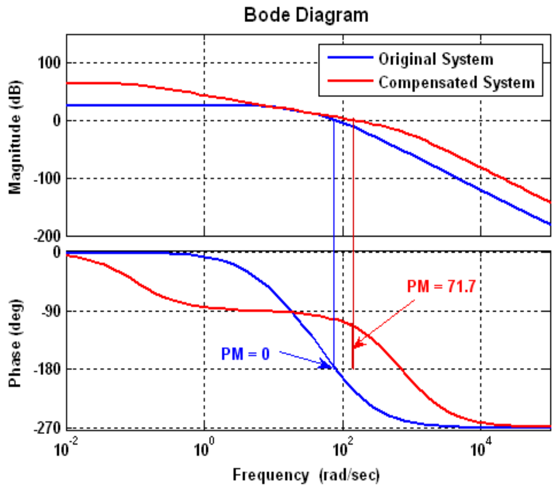

Fig. 6. *Case 2 Bode diagrams of the system before and after applying the full compensation*

The closed-loop system transient responses before and after full compensation are shown in Figure 7 and is obtained by the following code:

```
>> Gpfb=feedback(Gp,1)
Zero/pole/gain:
1000000
--------------------------------
(s+160.3) (s^2 - 0.3106s + 6550)
>> Gfb=feedback(G,1)
Zero/pole/gain:
80000000
-----------------------------------
(s+1116) (s^2 + 383.8s + 7.171e004)
>> step(Gpfb,Gfb)
```

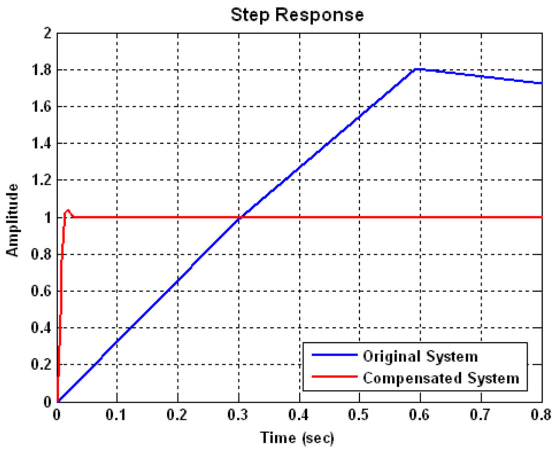

Fig. 7. *Case 2 Transient responses of the closed-loop system before and after the full compensation*

From the summary in Table II, it is obvious that the objectives are met. Again, the real results for the compensated control system are either close or better than the set specifications.

TABLE II. OBJECTIVES & REAL RESULTS FOR CASE 2

Specifications	Objectives	Real	Consideration
ζ	= 0.707	= 0.717	Close
PMO	≤ 4%	= 2.8%	Better
$t_{s(1\%)}$/tm	≤ 1.49	= 1.25	Better
$e_{ss}(t)$	< 1%	= 0.06%	Better

Case 3 Application of Rule 4 (Type 1 System)

In this case, the suggested application is for a plant with a marginal transfer function of Type 1, presented in its Bode form:

$$G_P(s) = \frac{70}{s(1+0.02s)(1+0.05s)} \quad (12)$$

By applying Rule 4, the values of the factors α_1 and α_2 are determined similarly as in Rule 1. Additional adjustment factor ε is introduced so that the real attenuation becomes $\varepsilon\alpha_1\alpha_2$. With the aid of a tracking procedure ε is determined for different marginal control systems of Type 1. To keep ζ = 0.707, when the ratio of the less significant to the most significant pole of the plant transfer function, r = p_1/p_2, varies from 50 to 1, the value of ε may vary from 0.1 to 1.27, as shown in Figure 8. When the ratio is r = 2.5, as in the case of Equation (19), then $\varepsilon = 1$. If r < 2.5, the attenuation should be adjusted within the limits ε = (1 to 1.27). If r >2.5, then the adjustment factor should be within the limits ε = (0.1 to 1).

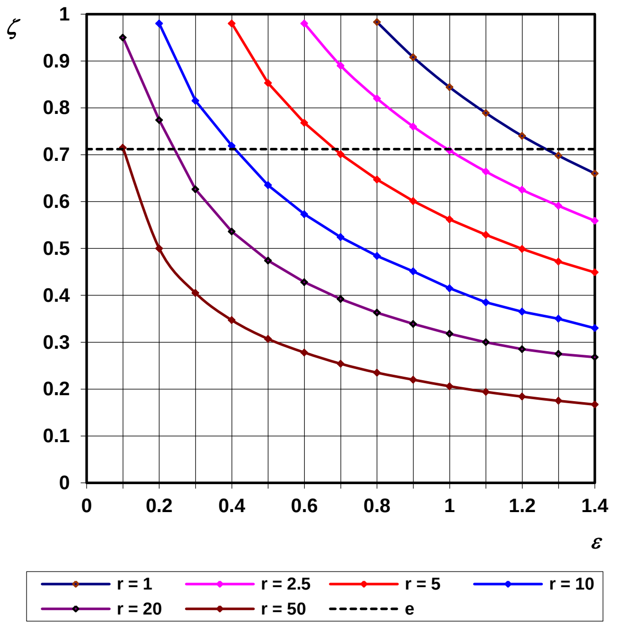

Fig. 8. *Relationship between the damping ratio ζ and the factor ε for different poles ratios r = p_1/p_2*

In this case, first, a two-stage lead compensation and attenuation is used. The two poles of Equation (12), to be cancelled, are p_1 = –50 and p_2 = –20. The lead compensation and attenuation employ transfer functions like those shown in Equations (3), (4). The factors used are α_1 = α_2 = 10 and $\varepsilon = 1$. Then, after applying Equation (7), the transfer function of the open-loop system becomes:

$$G(s) = \frac{70}{s(1+0.002s)(1+0.005s)} \quad (13)$$

The open-loop systems Bode diagrams, before and after applying the compensation technique, are achieved by the code as shown below and are presented in Figure 9.

```
>> z={0}
>> p={-50;-20;0}
>> k=70000
>> s = zpk('s');
>> Gp=70000/(s+50)/(s+20)/(s+0)
Zero/pole/gain:
70000
---------------
s (s+50) (s+20)
>> z={0}
>> p={-500;-200;0}
>> k=7000000
>> s = zpk('s');
>> G=7000000/(s+500)/(s+200)/(s+0)
Zero/pole/gain:
7000000
-----------------
s (s+500) (s+200)
>> bode(Gp,G)
```

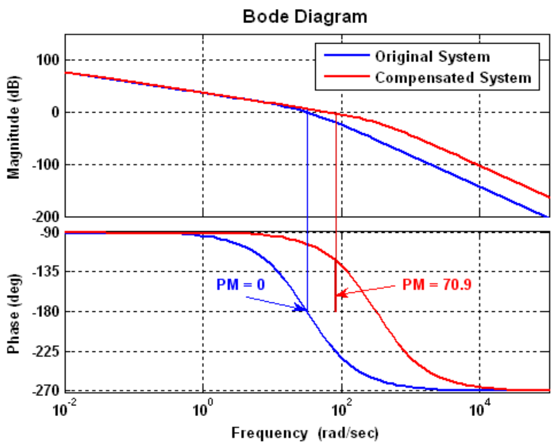

Fig. 9. *Case 3. Bode diagrams of the system before and after applying the full compensation*

The transient responses of the closed-loop system before and after applying the compensation are shown in Figure 10 and are obtained by the code:

```
>> Gpfb=feedback(Gp,1)
Zero/pole/gain:
70000
-------------------
(s+70) (s^2 + 1000)
>> Gfb=feedback(G,1)
Zero/pole/gain:
7000000
----------------------------------
(s+538.4) (s^2 + 161.6s + 1.3e004)
>> step(Gpfb,Gfb)
```

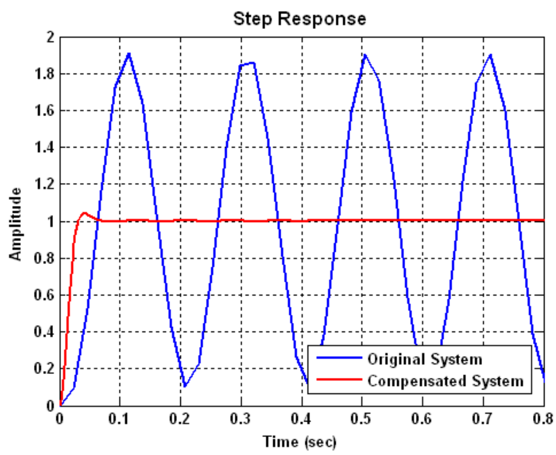

Fig. 10. *Case 3. Transient responses of the closed-loop system before and after the full compensation*

The real results of the compensated system are compared with the objectives for optimal performance and summarized in Table III. It can be seen that again the objectives are met.

TABLE III. OBJECTIVES & REAL RESULTS FOR CASE3

Specifications	Objectives	Real	Consideration
ζ	= 0.707	= 0.709	Close
PMO	≤ 4%	= 3.3%	Better
$t_{s(1\%)}$/tm	≤ 1.49	= 1.4	Better
$e_{ss}(t)$	=0%	= 0%	Matching

C. Application of the Compensation Technique with Changed Objectives

Some process control systems may require a very fast response, compromising with the PMO and $t_{s(1\%)}/t_m$ values. In practice the damping ratio may be modified [10], to be within the range of $\zeta = 0.30$ to 0.45. Then all the rules of the proposed method will stand, but additional amplification by a factor $\phi = 2$ is required. The value of ϕ is determined by tracking procedure for different transfer functions. For example, the system described by Equation (13) has $\zeta = 0.707$. Increasing its gain two times secures a damping ratio of $\zeta = 0.423$ and faster response, which is illustrated in Figure 11.

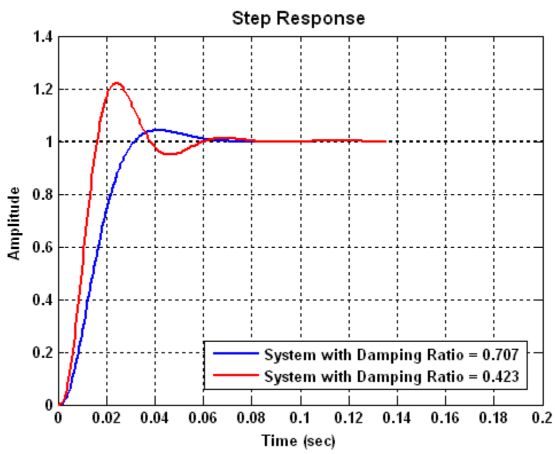

Fig. 11. *Comparison between the original and a faster transient response*

The transient responses are obtained with the aid of the following code:

```
>> G1=7000000/(s+500)/(s+200)/(s+0)
Zero/pole/gain:
7000000
----------------
s (s+500) (s+200)
>> G2=14000000/(s+500)/(s+200)/(s+0)
Zero/pole/gain:
14000000
----------------
s (s+500) (s+200)
>> G1fb=feedback(G1,1)
Zero/pole/gain:
7000000
----------------------------------
(s+538.4) (s^2 + 161.6s + 1.3e004)
>> G2fb=feedback(G2,1)
Zero/pole/gain:
14000000
-----------------------------------
(s+567.2) (s^2 + 132.8s + 2.468e004)
>> step(G1fb,G2fb)
```

D. Application of the Compensation Technique for Systems Different from Marginal

The suggested compensation technique can be also used for systems that are different from marginal. To apply the suggested compensation technique, such systems are initially brought to a marginal state by tuning their original steady-state gain *K*. For example, a system with a transfer function as shown in equation (14) is not marginal, but its performance is unacceptable due to large oscillations.

$$G_P(s) = \frac{50}{(1+0.02s)(1+0.05s)(1+s)} \tag{14}$$

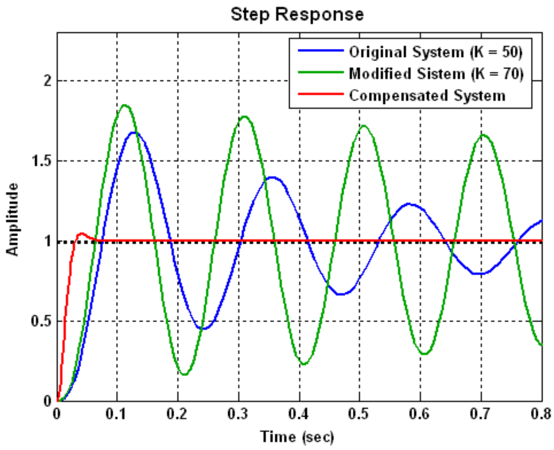

Fig. 12. *Application of the compensation technique for systems different from marginal*

By tuning the original gain from *K* =50 to *K'* = 70 and then applying Rule 1 and Rule 2 of the suggested compensation method, brings the system to the desirable performance as shown in Figure 12. The transient responses on the original system, the system after tuning and the compensated system are plotted by applying of the following code:

```
>> z={0}
>> p={-50;-20;-1}
>> k=50000
>> s = zpk('s')
>> Gp1=70000/(s+50)/(s+20)/(s+1)
Zero/pole/gain:
 50000
------------------
(s+50) (s+20) (s+1)
>> Gp1fb=feedback(G1p,1)
Zero/pole/gain:
 50000
---------------------------------
(s+66.29) (s^2 + 3.707s + 754.2)
>> z={0}
>> p={-50;-20;-1}
>> k=70000
>> s = zpk('s')
>> Gp2fb=feedback(Gp,2)
Zero/pole/gain:
 70000
---------------------------------
(s+70.17) (s^2 + 0.8292s + 1012)
>> z={0}
>> p={-500;-200;-0.1}
>> k=7000000
>> s = zpk('s')
>> G= 7000000/(s+500)/(s+200)/(s+0.1)
 Zero/pole/gain:
 7000000
----------------------
(s+500) (s+200) (s+0.1)
>> Gfb=feedback(G,1)
Zero/pole/gain:
 7000000
-------------------------------------
(s+538.4) (s^2 + 161.7s + 1.302e004)
>> step(Gp1fb,Gp2fb,Gfb)
```

III. Conclusion

Although the suggested method of multi-stage compensation is based on some known theoretical procedures, like the zero-pole cancellation, the lead and lag compensation, combining and analyzing them, results in development of some new ideas. The originality of the suggested technique is based on the statement of a number of rules, which are applied in a predetermined sequence.

The compensation equipment consists of three major parts. Its lead section eliminates all insignificant poles of the original plant transfer function and introduces new properly designed poles. This improves the transient response specifications, especially the damping ratio of the system. The lag section eliminates the most significant pole of the original plant transfer function and along with the amplifying section improves further the transient response and reduces considerably the steady-state error.

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