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Non Stationary Interaction of Elastic of Waves with Cylindrical Shells

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***Abstract*. We consider the interaction of non-stationary of waves, reinforced holes (cylindrical shells). Are considered infinitely long circular cylindrical shells are in infinitely elastic medium. Shell Manual- falls flat longitudinal wave voltages. With the help of the integral Laplace transform and Fourier An analytical expression of stress numerical results.**

Introduction. Investigation of the propagation explosive of non-stationary of waves in a piecewise-homogeneous of elastic and viscoelastic media represents a significant scientific and practical interest. This is due to necessity of studying the of possible damage of elements of underground constructions during the passage through them of the voltage pulse and with a variety of technological applications [1, 2, 3]. In particular, the stress waves are used for measuring of elastic constants the material cracks and detection of information transmission. Necessary to note that if the stress-strain state of elements of designs under static and quasi-static loading can be calculated using the well-established methods, the analysis of non-stationary of waves propagation in structures is a complex math problem [4]. In this paper we consider the problem of the dynamic behavior of cylindrical shells under the influence of non-stationary loads.

Statement of the problem and solution methods. Method of integral transforms built an approximate analytical solution of the problem on interaction of thin cylindrical shell with unpaved (elastic) environment at influence on seismic non-flat (of non-stationary) waves. The decision allows defining circumferential and longitudinal strain (and effort) shell.

In a flat statement of a similar problem was solved in [1, 2] - conditions of for hard contact shell and environment, and in [4] - to slippage conditions. The design scheme task is presented in Figure 1. A circular the cylindrical shell with a radius R and wall thickness h of the material has a density S_0, Poisson's ratio ν and Young modulus E, located in an elastic medium with a density ρ_0 and Lame coefficients λ, μ, accumulates at an angle γ to the axis of the shell is flat longitudinal wave Manual-voltage

$$q(r,\theta,z,t)=q_0 H(t)\ , \qquad (1)$$

Where q_0 - the amplitude of rolling wave; H - Heaviside function;

$$x=t-\frac{r(\cos\theta\sin r+\sin\theta)+z\cos r}{c_1}.$$

Having constructed solution of the problem for waves speed (1), using the Duhamel integral is easy to construct solution for non-stationary wave with an arbitrary time dependence. Timing is produced from the instant when rolling wave front (1) leads to the cross section of the shell $z=0$.

In the absence of static of mass forces vector system $\vec{u}=[u_r,u_\theta,u_z]^T$ in an elastic medium is determined by the equations

$$(\lambda+2\mu)grad\ div\vec{u}-\mu\ rotrot\vec{u}=\rho(\partial^2\vec{u}/\partial t^2) \qquad (2)$$

Lame by the displacement vector is expressed through the scalar $\vec{\psi}$ and vector potentials:

$$\vec{u}=grad\ \ \varphi+rot\vec{\psi}\ ,$$

Where φ – potential of longitudinal waves; $\vec{\psi}(\psi_r,\psi_\theta,\psi_x)$ - potentials of transverse waves. Potentials of functions in cylindrical coordinates satisfy the following wave equations

$$\nabla^2\varphi-\frac{1}{c_S^2}\frac{\partial^2\varphi}{\partial t^2}=0;$$
$$\nabla^2\psi_x-\frac{1}{c_S^2}\frac{\partial^2\psi_x}{\partial t^2}=0; \qquad (3)$$

$$\nabla^2\psi_\theta - \frac{\psi_\theta}{r^2} + \frac{2}{r^2}\frac{\partial \psi_r}{\partial \theta} - \frac{1}{c_s^2}\frac{\partial^2 \psi_\theta}{\partial t^2} = 0$$

$$\nabla^2\psi_r - \frac{\psi_r}{r^2} - \frac{2}{r^2}\frac{\partial \psi_\theta}{\partial \theta} - \frac{1}{c_s^2}\frac{\partial^2 \psi_r}{\partial t^2} = 0;$$

Where $c_p^2 = (\lambda + 2\mu)/\rho; \; c_s^2 = \mu/\rho;$ velocity respectively of propagation of longitudinal and transverse waves.

The displacement vector, of deformations and displacement potentials of in cylindrical coordinates are expressed in the form [1]:

$$u_r = \frac{\partial \varphi}{\partial r} + \frac{1}{r}\frac{\partial \psi_x}{\partial \theta} - \frac{\partial \psi_\theta}{\partial z};$$

$$u_\theta = \frac{1}{r}\frac{\partial \varphi}{\partial \theta} + \frac{\partial \psi_r}{\partial z} - \frac{\partial \psi_x}{\partial r};$$

$$u_z = \frac{\partial \varphi}{\partial z} + \frac{\partial \psi_\theta}{\partial r} + \frac{\psi_\theta}{r} - \frac{1}{r}\frac{\partial \psi_r}{\partial \theta};$$

$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial x^2},$$

Where f – arbitrary function of time.

The equation for the displacement vector shell $\bar{u}_0 = [u, \vartheta, \omega]^T$ received as

$$L\bar{u}_0 = R^2\left[\partial^2 \bar{u}_0/\partial t^2 + \vec{P}/(\rho_0 h)\right]/c^2,$$

$$c^2 = E_0/\rho_0(1-\nu^2),$$

Or in expanded form

$$\left.\begin{aligned}
&\frac{\partial^2 u}{\partial x^2} + \frac{1-\nu_0}{2R^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{1+\nu_0}{2R}\frac{\partial^2 \vartheta}{\partial x \partial \theta} + \\
&\frac{\nu_0}{R}\frac{\partial w}{\partial x} - \frac{1-\nu_0}{E_0 h_0}\rho_0\frac{\partial^2 u}{\partial t^2} + P_x = 0 \\
&\frac{1+\nu_0}{2R}\frac{\partial^2 u}{\partial x \partial \theta} + \frac{1-\nu_0}{2}\frac{\partial^2 \vartheta}{\partial x^2} + \frac{1}{R^2}\frac{\partial^2 v}{\partial \theta} + \\
&\frac{1}{R^2}\frac{\partial w}{\partial \theta} - \frac{1-\nu_0^2}{E_0 h_0}\rho_0\frac{\partial^2 \vartheta}{\partial t^2} + P_\theta = 0 \\
&\frac{\nu_0}{R}\frac{\partial u}{\partial x} + \frac{1}{R^2}\frac{\partial \vartheta}{\partial \theta} + \frac{h^2}{12}\Big(\frac{\partial^2 w}{\partial x^4} + \frac{2}{R^2}\frac{\partial^4 w}{\partial x^2 \partial \theta^2} + \\
&\frac{2}{R^4}\frac{\partial^4 w}{\partial \theta^4}\Big) - \frac{1-\nu_0^2}{E_0 h_0}\rho_0\frac{\partial^2 w}{\partial t^2} + P_r = 0
\end{aligned}\right\} \quad (4)$$

here R - the radius of the middle surface of the shell; h_0 - the thickness of the shell; E_0 - Young's modulus; □$_0$- Poisson's ratio; ρ_0- the density of the shell material; θ – angle reckoned from the initial generator; $\vec{P} = [P_x, P_\theta, P_r]^T$ – vector of the surface forces.

$$\begin{pmatrix} P_x \\ P_\theta \\ P_r \end{pmatrix} = \frac{1-\nu_0^2}{E_0 h_0}\begin{pmatrix} \sigma_{rx} \\ \sigma_{r\theta} \\ \sigma_{rr} \end{pmatrix}.$$

The initial conditions for the half-space accept the null; and the boundary conditions at r→∞ given rolling wave (1), at r→∞ zero. On the surface of the shell full slippage conditions are formulated:

- Continuity of normal stress components at $r = R$

$$P_r = -q_0\left[(1-\eta)\sin^2\gamma\cos^2\theta + \eta\right]A - \frac{\rho_0 h c^2}{R^2}\left[\lambda\left(\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{R}\frac{\partial \varphi}{\partial r} + \frac{1}{R^2}\frac{\partial^2 \varphi}{\partial t^2}\frac{\partial^2 \varphi}{\partial z^2}\right) + 2\mu\left(\frac{\partial^2 \varphi}{\partial r^2} - \frac{1}{R^2}\frac{\partial \psi_r}{\partial \theta} + \frac{1}{R}\frac{\partial^2 \psi_r}{\partial \theta} + \frac{\partial^3 \psi_\theta}{\partial r \partial \theta \partial z}\right)\right];$$

- The absence of shear stresses at $r = R$

$$P_0 = q_0 A(1-\eta)\sin^2\gamma\sin\theta\cos\theta - \frac{\rho_0 h c^2}{R^2}\mu\left(\frac{2}{R}\frac{\partial^2 \varphi}{\partial r \delta\theta} - \frac{1}{R}\frac{\partial \psi_r}{\partial r} - \frac{\partial^2 \psi_r}{\partial r^2} + \frac{1}{R^2}\frac{\partial^2 \psi_r}{\partial \theta^2} - \frac{2}{R^2}\frac{\partial^2 \psi_\theta}{\partial \theta \partial z} + \frac{2}{R}\frac{\partial^3 \psi_r}{\partial r \partial \theta \partial z}\right) = 0;$$

$$P_z = 0{,}5Aq_0(1-\eta)\sin 2\gamma\cos\theta - \frac{\rho_0 h c^2}{2R}\mu\left(2\frac{\partial^2 \varphi}{\partial r \partial z} + \frac{1}{R}\frac{\partial^2 \psi_r}{\partial \theta \partial z} + \frac{1}{R^2}\frac{\partial^2 \psi_\theta}{\partial r} - \frac{1}{R}\frac{\partial^2 \psi_\theta}{\partial r^2} - \frac{\partial^3 \psi_\theta}{\partial r^3} + \frac{2}{R^3}\frac{\partial^2 \psi_\theta}{\partial \theta^2} - \frac{1}{R^2}\frac{\partial^3 \psi_\theta}{\partial r \partial \theta^2} + \frac{\partial^3 \psi^2}{\partial r \partial \theta \partial z}\right) = 0,$$

The continuity of the displacement rates at $r = R$

$$\frac{\partial w}{\partial t} = -\frac{q_0}{\rho C_1}A\sin\gamma\cos\theta - \frac{\partial}{\partial t}\left(\frac{\partial \varphi}{\partial r} + \frac{1}{R}\frac{\partial \psi_r}{\partial \theta} + \frac{\partial^2 \psi_r}{\partial r \partial z}\right),$$

where $q_0 = \rho C_1 v_n; \; v_n = q_0/\rho C_1;$
$v_n = (q_0/\rho a)\sin\gamma \cdot \cos\theta$

Solution of the formulated problem by separating variables. Are applicable to equations (3) - (4) of the Laplace transform with respect to time, a finite cosine - or sinus - Fourier transform by corner of θ and the Laplace transform in the coordinate z, write them in the pictures

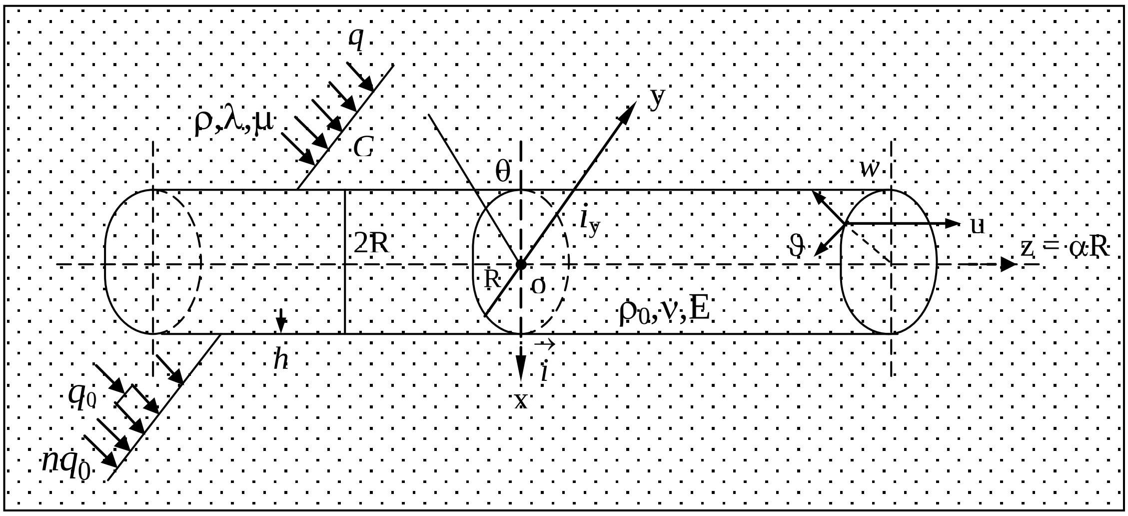


Fig.1. The settlement scheme**.**

$$\partial^2\varphi^{LcL}/\partial r^2+\partial\varphi^{LcL}/r\partial r-\left(n^2/r^2-s^2+p^{cl}\right)\varphi^{LcL}=0$$
$$\partial^2\psi_m^d/\partial r^2+\partial\psi_m^d/r\partial r-\left(n^2/r^2-s^2+p^2/\gamma_p^2\right)\psi_m^d=0, \qquad (5)$$

Where m=1,2

$$d=LsL \;\; at \;\; m=1, \quad d=LcL \;\; at \;\; m=2,$$

$$\left(S^2-n^2a\right)U^{LcL}+nsb\vartheta^{LsL}+\nu Sw^{LcL}=A^2p^2u^{LcL};$$

$$-bsnu^{LcL}-\left(n^2-as^2\right)\vartheta^{LsL}-nw^{LcL}=A^2p^2\vartheta^{LcL};$$

$$\nu s u^{LcL}+nu^{LcL}+\left[k^*\left(S^2-n^2\right)+1\right]w^{LcL}=A^2p^2w^{LcL};$$

$$Gq_0\left(\pi e^{-p\sin\gamma}\right)D\left(p,s,n,\gamma,I_n,\varphi,\psi_r,\psi_\theta,M,N\right)/\left[p/S+p\cos\gamma\right],$$

Where P, S – parameters of the Laplace transform in t $a=\frac{z}{R}$. n – Fourier transformation parameterθ; $I_u(x)$ – modified cylindrical function indices n. In equation (5) used the dimensionless variables

$$\varphi=\varphi/R^2;\; \psi_m=\psi_m/R^2;\; t=C_1t/R;\; r=r/R;\; q_0=q_0/\rho C_1^2;\; u=u/h;\; \vartheta=\vartheta/h;\; w=w/h.$$

Appeal images t performed using of Theorem of delay. Function deflections ω shell at ν = 0,3 given in divided of variables received in the form of:

$$w(t,\theta,z,\gamma)=\frac{q_0E}{5{,}46\rho C_1\rho_0R^2}\left[\beta^2\sin^2\gamma\cos\theta-\frac{(1-\eta)}{4AN}\sin^2\gamma\left(\beta_u-2\beta_3\sin\theta\sin^3\theta\right),\right] \qquad (6)$$

Where $\beta_2=\frac{(t-\omega_1)^3H(t-\omega_1)}{1{,}1(1{,}69A+\cos\gamma)}-\frac{(t-\omega_2)^3H(t-\omega_2)}{1{,}1(\cos\gamma-1{,}69A)}+\frac{(t-\omega_3)^3H(t-\omega_3)}{0{,}65(A+\cos\gamma)}-\frac{(t-\omega_4)^3H(t-\omega_4)}{0{,}65(\cos\gamma-A)};$

$$\beta_4=\frac{(t-\omega_4)^4H(t-\omega_1)}{1{,}1(1{,}69A+\cos\gamma)}-\frac{(t-\omega_2)^4H(t-\omega_2)}{1{,}1(\cos\gamma-1{,}69A)}+\frac{(t-\omega_3)^4H(t-\omega_3)}{0{,}65(A+\cos\gamma)}-\frac{(t-\omega_4)^4H(t-\omega_4)}{0{,}65(\cos\gamma-A)},$$

$$\omega_{1,2}=\sin\gamma(1-\cos\theta)\neq 1{,}69Az/R;\; \omega_{3,4}=\sin\gamma(1-\cos\theta)\pm Az/R.$$

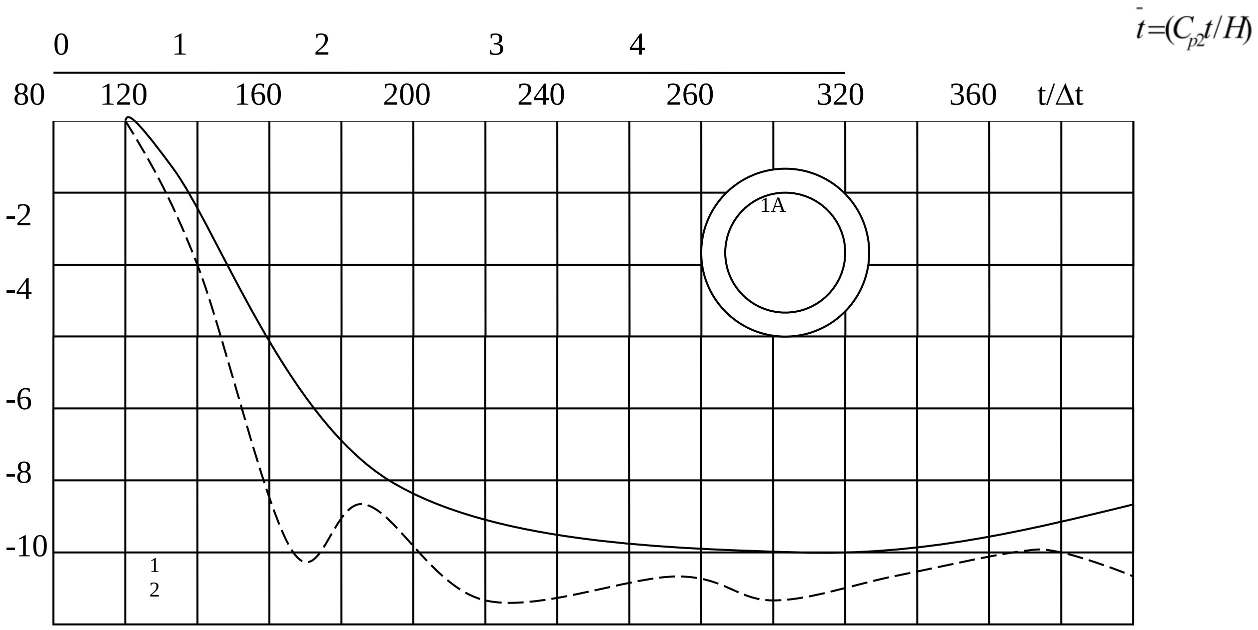
With the help of Duhamel integral and (6) we can construct a solution to the incident plane longitudinal wave (1).

To assess the accuracy of the constructed solution was calculated steel shells in an elastic medium on the initial data

$$\eta=0{,}5м,\;\; h=0{,}014м, \rho_0=7{,}8T/м, \nu=0{,}3, E=2{,}1\cdot10^5 мПа, \gamma=90^0, \eta=0{,}35;$$

$$\rho_0=1{,}6T/м^3, q=800м/c, C_2=400м/c, q_0=1{,}6МПа, \lambda=0{,}4 \;.$$

The dependence of the change in compressive of the elastic loop voltage $\bar{\sigma}_x$ 1A at the points in time $\bar{t}$ on the contour of free circular hole is shown in Fig.2.



$$\overline{\sigma_k} = \frac{\sigma_k}{|\sigma_0|}$$

Fig.2. Changing the compression of the elastic loop voltages $\overline{\sigma}_x$ 1A at the points in time $\bar{t}$ on the contour of free round holes: 1 Results of analytical solutions when exposed to of a plane longitudinal of the elastic wave type Heaviside certain functions; 2 the results present work p.

Thus the constructed solution allows to estimate a first approximation of the parameter space of movement and stress - strain state long (main) day labor pipelines and tunnels to join the rock mass lining when they overlap plane wave voltages.

Conclusions:

1. Analysis of the results shows that the maximum value of stresses in the shell is achieved at a certain time delay.

2. It was found that at the beginning of the hole and backed by the time the wave passes around its radius becomes almost uniformly uniform compression, then comes a qualitatively new phase of the movement, which outline voltages fed and appeared noticeable bending stresses rapidly grow to form a ring of five holes waves.

3. In contrast to the known solutions allows to calculate the change in deformation and curvature of the shell along the axis Oz, as well as longitudinal forces and bending moments.

References

1. Oscillations and waves in a dissipative inhomogeneous bodies and structures. Publisher Tashkent. 1992, 250p.

2. Baron M.L., Parnes R. Diffraction of a pressure wave by a cylindrical shell in an elastic-medium. Proc. 4 the U.S. Nat. Congr. Appl. Mech., 1962 №1, p.63-76.

3. Guz A. N., Kubenko V.D., Cherenkov M.A., Diffraction of elastic waves. - Kiev: Science: Dumka. 1978. -307p.

4. Safarov I.I. Boltayev M.B. The analytical solution of the problem in blasting rocks impact loads on the construction of pipelines and tunnels attachment. Mining Bulletin of Uzbekistan. №29, 2, 2007, p. 109-112.