Advanced D-Partitioning Analysis and its Comparison with the Kharitonov's Theorem Assessment

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***Abstract*—This paper contributes for further advancement of the D-Partitioning analysis applied to systems with multivariable parameters. It also explores the effects of simultaneous system uncertainties by determining graphically regions of stability in the space of the system's parameters. The interaction between the varying parameters will also bring a new light in the graphical solution of the problem of stability. Considerable advantages of the suggested advanced D-partitioning analysis tool are illustrated comparing it with the Kharitonov's theorem assessment. The advanced D-partitioning is beneficial for further development of control theory in the area of systems stability analysis.**

Keywords—parameter; D-Partitioning; stability; Kharitonov's theorem; analysis;

I. INTRODUCTION

Following some initial ideas of Neimark [1], [2], [3] the D-partitioning method was better clarified and further advanced by the author in previous published work [4], [5], [6], [7]. It enables a quick and convenient determination of the regions of stability in case of variation of system's parameters.

The method of the D-partitioning is a powerful tool for system analysis. It can be easily implemented and has a considerable of advantages compared to other stability analysis methods. Basically, it has the advantage of a clear graphical display of the variation of each parameter and its effect on the system's stability. MATLAB software package can be employed for automatically plotting of the regions of stability. The objective of this research is to demonstrate the application of the developed by the author advanced D-partitioning method for cases of simultaneous variation of two system parameters and its advantages compared to other well-known methods, as the Kharitonov's Theorem assessment [8], [9], [10].

II. ADVANCED D-PARTITIONING IN CASE OF TWO SIMULTANEOUSLY VARIABLE PARAMETERS

To implement the method of the D-partitioning, a general characteristic equation is presented in the format:

$$G(s) = a_0 s^n + a_1 s^{n-1} + \ldots + a_n = 0 \qquad (1)$$

A characteristic equation of a hypothetical third order unity feedback system can be presented as follows:

$$G(s) = (T_1 s + 1)(T_2 s + 1)(T_3 s + 1) + K = 0 \qquad (2)$$

It is suggested that simultaneously two of the system's parameters are variable:

$$T_1 = \mu \text{ (time-constant)}, \qquad K = \gamma \text{ (gain)} \qquad (3)$$

The initial objective is to determine the regions of variation of these two parameters, for which the system will be stable.

Equations (3) are substituted in (2), from where:

$$\begin{aligned} G(s) = \mu[T_2 T_3 s^3 + (T_2 + T_3)s^2 + s] + \gamma + \\ + T_2 T_3 s^2 + (T_2 + T_3)s + 1 = 0 \end{aligned} \qquad (4)$$

By substituting $s = j\omega$, the equation (4) could be presented in the detailed form:

$$\left.\begin{aligned} P(j\omega) &= [T_2 T_3 (j\omega)^3 + (T_2 + T_3)(j\omega)^2 + j\omega] \\ Q(j\omega) &= 1 \\ R(j\omega) &= T_2 T_3 (j\omega)^2 + (T_2 + T_3) j\omega + 1 \end{aligned}\right\} \qquad (5)$$

Where

$$\left.\begin{aligned} P(j\omega) &= P_1(\omega) + jP_2(\omega) \\ Q(j\omega) &= Q_1(\omega) + jQ_2(\omega) \\ R(j\omega) &= R_1(\omega) + jR_2(\omega) \end{aligned}\right\} \qquad (6)$$

Then the equation (4) can be presented by a set of two equations:

$$\left.\begin{aligned} \mu P_1(\omega) + \gamma Q_1(\omega) + R_1(\omega) = 0 \\ \mu P_2(\omega) + \gamma Q_2(\omega) + R_2(\omega) = 0 \end{aligned}\right\} \qquad (7)$$

Considering equations (7), the variable parameters can be determined as:

$$\left.\begin{aligned} \mu &= \frac{T_2 + T_3}{T_2 T_3 \omega^2 - 1} \\ \gamma &= \frac{(T_2 T_3 \omega^2 - 1)^2 + (T_2 + T_3)^2 \omega^2 + 1}{T_2 T_3 \omega^2 - 1} \end{aligned}\right\} \tag{8}$$

The determinant of equation (8) is:

$$\Delta = T_2 T_3 \omega^2 - 1 \tag{9}$$

The determinant becomes $\Delta = 0$ at a specific frequency $\omega = \omega_\infty$ that can be found out from equation (8) as:

$$\omega = \omega_\infty = \sqrt{\frac{1}{T_2 T_3}} \tag{10}$$

If $\Delta = 0$, both system parameters are approaching infinity:

$$\mu(\omega_\infty) \to \infty, \quad \gamma(\omega_\infty) \to \infty, \tag{11}$$

This implies that the main D-Partitioning curve has an interruption, or a breakdown, at a frequency $\omega = \omega_\infty$. It consists of two parts, the first one is plotted within the frequency range $0 < \omega < \omega_\infty$, while the second one is obtained for $\omega_\infty < \omega < \infty$.

For a better clarification, first the functions $\mu(\omega)$ and $\gamma(\omega)$ are plotted, as shown in Fig. 1 (a) and Fig. 1(b).

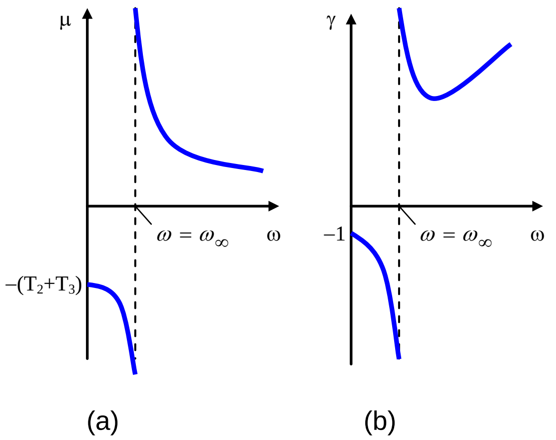

Fig. 1: *The graphical presentations of $\mu(\omega)$ and $\gamma(\omega)$ showing the interruption*

of the curves at a frequency $\omega = \omega_\infty$

The regions of the D-partitioning also depend on two straight lines in the (μ, γ) plane, considered as *special lines*. The *special lines* are plotted for the two border frequencies $\omega = 0$ and $\omega = \infty$. The equations of the special lines are obtained from equation (1) by:

$$a_n = 0, \text{ at } \omega = 0; \quad a_o = 0, \text{ at } \omega = \infty \tag{12}$$

The special lines are determined by comparing the equations (1) and (4) identifying the coefficients a_n and a_o and equalizing them to zero:

$$\left.\begin{aligned} &\mu T_2 T_3 = 0, \qquad \gamma + 1 = 0 \\ &\text{or} \\ &\mu = 0 = Const. \quad \gamma = -1 = Const. \end{aligned}\right\} \tag{13}$$

The regions of stability are determined by the D-Partitioning curve, defined by equations (8) and the special lines, defined by equations (13). The D-Partitioning regions could be determined by plotting the main D-Partitioning curve, together with the special lines on the (μ, γ) plane.

The locked regions between these parts of the curve, corresponding to realistic physically realized system parameters and the special lines are identified as the regions of stability. The realistic stable regions are also always located on the left-hand side of the D-Partitioning curve, following the frequency increment. Finally, by combining the curves $\mu(\omega)$, $\gamma(\omega)$ from Figure 1(a) and Figure 2(b) and the special lines, the D-Partitioning is obtained in the (μ, γ) plane as seen in Figure 2.

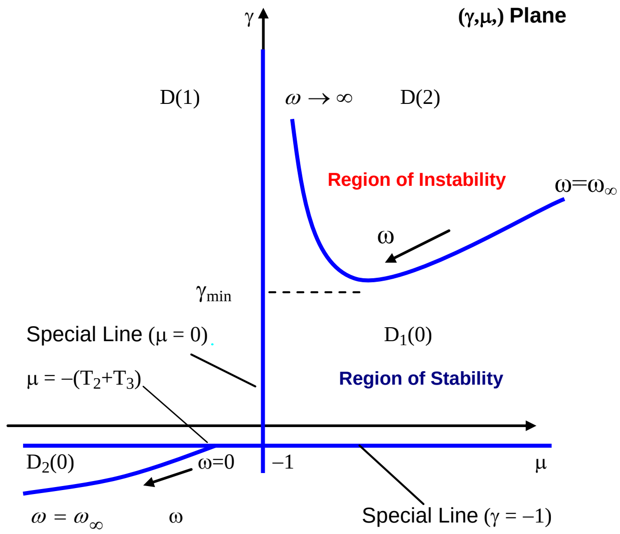

Fig. 2. *Advanced D-Partitioning, Defining the Regions of Stability and the Regions of Instability*

Considering equation (8), the variable parameters μ and γ are considered as even functions. It follows that each one of the parameters μ and γ has over-tracing values within the frequency region $-\infty \le \omega \le +\infty$, as seen from equation (14):

$$\begin{aligned} \mu(+\omega) &= \mu(-\omega) \\ \gamma(+\omega) &= \gamma(-\omega) \end{aligned} \tag{14}$$

Then, if in the plain (μ, γ), the D-Partitioning curve is plotted following the frequency increment from $-\infty$ to 0, the rest part of the curve, plotted for frequency increment from 0 to $+\infty$ is over-tracing the already plotted curve in reverse order.

Taking into account that $\mu = T_1$ is a time-constant and it can adopt only positive values, practically only the stable region $D_1(0)$ should be considered. The region of stability $D_1(0)$ is locked within the left-hand side of the D-Partitioning curve, corresponding to frequency rise from $\omega = \omega_\infty$ to $\omega \to \infty$ and the special line $\gamma = -1$.

Since the gain $K = \gamma$ may also adopt only positive values, the realistic border of the stable region $D_1(0)$ should be considered $K = \gamma = 0$. Further, the conclusion is that for small values of the gain $K < \gamma_{min}$, the system is stable for any values of the time-constant $\mu = T_1$.

As an example, a system consisting of an armature-controlled dc motor and a type-driving mechanism is suggested to illustrate the application of the advanced D-Partitioning analysis in case of variable gain and variable time-constant. Two of the motor time-constants $T_2 = 0.5$ sec and $T_3 = 0.8$ sec are known and constant values. The variation of the ambient temperature may cause the change of the system gain K, while the variation of the load causes change of the mechanism time-constant T_1. The transfer function of the open loop system is presented as:

$$G_O(s) = \frac{K}{(1+T_1 s)(1+T_2 s)(1+T_3 s)} = \frac{K}{(1+T_1 s)(1+0.5s)(1+0.8s)} = \frac{K}{0.4T_1 s^3 + (1.3T_1 + 0.4)s^2 + (T_1 + 1.3)s + 1} \quad (15)$$

Then, the characteristic equation of the unity feedback control system is determined as follows:

$$G(s) = 0.4T_1 s^3 + (1.3T_1 + 0.4)s^2 + (T_1 + 1.3)s + 1 + K \quad (16)$$

By substituting $s = j\omega$ and $T_1 = T$, equation (16) is modified to:

$$K = -1 + (1.3T + 0.4)\omega^2 + j\omega(0.4T\omega^2 - 1.3 - T) \quad (17)$$

Since the gain may obtain only real values, the imaginary term of equation (17) is set to zero, then:

$$\omega^2 = \frac{1.3 + T}{0.4T} \quad (18)$$

The result of (18) is substituted into the real part of equation (17), from where:

$$K = \frac{1.3T^2 + 1.69T + 0.52}{0.4T} = 3.25T + 4.225 + \frac{1.3}{T} \quad (19)$$

The D-partitioning curve $K = f(T)$ defines the border between the region of stability D(0) and instability D(1) for the case of simultaneous variation of the two system parameters.

The D-partitioning curve $K = f(T)$, as presented in Fig. 3, is plotted with the aid of the following MATLAB code:

```
>> T = 0:0.1:5;
>> K = 3.25.*T+4.225+1.3./T

K =
Columns 1 through 10
Inf  17.5500  11.3750  9.5333  8.7750  8.4500  8.3417
8.3571  8.4500  8.5944
Columns 11 through 20
8.7750  8.9818  9.2083  9.4500  9.7036  9.9667
10.2375  10.5147  10.7972  11.0842
Columns 21 through 30
11.3750  11.6690  11.9659  12.2652  12.5667  12.8700
13.1750  13.4815  13.7893  14.0983
Columns 31 through 40
14.4083  14.7194  15.0313  15.3439  15.6574  15.9714
16.2861  16.6014  16.9171  17.2333
Columns 41 through 50
17.5500  17.8671  18.1845  18.5023  18.8205  19.1389
19.4576  19.7766  20.0958  20.4153
Column 51
20.7350
>> plot(T,K)
```

Each point of the D-partitioning curve represents also the marginal values of the two simultaneously variable parameters, which is a unique advancement and an innovation in the theory of control systems stability analysis.

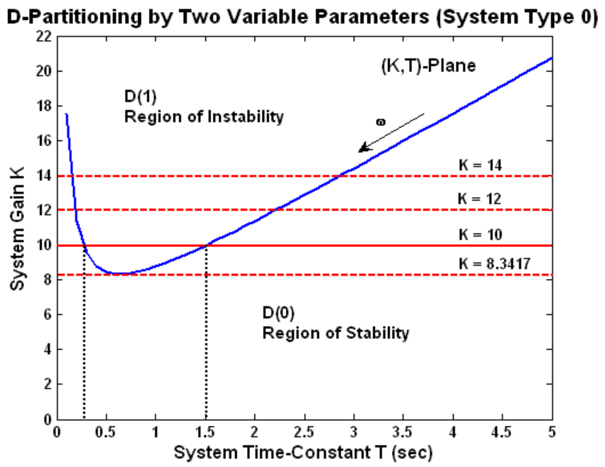

Fig. 3: *Advanced D-Partitioning in Terms of Two Variable Parameters*

Initially, the illustration of the system performance in case of variation of the time-constant T can be done when the gain set to $K = 10$. Then, if the time-constant is within the ranges 0 < T < 0.25 sec and T > 1.5 sec the system is stable. But for the same value of the gain K = 10, the system becomes unstable if the time-constant is in the range 0.25 sec < T < 1.5 sec.

The system performance can also be investigated for any other values of the variable gain K, like K = 12, K = 14, etc.

It is obvious that if K is varied, this affects the values of T at which the system may become unstable. Higher values of K, enlarge the range of T at which the system will fall into instability.

If K < 8.3417, a limit determined with the aid of a MATLAB procedure, the system is stable for any value of the T. It is obvious that the system performance and stability depends on the interaction between the two simultaneously varying parameters.

III. COMPARIZON OF THE ADVANCED D-PARTITIONING WITH THE KHARITONOV'S THEOREM ASSESSMENT

The well-known and popular Kharitonov's theorem assessment can be used in the case where the coefficients are only known to be within specified ranges [8], [12]. It provides a test of stability for a so-called interval polynomial. An interval polynomial is the family of all polynomials:

$$P(s) = a_0 + a_1 s^1 + a_2 s^2 + \ldots + a_n s^n = 0 \qquad 20)$$

The interval polynomial (20) is the characteristic equation of a control system with variable parameters, where each of its coefficients a_i can take any value in the specified intervals $a_i \in [a_i^-, a_i^+]$, or $a_i^- \le a_i \le a_i^+$. The notation a_i^- represents the lower limit of the variable coefficient, while a_i^+ represents the upper limit of the variable coefficient.

An interval polynomial characterized by equation (20) is stable (i.e. all members of the family are stable) if and only if the four so-called Kharitonov polynomials represented in equation (21) are stable [8], [12].

$$\left.\begin{aligned}
P_1(s) &= a_0^- + a_1^- s^1 + a_2^+ s^2 + a_3^+ s^3 + \ldots = 0 \\
P_2(s) &= a_0^+ + a_1^+ s^1 + a_2^- s^2 + a_3^- s^3 + \ldots = 0 \\
P_3(s) &= a_0^+ + a_1^- s^1 + a_2^- s^2 + a_3^+ s^3 + \ldots = 0 \\
P_4(s) &= a_0^- + a_1^+ s^1 + a_2^+ s^2 + a_3^- s^3 + \ldots = 0
\end{aligned}\right\} \quad (21)$$

It is obvious that while there is similarity in the four Kharitonov polynomials, at the same time, there is a specific arrangement of the lower limit and upper limit coefficients at each one of these polynomials. What is also extraordinary about Kharitonov's result is that although in principle an infinite number of polynomials are tested for stability, in fact only four polynomials need to be tested.

Further, each of the four Kharitonov polynomials is tested for stability with the aid of the well-known Routh-Hurwitz stability criterion. The results are placed in tables for final assessment of the system's stability described by the Kharitonov's interval polynomial.

The objective of this discussion is the suggested in this research advanced D-Partitioning analysis, to be compared with the Kharitonov's assessment when the variation of the system's uncertain parameters is defined within specific limits [8], [12].

To validate this comparison, the control system of the armature-controlled dc motor with a type-driving mechanism is considered once again. In that case, the characteristic equation (16) of the system can be presented as an interval polynomial, while the variable gain K and the variable time-constant T_1 are defined within specific limits.

To demonstrate the application of the Kharitonov's Theorem assessment of the system's stability, two cases are presented as follows:

Case 1: The original characteristic equation (16) is modified to the interval polynomial, shown in equation (22), now being a family of all polynomials:

$$\left.\begin{aligned}
&P_1(s) = K + 1 + (T_1 + 1.3)s + \\
&\qquad + (1.3T_1 + 0.4)s^2 + 0.4T_1 s^3 \\
&K \in [8,10], \text{ or } 8 < K < 10 \\
&T_1 \in [1,2], \text{ or } 1 \text{ sec} < T_1 < 2 \text{ sec}
\end{aligned}\right\} \quad (22)$$

The interval polynomial $P_1(s)$ is stable, (i.e. all members of the family are stable) if and only if the four so-called Kharitonov's polynomials are stable:

$$\left.\begin{aligned}
&k_1(s) = 10 + 1 + (1 + 1.3)s + \\
&\qquad + (1.3 + 0.4)s^2 + 0.4s^3 \\
&k_2(s) = 10 + 1 + (2 + 1.3)s + \\
&\qquad + (1.3 \times 2 + 0.4)s^2 + 0.4 \times 2s^3 \\
&k_3(s) = 8 + 1 + (2 + 1.3)s + \\
&\qquad + (1.3 \times 2 + 0.4)s^2 + 0.4 \times 2s^3 \\
&k_4(s) = 8 + 1 + (1 + 1.3)s + \\
&\qquad + (1.3 + 0.4)s^2 + 0.4s^3
\end{aligned}\right\} \quad (23)$$

After the calculation, the Kharitonov's polynomials are presented in the proper state for assessment:

$$\left.\begin{aligned} k_1(s) &= 27.5 + 5.75s + 4.25s^2 + s^3 \\ k_2(s) &= 13.75 + 4.125s + 3.75s^2 + s^3 \\ k_3(s) &= 11.25 + 4.125s + 3.75s^2 + s^3 \\ k_4(s) &= 22.5 + 5.75s + 4.25s^2 + s^3 \end{aligned}\right\} \quad (24)$$

Taking into account any of the third order equations of (21), representing the Kharitonov polynomials, in order to apply the Routh-Hurwitz stability criterion [13], [14] the following table is created:

TABLE I ARRAY OF THE ROUTH-HURWITZ STABILITY TEST

(CASE OF A THIRD ORDER SYSTEM

k_i (s)			
s^3	a_n	a_{n-2}	0
s^2	a_{n-1}	a_{n-3}	0
s^1	b_1	0	0
s^0	c_1	0	…

where

$$b_1 = \frac{(a_{n-1} \times a_{n-2}) - (a_n \times a_{n-3})}{(a_{n-1} \times a_{n-2})} \quad (25)$$

$$c_1 = \frac{(b_1 \times a_{n-3}) - (a_{n-1} \times 0)}{b_1} \quad (26)$$

The Routh-Hurwitz stability criterion is applied to all these polynomials $k_i(s)$,where *(i = 1, 2, 3, 4).*

TABLE II RESULTS FROM THE FOUR KHARITONOV POLYNOMIALS (CASE 1)

k_1 (s)		k_2 (s)		k_3 (s)		k_4 (s)	
1	5.75	1	4.13	1	4.13	1	5.75
4.25	27.5	3.75	13.75	3.75	11.25	4.25	22.5
-0.72		0.46		1.13		0.46	
27.5		13.75		11.25		22.5	

In this particular case, the first column of the Routh array for the three polynomials $k_2(s)$, $k_3(s)$ and $k_4(s)$ are all positive (that is, there is no change of sign in the first column).

But the polynomial $k_1(s)$ has change of sign in the first column of the Routh array. This means that the closed-loop system will be unstable for the given set of coefficients variations.

Case 2: Although the polynomial $k_1(s)$ in Table II have change of sign in the first column and one of the components of the Routh array is negative, its value is close to zero. This means that the closed-loop system is close to the state of margin of stability.

If the set of parameter variations is changed, the closed-loop system may become stable. This is demonstrated with the following changes of the parameters variation limits in the characteristic interval polynomial, as shown in equation (23):

$$\left.\begin{aligned} & P_2(s) = K + 1 + (T_1 + 1.3)s + \\ & \qquad + (1.3T_1 + 0.4)s^2 + 0.4T_1 s^3 \\ & K \in [4,6], \text{ or } 4 < K < 6 \\ & T_1 \in [2,4], \text{ or } 2 \text{ sec} < T_1 < 4 \text{ sec} \end{aligned}\right\} \quad (27)$$

Similarly, the interval polynomial $P_2(s)$ is stable if and only if the four so-called Kharitonov's polynomials are also stable:

$$\left.\begin{aligned} k_1(s) &= 6 + 1 + (2 + 1.3)s + \\ &\quad + (1.3 \times 2 + 0.4)s^2 + 0.4 \times 2s^3 \\ k_2(s) &= 6 + 1 + (4 + 1.3)s + \\ &\quad + (1.3 \times 4 + 0.4)s^2 + 0.4 \times 4s^3 \\ k_3(s) &= 4 + 1 + (4 + 1.3)s + \\ &\quad + (1.3 \times 4 + 0.4)s^2 + 0.4 \times 4s^3 \\ k_4(s) &= 4 + 1 + (2 + 1.3)s + \\ &\quad + (1.3 \times 2 + 0.4)s^2 + 0.4 \times 2s^3 \end{aligned}\right\} \quad (28)$$

After further calculation, all of the Kharitonov's polynomials are in the proper state for assessment:

$$\left.\begin{aligned} k_1(s) &= 8.75 + 2.875s + 3.75s^2 + s^3 \\ k_2(s) &= 4.375 + 3.3125s + 3.5s^2 + s^3 \\ k_3(s) &= 3.125 + 3.3125s + 3.5s^2 + s^3 \\ k_4(s) &= 6.25 + 2.875s + 3.75s^2 + s^3 \end{aligned}\right\} \quad (29)$$

Again the Routh-Hurwitz stability criterion is applied to all these polynomials $k_i(s)$,*(i = 1, 2, 3, 4)* taking into account Table I.

TABLE III RESULTS FROM THE FOUR KHARITONOV POLYNOMIALS (CASE 2)

$k_1(s)$		$k_2(s)$		$k_3(s)$		$k_4(s)$	
1	2	1	3	1	3	1	2
3	8	3	4	3	3	3	6
0		2		2		1	
8		4		3		6	

Since the first column of each Kharitonov's Polynomial, shown in Table III, contains no change in sign and all its components are positive, the conclusion is that all of the roots of each $k_i(s)$, $(i = 1, 2, 3, 4)$ polynomial have negative real parts. Therefore the closed-loop control system is stable for all coefficient values in the specific ranges. That is, the feedback control system is guaranteed asymptotically stable.

The Kharitonov's assessment can be useful for determining system's stability in the cases of variation of large number of the system's parameters when defined within specific limits.

At the same time the Kharitonov's assessment method has substantial disadvantages. It is short of determination of the parameter marginal values of stability, also the results are achieved after considerable calculations and there is lack of any graphical display visualizing these results.

The Kharitonov's assessment is also applicable only for a prearranged and specified set of system's parameter variations.

The major disadvantage of the Kharitonov's method is that the Kharitonov polynomials deal with the coefficients variations of the Kharitonov characteristic interval polynomial, rather than directly with the system's parameter variations. The variations of the system's parameters remain in a hidden mode. These variations cannot be directly observed from the four Kharitonov polynomials.

Alternatively, the advanced D-Partitioning analysis, presented in this research, has considerable advantages, compared with the Kharitonov's theorem assessment.

The advanced D-Partitioning analysis does not need a specified set of limits of parameter variations. It is applicable generally and can deliver results representing the exact marginal values of the multivariable parameters.

The D-Partitioning analysis results are obtained easily with the aid of the interactive MATLAB procedure. The D-Partitioning curve in terms of the two variable parameters is plotted by the simple MATLAB code, as already demonstrated. The clear graphical display of the regions of stability and instability is another significant advantage of the D-partitioning.

A graphical evaluation between the two methods of analysis, as seen from Fig. 4, is validating the considerable advantage of the achieved advanced D-Partitioning analysis compared with the Kharitonov's assessment.

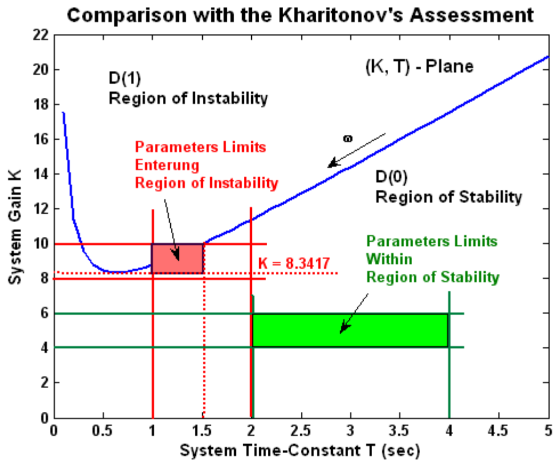

Fig. 4: *Advanced D-Partitioning Analysis compared with the Kharitonov's Assessment in Terms of Two Simultaneously Variable Parameters*

(Stability Assessment for Case 1 and Case 2)

The graphical result of the advanced D-Partitioning analysis is illustrating immediately the region of stability D(0) and the region of instability D(1) that can be used for the complete general assessment of the closed-loop system stability.

It is obvious from Fig. 4 that for Case 1, when the system gain is within the limits $K \in [8,10]$, and simultaneously the system time-constant is within the limits $T_1 \in [1,2]$, these parameter limits are entering the region of instability D(1) and the system will become asymptotically unstable.

For case 2, when the two variable parameters are within the limits $K \in [4,6]$ and $T_3 \in [2,4]$, these parameters limits are entirely within the region of stability D(0) and therefore the feedback control system will be guaranteed asymptotically stable.

This distinctive phenomenon is demonstrating the considerable advantage of the D-Partitioning analysis in comparison with the Kharitonov's assessment. By applying the D-Partitioning analysis and implementing a simple interactive MATLAB procedure, the system's asymptotic stability can be promptly determined and it can be graphically demonstrated, avoiding the significant calculations needed for the Kharitonov's theorem assessment.

III. Conclusion

Contribution of this research is the application of the further advancement of the D-partitioning stability analysis, accomplished by applying the method in case of multivariable system parameters. *The advancement of the D-partitioning stability analysis, developed by the author, proved to be a unique method that introduces a clear graphical display of the system's parameters variation and their interaction.* As a result, in case of two simultaneously variable parameters, regions of stability and instability are determined in the parameters' plane.

Each point on the D-Partitioning curve represents the marginal values of the two simultaneously variable parameters, being a unique property of the advanced D-Partitioning stability analysis that is not offered by the Kharitonov's assessment or any other known stability analysis method.

Also, by applying the D-Partitioning analysis, the system stability can be assessed immediately for any simultaneous variation of the two variable parameters without the need of determining the Kharitonov polynomials and calculating the values of the Routh array columns.

This research is worth achieving it, not only because it advances knowledge. It has a substantial practical aspect as well, since it can be used for analysis of a lot of industrial control systems that have uncertain or variable parameters due to various ambient conditions.

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