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Mathematical Modeling and Simulation Study of *SEIR* disease and Data Fitting of Ebola Epidemic spreading in West Africa

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***Abstract*—In this paper we have considered two mathematical models of epidemics viz., $SEII_hR$ and SEIR. In case of SEIR model, simulation studies and data fitting of Ebola epidemic is taken up as the issue has become a hot and burning issue around the globe. The registered data up to December 14, 2014, both the infected and death cases, of Ebola epidemic in the countries Guinea, Liberia and Sierra Leone is used for data fitting. The projected number of infected and death cases up to April 7, 2015 are computed using simulation study and incorporated.**

Keywords—Ebola, SEIR model, Simulation study, Data fitting

1. Introduction

It is well known that the infectious diseases are caused by pathogenic microorganisms, such as bacteria, viruses, fungi and parasites. The infectious diseases will spread directly or indirectly from person to person and / or from animals and birds to human beings. These infectious diseases are a potential cause of deaths worldwide [1] as the medical advancement is not up to the requirement level, medical administer is not proper and costs involved are not reachable to a common man [2].

It has been proved and accepted that the mathematical modeling is a valuable tool to understand the dynamics of infectious diseases. Further, the mathematical modeling and tools will support in developing the control strategies of infectious diseases. Ebola virus is one of the infectious diseases that causes high death rate. Ebola causes hemorrhagic fever syndrome in human beings. The infectious Ebola virus ($EBOV$) outbreak began in Guinea in December 2013 and later spread to Sierra Leon, and Liberia [3-5].

In this paper, we considered $SEIR$ (Susceptible-Expose-Infected—Removed) epidemic model and discussed the mathematical analysis. Also, simulation study is conducted and fitted the available data of Ebola infectious disease. For this purpose we used the most recent data made available with WHO (World Health Organization). The data includes both the infected and death cases in the three West African bordered countries viz., Guinea, Sierra Leon and Liberia [4-6]. The numbers of both the infected and death cases of $EBOV$ continue to increase rapidly in West Africa and likely to spread to other parts of the world [6]. The simulation studies of the model with variable values of sensitive parameter of the spread of the outbreak are undertaken and incorporated here in this paper.

2. Ebola epidemic and the facts

In this section we briefly introduce the Ebola epidemic, provide signs and symptoms of the disease and the prevention measurements.

2.1 Ebola epidemic

The Ebola epidemic is in earlier times known as Ebola hemorrhagic fever. It is a rare and deadly disease caused by infection with one of the Ebola virus species. Various Ebola viruses so far identified including Zaire Ebola Virus (ZEBOV), Sudan Ebola Virus (SUDV), Reston Ebola Virus (RESTV), Tai Forest Ebola Virus (TAFV), and Bundibugyo Ebola Virus (BDBV). Ebola viruses are being found at all most all the parts of the world mainly in several African countries. Ebola was first discovered in 1976 near the Ebola River which is now in the central African country Democratic Republic of the Congo. Ebola can cause disease in both humans and nonhuman primates, the highest order of mammals supposed to be fore runner of man, those include Monkeys, Gorillas, and Chimpanzees etc. The virus is capable to transmit through blood, body fluids of an infected person or animal and also through contact with contaminated objects [7-9].

2.2. Signs and Symptoms of Ebola disease

It is to be noted that either a person or an animal infected with Ebola virus is not contagious until explicit symptoms appear. The signs and symptoms of Ebola in human beings include fever, severe headache, fatigue, muscle pain, weakness, diarrhea, vomiting, abdominal or stomach pain, and unexplained hemorrhage including bleeding or bruising. Explicit

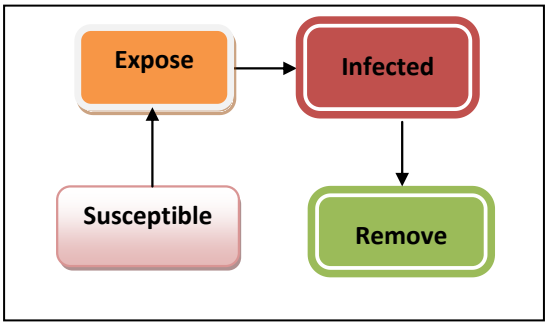
symptoms may appear in human beings at any time during two to twenty one days after exposure to Ebola virus with an average period of eight to ten days.

Recovery from the attack of Ebola virus is dependent on the level of medical care administered to and immunity of the patient. It is estimated in general that the patients who recover from Ebola infection will develop antibodies, blood proteins produced by a counteracting agent, so that they will not be again exposed to the infection for at least ten years and possibly more number of years. The probe to calculate the exact amount of immunity power is still under the way. It is to verify whether the patients recovered from Ebola virus develop immunity against the same virus or against all the Ebola viruses and also whether the immunity works either for life time or for a certain period of time [7-9].

2.3 Prevention measurements against Ebola virus

It is to be noted that, so far in the world, there has been no FDA (Food and Drug Administration, USA) approved vaccine available for Ebola. A person who goes on travelling to or living in the area where Ebola epidemic out broke is suggested to take the following prevention measurements so as not be affected: (i) Keep up always careful hygiene of the body including washing hands with soap water or an alcohol – based sanitizer, (ii) Keep away from the contact of body fluids of sick persons. The body fluids to be avoided from contacting include blood, urine, feces, saliva, vomit, sweat and semen, (iii) Avoid handling the items those have already been contacted the body fluids of an infected person, (iv) Handling the dead body of a person who died with Ebola disease during funeral or burial rituals are to be avoided (v) The blood, body fluids and raw meat of bats and nonhuman primates are capable of propagating Ebola virus, so avoid their contact and (vi) for the next 21 days after returned back from journey the traveler is required to monitor his health. If any symptoms of Ebola virus are developed the medical care must be sought immediately [4, 7, and 10].

3. Mathematical modeling of $SEIR$ epidemics



In the literature a lot many varieties of mathematical models have been proposed to describe the dynamics of infectious diseases. Such mathematical models include SIR, $SEIR$ and $SEII_hR$. Here in this paper we have briefly introduced both $SEII_hR$ and $SEIR$ models.

Some concepts of $SEII_hR$ model are discussed. In the literature a lot many mathematical models describing the evolution and dynamics of infectious diseases have been proposed. In this present study we consider the simple $SEIR$ model. We present mathematical and stability analysis of the model besides used the data of Ebola affected patients to fit in the $SEIR$ model. We also have introduced briefly still sententious model known as $SEII_hR$ model but fitting the data of Ebola epidemic and drawing inferences would be taken up in our subsequent work.

3.1. $SEI\ I_hR$ Model

The $SEIR$ model ignores the feature of possible isolation of an infected patient from other compartments of people so as to restrict the Ebola virus propagation. This feature has been incorporated in modeling of infectious diseases and the new emerging model is named as $SEI\ I_hR$.

3.1.1 Mathematical formulation of $SEII_hR$ model

The mathematical model describing $SEII_hR$ model is expressed as systems of differential equations as follows:

$$\frac{dS}{dt} = \mu N - \frac{\beta(t)SI}{N} - \mu S \quad (1a)$$

$$\frac{dE}{dt} = \frac{\beta(t)SI}{N} - \sigma E - \lambda E - \mu E \quad (1b)$$

$$\frac{dI}{dt} = \sigma E - \gamma I - \alpha I - \mu I \quad (1c)$$

$$\frac{dI_h}{dt} = \lambda E + \alpha I - \omega I_h - \mu I_h \quad (1d)$$

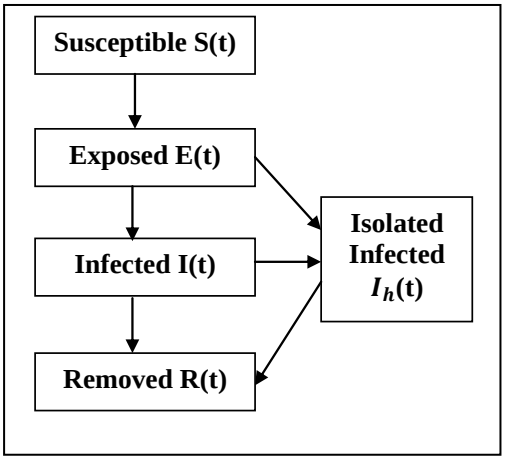
$$\frac{dR}{dt} = \gamma I + \omega I_h - \mu R \quad (1e)$$

Here the parameter $N(t) = S(t) + E(t) + I(t) + I_h(t) + R(t)$ representing the total population is considered to be a constant while both the numbers of births and deaths are equal. These assumptions about N, births and deaths are made since the Ebola epidemic outbreak time is considerably short [3, 11] during which the population size remains the same.

3.1.2 Description of $SEII_hR$ Model

Since the Ebola outbreak time is assumed to be short, and during this small amount of time period the number of births and deaths occurring are considered to be equal and hence the total population represented by N may be considered as a constant. The model assumes that the total population N is divided in to five groups which we call here as compartments. Each of the five compartments is represented by one letter in the name of the model sequence $SEII_hR$ at the time t . That is, the names of the compartments are S, E, I, I_h and R respectively: $S(t)$ is the number of people in the susceptible compartment where the people are capable of becoming infected, $E(t)$ is the number of people in the exposed compartment where the people are incubating the infection, $I(t)$ is the number of people in the infected compartment where the people are infected or infectious with the virus [12],

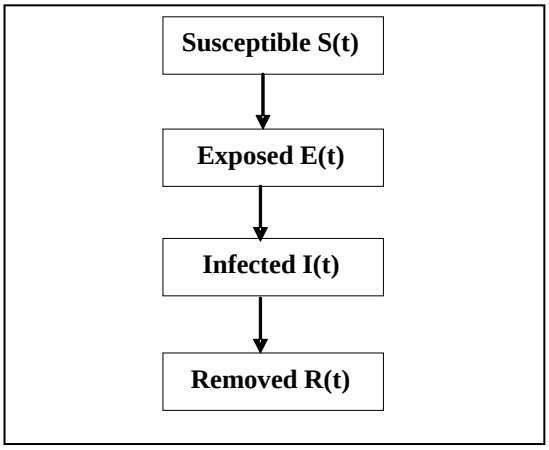
$I_h(t)$ represents the number of people in the isolated infectious compartment where the infected and exposed people have been isolated from the susceptible population and $R(t)$ is the number of people in the removed compartment where the people are considered to be recovered or died. The transmission of EBOV is considered to follow the multistoried sequence:



The parameter μ represents the death rate or equivalently the birth rate of the population during the short span of the epidemic out break and β represents the transmission rate of the disease that is the rate of transferring people from the compartment $S(t)$ to $E(t)$. Similarly, $1/\sigma$ and $1/\gamma$ represent the average durations of stay in the compartments of $E(t)$ and $I(t)$. Also the parameter $1/\omega$ represent the average time that takes for a patient to transfer from isolation to death that is to go from $I_h(t)$ to $R(t)$. The parameters λ and α are the probabilities of isolated individual from exposed and infected compartments respectively. We will take up the mathematical analysis, simulation study, real data fitting, Ebola data fitting of $SEII_hR$ model in our subsequent work. In the present paper for the above purpose we consider still the simpler model as described in the next subsection.

3.2 $SEIR$ Model and its Description

The persons affected by epidemics can be comfortably and for modeling purpose, divided into various compartments. Here we mainly consider those diseases which have only four compartments viz., Susceptible, Exposed, Infected and Removed (SEIR). The $SEIR$ mathematical model is widely used in the field of epidemiology to analyze the infectious diseases. The infectious diseases spread from an infected individual to other susceptible individuals in the surroundings. The total population at time t represented by $N(t)$ is considered as a constant and is the sum of the populations in the compartments $S(t), E(t), I(t)$ and $R(t)$. The transmission of $SEIR$ $EBOV$ is considered to follow the forward sequence:



The governing equations describing the evolution and dynamics of SEIR model can be described by a set of ordinary differential equations as follows [13-14]:

$$\frac{dS}{dt} = \frac{-\beta(t)SI}{N} \quad (2a)$$

$$\frac{dE}{dt} = \frac{\beta(t)SI}{N} - \sigma E \quad (2b)$$

$$\frac{dI}{dt} = \sigma E - \gamma I \quad (2c)$$

$$\frac{dR}{dt} = \gamma I \quad (2d)$$

Here above we assume that $N(t) = S(t) + E(t) + I(t) + R(t)$ is constant and denotes the total number of population in the system. That is, the population is closed in the sense that the immigrations, new births and deaths of people are not considered. The quantity $S(t)$ denotes the number of individuals those are susceptible to the disease but not infected at time t. The quantity $E(t)$ denotes the number of individuals those are exposed to the virus or infected but not yet infectious. The quantity $I(t)$ denotes the number of infected individuals who are able to spread the disease through contact with susceptible, and $R(t)$ denotes the number of individuals those have successfully gained immunity from the disease and / or removed by death. After transmission of the virus, susceptible individuals $S(t)$ enter the exposed class $E(t)$ before they become infectious individuals and later either recover or die. The parameter β is transmission rate of disease from susceptible to exposed. Similarly, $1/\sigma$ and $1/\gamma$ are the average durations of incubation and infectiousness respectively [3, 11, 13-14]. For better mathematical analysis we use dimensionless or scaled system of equations with new variables as $S = u\,N, E = v\,N, I = w\,N. R = z\,N$ and $\tau = \gamma\,t$ as follows:

$$\frac{du}{d\tau} = -R_0\,u\,w \quad (3a)$$

$$\frac{dv}{d\tau} = R_0 u\,w - K\,v \quad (3b)$$

$$\frac{dw}{d\tau} = K\,v - w \quad (3c)$$

$$z = 1 - u - v - w \quad (3d)$$

Where $R_0 = \frac{\beta}{\gamma}$, and $K = \sigma/\gamma$. The equations (3a) to (3d) represent the dimensionless equations of the $SEIR$ epidemic model.

3.3 Analysis of $SEIR$ model

Here we consider $SEIR$ mathematical model assuming that the numbers of both births and deaths are equal since the time duration under consideration is quite short and as a result population size remains to be a constant during this period of time. This assumption is made since the Ebola epidemic outbreak occurs in short time duration [15-16].

Considering the biological interpretation of system of ordinary differential equations given in (2), we can easily understand that the feasible region for system is R_+^4, the four dimensional space surrounded by only the positive axes. On summing up all the individual equations of the system (2), it is very easy and straight forward in getting $\frac{dS}{dt} + \frac{dE}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = 0$. It can be interpreted that this restriction on the variables S, E, I and R simplifies the 4 – dimensional world and can be represented by the simplex $\mathcal{E} = \{(S, E, I, R) \epsilon R^4_+ : S + E + I + R = N\}$. Further, it can be noted that $\mathcal{E}$ is positively invariant and on which $R(t) = N - S(t) - E(t) - I(t)$ is satisfied everywhere [17].

It can be noticed that disease-free equilibrium state exists. In the disease-free equilibrium state absence of infection occurs. Thus, all the infected classes, except the susceptible class, will be zero and the entire population will comprise of only infection free susceptible individuals [14, 18].

Here in what follows equilibrium points of system (2) are found, analyzed and discussed. For this very purpose here Jacobian stability approach is used and shown that the stability of the disease-free equilibrium state occurs. The disease free equilibrium (DFE) point of the model which we have discussed above can be computed as

$$\frac{dS}{dt} = \frac{dE}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = 0 \quad (4a)$$

$$\frac{-\beta SI}{N} = 0 \quad (4b)$$

$$\frac{\beta SI}{N} - \sigma E = 0 \quad (4c)$$

$$\sigma E - \gamma I = 0 \quad (4d)$$

On solving the above equations we get the disease free equilibrium point $E_0 = (N, 0, 0)$ and at this equilibrium point the basic reproductive number takes the value $R_0 = (\beta/\gamma) < 1$. This shows that the susceptible individuals are stable at the equilibrium point and it can be interpreted as the epidemic is died out. Note that at any equilibrium point if the reproductive number assumes the value $R_0 > 1$ then the equilibrium point is considered to be unstable and it means that the epidemic spread continuous [17-19].

4. Simulation study and Data fitting

In this section we have taken up the simulation study and data fitting of the Ebola epidemic. For this purpose we have considered the number of infected and death cases of Ebola disease registered up to December 14, 2014 in the three countries Guinea, Liberia and Sierra Leone. The results of the simulation study are incorporated graphically and also tabulated.

4.1 Simulation study

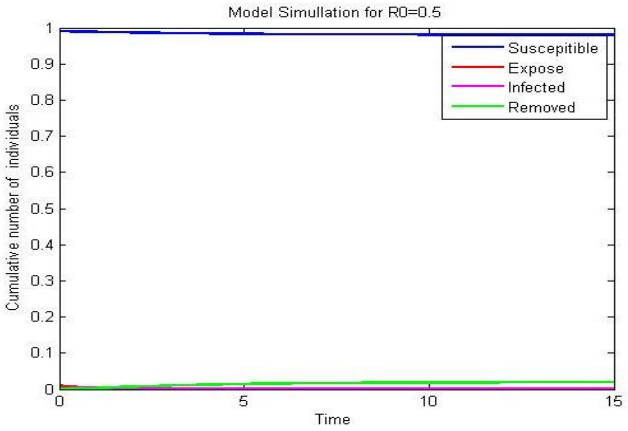


Figure 1: For $R_0 = 0.5$, The population size of susceptible compartment decreases from 0.99 to 0.9803 while that of exposed and infected compartments decrease from 0.009 and 0.001 to asymptotically zero respectively. The removed compartment size increases from zero to 0.01964. The complete population sizes of these compartments are the multiples of the respective fractions and the total population N.

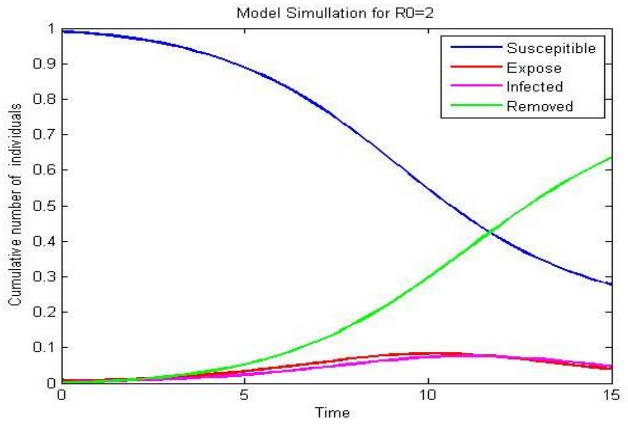


Figure 2: For $R_0 = 2$ the figure shows that the population size of susceptible compartment decreases from 0.99 to 0.28901. The population size of exposed compartment rose from 0.009 to 0.08348 and then fall down to 0.04016. The population size of infected compartment rose up from 0.001 to 0.07626 and then fell down to 0.0403. But the population size of removed compartment increases up from zero to 0.635. The intersection point of the curves representing both the susceptible and removed compartments occurs at 0.4248. Here in this figure too, the population sizes of all the compartments are normalized from N to 1.

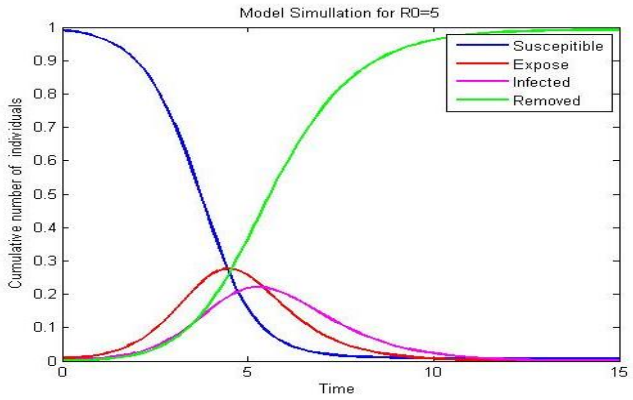


Figure 3: For $R_0 = 5$ the population size of susceptible compartment decreases from 0.99 to 0.06916. The population size of exposed compartment rises up from 0.009 to 0.2765 and then fall down to 0.0001292. The population size of infected compartment rises up from 0.001 to 0.2213 and then fall down to 0.000539. The population size of removed compartment increases from zero to 0.9924. The curve representing susceptible compartment has an intersecting point with the curves representing exposed, infected, removed compartments at 0.2765, 0.2132 and 0.2601 respectively. The intersection point of infected and exposed compartment curves is 0.2196, infected and removed compartment curves is 0.1134 and exposed and removed compartment curves is 0.2765.

4.2 Data analysis and fitting of Ebola epidemic cases

In this section we present the data containing both the sizes of Ebola infected persons and deaths and represent them graphically. We have collected the data of the three countries viz., Guinea, Liberia and Sierra Leone [4-6] and presented here. The cumulative cases of Ebola epidemic identified registered and reported from these three countries up to December 14, 2014 is 18590 and similarly the total cumulative number of death report is 7288. But, initially up to March 22, 2014 a total number of 49 infected cases and 29 death cases were recorded in the country Guinea and no cases were identified elsewhere. But, it can be expected that there might have been 3 times more cases than officially recorded and reported at any point of time. Here we have plotted the graphs of Ebola infected cases and death cases country wise and cumulatively. The graphical representations and the respective descriptions are as follows:

The Ebola epidemic actually started to spread on December 2013 [20] but the WHO started registering the infected and the death cases after 81 days later and hence the registered data is available only from March 22, 2014. The Ebola epidemic initially started in the country Guinea and later spread to the neighboring countries including Liberia and Sierra Leone. But as on today the most affected among these three countries is Sierra Leone.

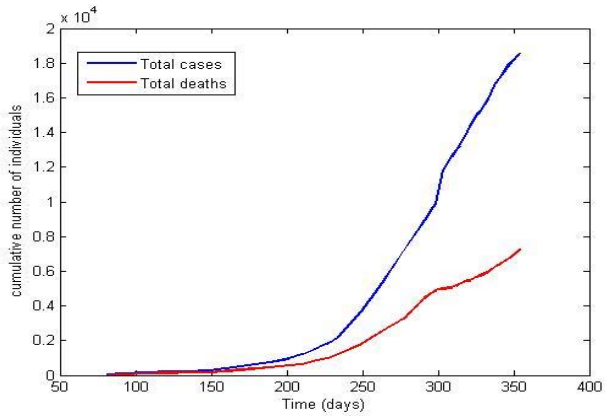


Figure 4: Blue and red curves represent the cumulative number of Ebola infected cases and number of death cases respectively in all the three countries viz., Guinea, Liberia, and Sierra Leone put together.

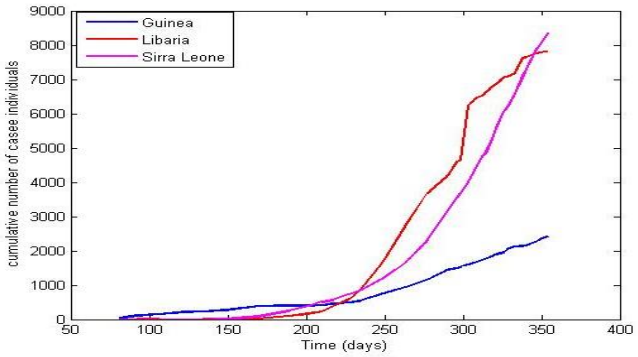


Figure 5: The number of Ebola cases registered in the three countries till December 14, 2014 has been represented graphically. It can be observed that the Ebola epidemic initially started from Guinea and later spread to other countries but the epidemic is under control in Guinea in comparison with the neighboring countries. The epidemic spread is taken over by Sierra Leon and where the reported cases are much more than the other two countries viz., Guinea and Liberia.

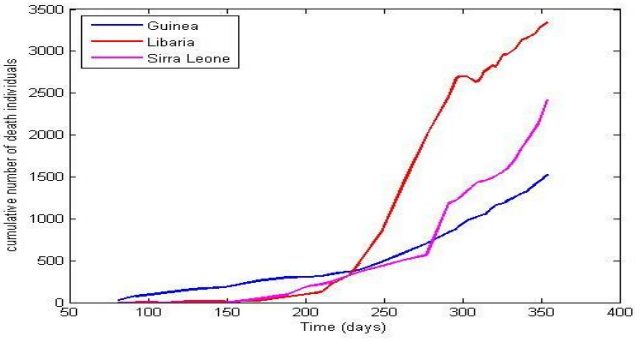


Figure 6: The death cases due to Ebola epidemic reported up to December 14, 2014 separately in the three countries viz., Guinea, Liberia and Sierra Leone have been plotted here. It can be observed that the death cases reported is increasing in Liberia much more than the remaining two countries Guinea and Sierra Leone.

4.3. Data fitting

Here in this section we consider simulated curves and fit the real data of Ebola epidemic. For this purpose both the infected and death cases of all the three countries are considered. The graphical representations of the simulation study and the data fittings are presented here under:

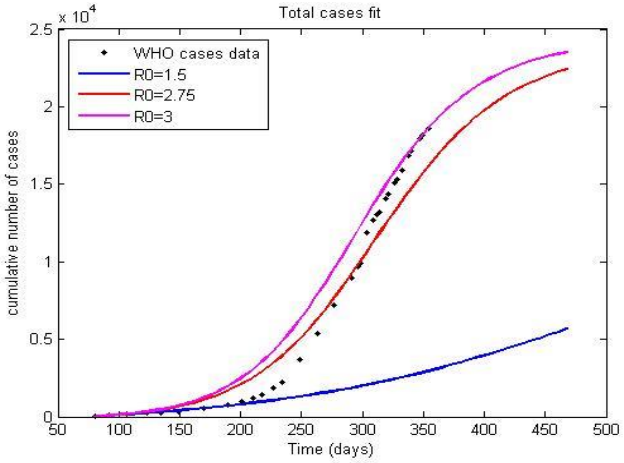


Figure 7: Curves represent the simulation study of basic reproductive number R_0 with three different values $R_0 = 1.5, 2.75\ and\ 3$. Also actual data of Ebola is fitted. The total number of Ebola infected cases till December 14, 2014 is 18590. But, the simulation study guesses that the number of infected cases will fall or rise up to 5679, 22420, and 23510 cases by April 2015 if the Ebola epidemic spreads with an order proportional to $R_0 = 1.5, 2.75\ and\ 3$. Further in the plot it can be seen that the real data fitting closely matches with $R_0 = 3$ and hence it can be interpreted that the infected cases are rising accordingly. The best fit of the model projects with 23420 infected cases by April 7, 2015.

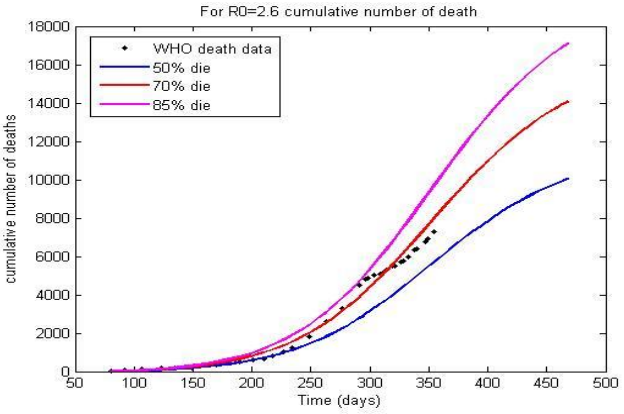


Figure 8: The graph represents plots representing the population size of the removed compartment with respect to the basic reproductive number $R_0 = 2.6$ while with the variable death rates of 50%, 70% and 85%. We have also fitted the data of cumulative death cases which is 7288 as on December 14, 2014. But, the simulation study guesses that the number of death cases will rise up to 10050, 14070 and 17080 by April 2015 if the death rates are respectively 50%, 70% and 85%. The cumulative number of death cases in all the three countries put together is considered in this plot. Further in the plot it can be seen that the real data fitting closely matches with 70% and hence it can be interpreted that the death cases is rising accordingly. The best fit of the model projects with 14070 death cases by April 7, 2015.

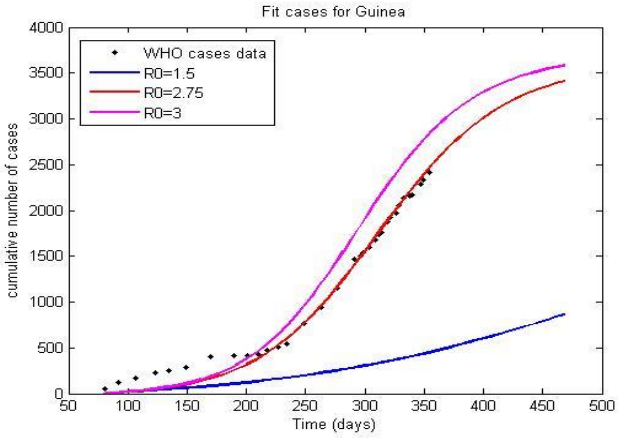


Figure 9: The cumulative Ebola infected cases 2415 in the country of Guinea up to December 14, 2014 have been fitted in the plot. The continuous curves represent simulations of basic reproductive number with three values $R_0 = 1.5, 2.75\ and\ 3$. The simulation study suggests that the cumulative Ebola infected cases by April 2015 will be 864.6, 3414 and 3580 if the infected cases grow according as $R_0 = 1.5, 2.75\ and\ 3$. Further in the plot it can be seen that the real data fitting a closely matching with $R_0 = 2.75$ and hence it can be interpreted that the infected cases in Guinea is rising accordingly. The best fit of the model projects with 3414 infected cases in Guinea by April 7, 2015.

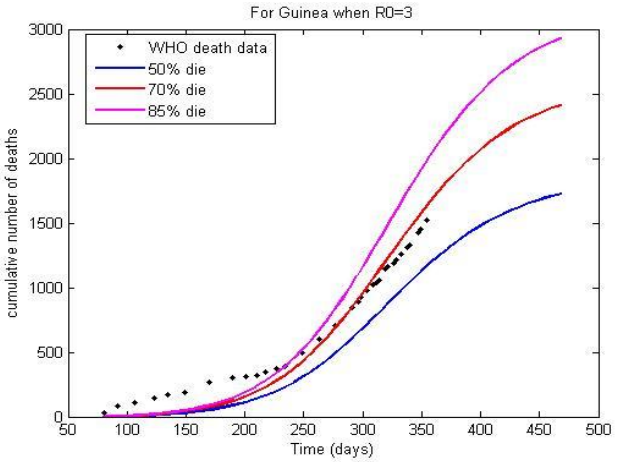


Figure 10: The cumulative Ebola death cases 1525 in the country of Guinea up to December 14, 2014 have been fitted in the plot. The continuous curves represent simulation study of basic reproductive number $R_0 = 3$ and the respective death rates from the removed compartment are $50\%, 70\%\ and\ 85\%$. The simulation study suggests that the cumulative Ebola death cases by April 2015 will be 1725, 2415 and 2932 corresponding to 50%,

70% and 85%. Further in the plot it can be seen that the real data fitting a closely matching with 70% and hence it can be interpreted that the death cases in Guinea is rising accordingly. The best fit of the model projects with 2415 death cases in Guinea by April 7, 2015.

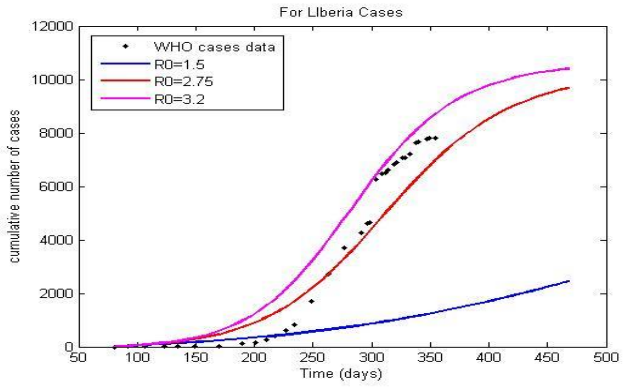


Figure 11: The cumulative Ebola infected cases 7819 in the country of Liberia up to December 14, 2014 have been fitted in the plot. The continuous curves represent simulations of basic reproductive number with three values $R_0 = 1.5, 2.75\ and\ 3.2$. The simulation study suggests that the cumulative Ebola infected cases by April 2015 will be 2450, 9656 and 10400 if the infected cases grow according as $R_0 = 1.5, 2.75\ and\ 3.2$. Further in the plot it can be seen that the real data fitting is closely matching with $R_0 = 3.2$ and hence it can be interpreted that the infected cases in Liberia is rising accordingly. The best fit of the model projects with 10400 infected cases in Liberia by April 7, 2015.

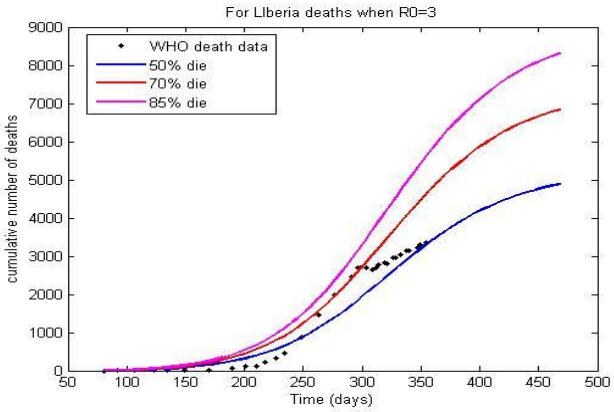


Figure 12: The cumulative Ebola death cases 3346 in the country of Liberia up to December 14, 2014 have been fitted in the plot. The continuous curves represent simulation study of basic reproductive number $R_0 = 3$ and the respective death rates from the removed compartment are $50\%, 70\%\ and\ 85\%$. The simulation study suggests that the cumulative Ebola death cases by April 2015 will be 4888, 6843 and 8309 corresponding to 50%, 70% and 85%. Further in the plot it can be seen that the real data fitting a closely matching with 50% and hence it can be interpreted that the death cases in Liberia is rising accordingly. The best fit of the model projects with 4888 death cases in Liberia by April 7, 2015.

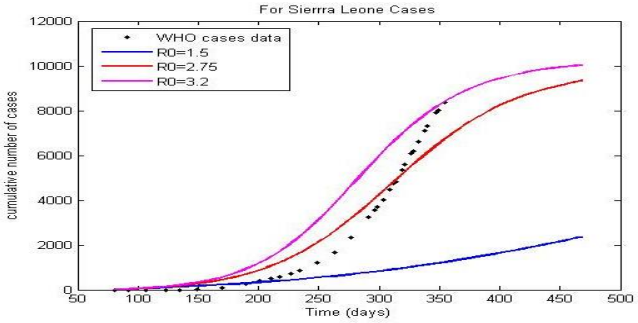


Figure 13: The cumulative Ebola infected cases 8356 in the country of Sierra Leone up to December 14, 2014 have been fitted in the plot. The continuous curves represent simulations of basic reproductive number with three values $R_0 = 1.5, 2.75\ and\ 3.2$. The simulation study suggests that the cumulative Ebola infected cases by April 2015 will be 2364, 9335 and 10030 if the infected cases grow according as $R_0 = 1.5, 2.75\ and\ 3.2$. Further in the plot it can be seen that the real data fitting is closely matching with $R_0 = 3.2$ and hence it can be interpreted that the infected cases in Sierra Leone is rising accordingly. The best fit of the model projects with 10030 infected cases in Sierra Leone by April 7, 2015.

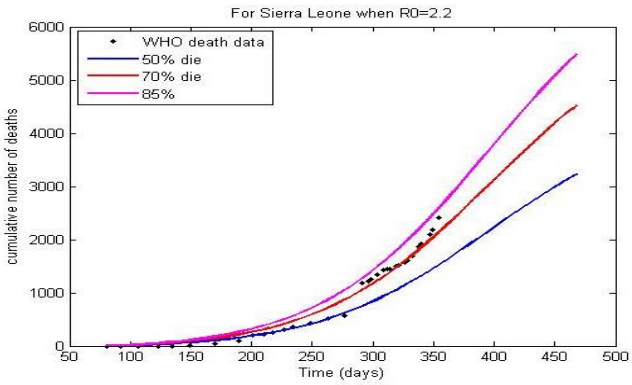


Figure 14: The cumulative Ebola death cases 2417 in the country of Sierra Leone up to December 14, 2014 have been fitted in the plot. The continuous curves represent simulation study of basic reproductive number $R_0 = 2.2$ and the respective death rates from the removed compartment are $50\%, 70\%\ and\ 85\%$. The simulation study suggests that the cumulative Ebola death cases by April 2015 will be 3228, 4520 and 5488 corresponding to 50%, 70% and 85%. Further in the plot it can be seen that the real data fitted is a closely matching with 75% and hence it can be interpreted that the death cases in Sierra Leone is rising accordingly. The best fit of the model projects with 4843 death cases in Sierra Leone by April 7, 2015.

The observations of the above simulation studies and the data fittings are tabulated as follows:

S. No.	Name of the country and cumulative number of infected cases identified till December 14, 2014	Basic reproductive number and the corresponding projected infected cases			Best fit up to April 7, 2015
1.	Guinea (2415)	$R_0 = 1.5$ (864.6)	$R_0 = 2.75$ (3414)	$R_0 = 3$ (3580)	$R_0 = 2.75$ (3414)
2.	Liberia (7819)	$R_0 = 1.5$ (2450)	$R_0 = 2.75$ (9656)	$R_0 = 3.2$ (10400)	$R_0 = 3.2$ (10400)
3.	Sierra Leone (8356)	$R_0 = 1.5$ (2364)	$R_0 = 2.75$ (9335)	$R_0 = 3.2$ (10030)	$R_0 = 3.2$ (10030)
4.	All the above three countries (18590)	$R_0 = 1.5$ (5679)	$R_0 = 2.75$ (22420)	$R_0 = 3$ (23510)	$R_0 = 3$ (23510)

Table 1: Country wise Ebola epidemic infected cases and projected cases with varied R_0 and the best projected number of cases.

S. No.	Name of the country and cumulative number of death cases identified till December 14, 2014	Rate of death from removed compartment and the corresponding projected infected cases			Best fit up to April 7, 2015
1.	Guinea (1525)	**50%** (1725)	**70%** (2415)	**85%** (2932)	**70%** (2415)
2.	Liberia (3346)	**50%** (4888)	**70%** (6843)	**85%** (8309)	**50%** (4888)
3.	Sierra Leone (2417)	**50%** (3228)	**70%** (4520)	**85%** (5488)	**75%** (4843)
4.	All the above three countries (7288)	**50%** (10050)	70% (14070)	**85%** (17080)	70% (14070)

Table 2: Country wise Ebola epidemic death cases and projected cases with varied percentages and the best projected percent of death cases.

5. Conclusions

Mathematical model is the basic tool to solve real world problems. We have studied simulation of SEIR epidemic model, plotted the simulated curves and the data of Ebola infected and death cases reported by WHO are fitted. For this purpose the data of the three countries Guinea, Liberia and Sierra Leone up to December 14, 2014 is considered and used.

Simulations study in Figure 1 shows that if the basic reproductive number R_0 is less than unity then the spread of epidemic is under control and eventually stop. Simulation study in figures 2 and 3 shows that if the basic reproductive number R_0 is not less than unity then the infection will continue to spread in the population and also the susceptible population converges to zero as time diverges.

Data presented in figure 5 and 6 show that the epidemic started earlier in Guinea and later spread to Liberia and Sierra Leone. Figure 5 shows that Ebola outbreak cases increased rapidly in Liberia and Sierra Leone and at the recent times cases in Liberia exceeded Guinea but not Sierra Leone. Data of death curves support that the death cases of Ebola epidemic in Liberia is needed a series attention because it is more in comparison with the remaining two countries, see Figure 6. We fit the data of Ebola infected and death cases registered up to December 14, 2014 and observed the feature projection of the Ebola outbreak in West Africa, see Figures 7 to 14. As per the records available the cumulative infected cases roses from 49 at March 22, 2014 to 18590 at December 14, 2014, but the simulation study of SEIR model predicts the number to 23510 up to April 7, 2015, see Figure 7. As per the records available the cumulative death cases roses from 29 at March 22, 2014 to 7288 at December 14, 2014, but the simulation study of SEIR model predicts the number to 14870 up to April 7, 2015, see Figure 8. The current simulation and data fitting of Ebola outbreak in the whole world requires a series attention so as to control the spread in a possible short time.

Since for the Ebola epidemic till now there has not been any tested and verified vaccination or medical treatments the best way to control the outbreak is to isolate the infected person as soon as identified. Hence the extension of $SEIR$ model, known as $SEII_hR$ model, is expected to be a better suitable one to study the control mechanism of Ebola epidemic. The new compartment I_h accommodates the infected, identified and isolated individuals those come from exposed and infected compartments before transmitted the epidemic to a susceptible individual. In the next paper we will apply this concept and study the propagation of various epidemics including Ebola in detail. Further, we also will take up parameter estimation for better projected values.

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