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An EOQ Inventory Model with Ramp Type Demand with Shortages

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***Abstract*—In this paper an EOQ inventory model is developed in which inventory is depleted not only by demand but also by deterioration at a generalized weibull distributed rate, assuming the demand rate a ramp type function of time. A brief analysis of the cost involved is carried out by an example.**

Keywords: EOQ model, Ramp type demand, Weibull distribution, Shortages, Deterioration.

AMS Subject Classification: 90B05

1. Introduction:

Harris and Wilson's [1] classical inventory model assumes that the depletion of inventory is due to a constant demand rate. But in many inventory models, the effect of deterioration is very important. All most items deteriorate with time. All food items, drugs, pharmaceuticals and radioactive substances are few examples of items in which deterioration take place during the normal storage period so this loss must be taken into account when analyzing the inventory system, and then the problem of decision makers is how to control and maintain inventories of deteriorating items. Many researchers like Ghare and Schrader [2], Goel and Aggrawal [3], Covert and Philip [4], Aggrawal [5], Cohen [6], Mishra [7] are very important in this connection. As the time progressed, several other researchers developed inventory models with deteriorating items with time dependent demand rates. In this connection, related works may refer to Ritchie [8], Tadikamalla [9], Ghose and Chaudhari [10], Donaldson [11], Silver [12], Datta and pal [13], Deb and Chaudhari [14], Pal and mandal [15]. Mishra [7] developed a two parameter Weibull distribution deterioration for an inventory model. This was followed by many researchers like Dev and Patel [16], Shah and Jaiswal [17], Jalan et al. [18], Giri and Goyal [19], Singh and Sehgal [20] etc.

In the present paper, efforts have been made to analyze an EOQ inventory models for items that deteriorate at a generalized Weibull distributed rate, assuming the demand rate a function of time. Shortages are allowed. Such demand pattern is generally seen in newly launched fashion goods, cosmetics, garments, automobiles etc; for which the demand increases with time as they launched into the market and after some time it becomes constant. The numerical examples is presented .

2. Assumptions and Notations:

The fundamental assumptions and notations used in this paper are given as follows:

(a) Replenishment size is constant and its rate is infinite.

(b) Lead time is zero.

(c) T is the fixed length of each production cycle.

(d) C_h is the inventory holding cost per unit per unit time.

(e) C_s is the shortage cost per unit per unit time.

(f) C_d is the unit deterioration cost.

(g) The deterioration rate function follows a generalized Weibull* distribution

$$Z(t) = \lambda^2 \beta t^{\beta-1} e^{-\lambda^2 t^\beta}, 0 < \lambda << 1, \beta > 0, t > 0$$

Where λ is the scale parameter, β is the shape parameter and t is the time of deterioration.

(h) Shortages are allowed and completely backlogged.

(i) S is the maximum inventory level of each ordering cycle.

(j) The demand rate R(t) is assumed to be a ramp type function of time:

$R(t) = D_0[t-(t-\mu)H(t-\mu)]$, $D_0 > 0$

Where $H(t-\mu)$ is the well-known Heaviside's function defined as follows:

$H(t-\mu) = 1, t \geq \mu$

$\quad = 0, t < \mu$

3. Model development:

Let Q be the total amount of inventory produced or purchased at the beginning of each period and after fulfilling backorders let us assume we get an amount S (>0) as initial inventory. Due to reasons of market demand and deterioration of the items, the inventory level gradually diminishes during the period (0, t_1) and finally falls to zero at $t = t_1$. Shortages are allowed during the period (t_1, T), which are completely backlogged.

The inventory level I(t) of the system at any time t over [0, T] can be described by the following equations:

$$\frac{dI(t)}{dt}+Z(t)\,I(t)=-R(t),\ 0\le t\le t_1 \qquad \textbf{(3.1)}$$

and

$$\frac{dI(t)}{dt}=-R(t),\ t_1\le t\le T \qquad \textbf{(3.2)}$$

The boundary conditions are

$I(t_1) = 0$ and $I(0) = S$ **(3.3)**

By assumptions of (g) and (j) in section (2) and assuming $\mu < t_1$, the two governing Eqs. (3.1) and (3.2) becomes:

$$\frac{dI(t)}{dt}+\lambda\beta t^{\beta-1}e^{-\lambda t^\beta}\,I(t)=-D_0(t),\ 0\le t\le \mu \qquad \textbf{(3.4)}$$

$$\frac{dI(t)}{dt}+\lambda\beta t^{\beta-1}e^{-\lambda t^\beta}\,I(t)=-D_0\mu,\ \mu\le t\le t_1 \qquad \textbf{(3.5)}$$

and

$$\frac{dI(t)}{dt}=-D_0\mu,\ t_1\le t\le T \qquad \textbf{(3.6)}$$

Now by using the boundary conditions (3.3), the solution of above three equations are respectively given as:

$$I(t)=S\,e^{-\left(1-e^{-\lambda t^\beta}\right)}-D_0\,e^{-\left(1-e^{-\lambda t^\beta}\right)}\int_0^t t\,e^{\left(1-e^{-\lambda t^\beta}\right)}dt,\ 0\le t\le \mu \qquad \textbf{(3.7)}$$

$$I(t)=S\,e^{-\left(1-e^{-\lambda t^\beta}\right)}-D_0\,e^{-\left(1-e^{-\lambda t^\beta}\right)}\left[\int_0^\mu t\,e^{\left(1-e^{-\lambda t^\beta}\right)}dt+\mu\int_\mu^t e^{\left(1-e^{-\lambda t^\beta}\right)}dt\right],\ \mu\le t\le t_1 \qquad \textbf{(3.8)}$$

And

$$I(t)=-D_0\mu(t-t_1),\ t_1\le t\le T \qquad \textbf{(3.9)}$$

When $0 < \lambda < 1$, we neglect the second and higher terms of λ, equation (3.7) and (3.8) becomes:

$$I(t)=S\left(1-\lambda t^\beta\right)-\frac{D_0t^2}{2}\left(1-\frac{\lambda\beta t^\beta}{\beta+2}\right),\ 0\le t\le\mu \qquad \textbf{(3.10)}$$

and

$$I(t)=S\left(1-\lambda t^\beta\right)-D_0\mu t\left(1-\frac{\lambda\beta t^\beta}{\beta+1}\right)+\frac{D_0\mu^2}{2}\left[1-\lambda t^\beta+\frac{2\lambda\mu^\beta}{(\beta+1)(\beta+2)}\right],\ \mu\le t\le t_1 \qquad \textbf{(3.11)}$$

Since $I(t_1) = 0$, we get from equation (3.8) with neglecting second and higher order terms of α as:

$$S=D_0\mu t_1\left(1+\frac{\lambda t_1^\beta}{\beta+1}\right)-\frac{D_0\mu^2}{2}\left[1+\frac{2\lambda\mu^\beta}{(\beta+1)(\beta+2)}\right] \qquad \textbf{(3.12)}$$

Hence the total number of deteriorated units during $[0, t_1]$ is

D_t = Initial inventory – Total demand during $[0, t_1]$

$$=S-\int_0^{t_1}D(t)dt$$

$$=S-\left[\int_0^\mu D_0t\,dt+\int_\mu^{t_1}D_0\mu\,dt\right]$$

Evaluating the above two integrals and using equation (3.12), we get as

$$D_t=\frac{D_0\mu\lambda}{\beta+1}\left(t_1^{\beta+1}-\frac{\mu^{\beta+1}}{\beta+2}\right) \qquad \textbf{(3.13)}$$

The total number of inventory holding during the period $[0, t_1]$ is

$$I_1=\int_0^{t_1}I(t)dt$$

$$=\int_0^\mu I(t)dt+\int_\mu^{t_1}I(t)dt$$

$$=\int_0^\mu\left[S\,e^{-\left(1-e^{-\lambda t^\beta}\right)}-D_0\,e^{-\left(1-e^{-\lambda t^\beta}\right)}\int_0^t t\,e^{\left(1-e^{-\lambda t^\beta}\right)}dt\right]dt$$

$$+\int_\mu^{t_1}\left[D_0\mu e^{-\left(1-e^{-\lambda t^\beta}\right)}\int_t^{t_1}e^{\left(1-e^{-\lambda t^\beta}\right)}dt\right]dt$$

[From (3.7) & (3.8)]

Evaluating the above integrals and neglecting the second and higher order of α, we get as:

$$I_1=D_0\mu\left[-\frac{\mu^2}{6}+\frac{t_1^2}{2}-\frac{\lambda\beta\mu^{\beta+2}}{(\beta+1)(\beta+2)(\beta+3)}+\frac{\lambda\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right] \qquad \textbf{(3.14)}$$

The total shortage quantity during the interval $[t_1, T]$ is

$$I_2=-\int_{t_1}^T I(t)dt$$

$$=\int_{t_1}^T D_0\mu(t-t_1)dt \quad [\text{from}(9)] \qquad \textbf{(3.15)}$$

$$=\frac{1}{2}D_0\mu(T-t_1)^2$$

Then the average total cost per unit time is given by

$$TC(t_1) = \frac{C_d D_t}{T} + \frac{C_h I_1}{T} + \frac{C_s I_2}{T}$$

Now substituting the expressions for D_t, I_1, I_2 given by the equations (3.13), (3.14) and (3.15) respectively and then eliminating S by the equation (3.12), we get as:

$$TC(t_1) = \frac{D_0 \mu C_1}{T}\left[\frac{t_1^2}{2} - \frac{\mu^2}{6} - \frac{\lambda\beta\mu^{\beta+2}}{(\beta+1)(\beta+2)(\beta+3)} + \frac{\lambda\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right] + \frac{C_2 D_0 \mu}{2T}(T-t_1)^2 + \frac{C_3 D_0 \mu\lambda}{T(\beta+1)}\left(t_1^{\beta+1} - \frac{\mu^{\beta+1}}{\beta+2}\right)$$

(3.16)

To minimize the average total cost per unit time, the optimal value of t_1, say t_1^* can be obtained by solving the following equation

$$\frac{dTC(t_1)}{dt_1} = 0$$

Which also satisfies the condition

$$\left.\frac{d^2C(t_1)}{dt_1^{\,2}}\right|_{t=t_1^*} > 0$$

After solving, the condition $\frac{dTC(t_1)}{dt_1} = 0$ gives the equation:

$$C_3\lambda t_1^{\beta} + C_1\left(t_1 + \frac{\lambda\beta t_1^{\beta+1}}{\beta+1}\right) + C_2(t_1 - T) = 0, [\text{say } \phi(t_1)]$$

(3.17)

Since $\phi(0) < 0$ and $\phi(T) > 0$, then $\phi(0).\phi(T) < 0$. So there exits one solution $t_1 = t_1^* \in (0, T)$ of equation (3.17), which can be easily solved by Newton-Raphson method.

Also,

Substituting $t_1 = t_1^*$ in equation (3.12), we find the optimum value of S, given as:

$$S^* = D_0 \mu t_1^*\left(1 + \frac{\alpha t_1^{*\,\beta}}{\beta+1}\right) - \frac{D_0\mu^2}{2}\left[1 + \frac{2\alpha\mu^{\beta}}{(\beta+1)(\beta+2)}\right]$$

(3.18)

Again the total amount of backorder at the end of the cycle is $D_0\mu(T - t_1)$.

Therefore

$$Q = S + D_0\mu(T - t_1).$$

So the optimal value of Q is given by

$$Q^* = S^* + D_0\mu(T - t_1^*)$$

(3.19)

$$= D_0\mu\left[\frac{\lambda t_1^*}{\beta+1} - \frac{\mu}{2} - \frac{\lambda\mu^{\beta+1}}{(\beta+1)(\beta+2)} + T\right]$$

And the minimum value of the average total cost $TC(t_1)$ is $TC(t_1^*)$ as from equation (3.16).

4. Numerical example

Let the values of parameters of the inventory model be

C_1 = Rs 3 per unit per year, C_2 = Rs 15 per unit per year, C_3 = Rs 5 per unit, α = 0.001, β = 2,

D = 50 units, n = 0.5 and T = 1 year.

Under the above parameter values and according to equation (3.13), we obtain the optimum value of t_1 as $t_1^* = 0.83308$ year.

This value of t_1 also satisfies the sufficient condition for optimality.

Taking $t_1^* = 0.83308$ year, we get the following optimal values for total purchase quantity and the initial inventory

$Q^* = 50.01204$ units and $S^* = 34.71315$ units.

The minimum average total cost per unit time is found to be $C^* = Rs\,76.46476$ per year.

5. Conclusion

In this paper, we find that the optimal reorder time (t_1^*) of the proposed EOQ model is unique and is independent of μ and D_0 [by eq. (3.17)]. But the minimum average total cost per unit time $TC(t_1^*)$, the optimal initial inventory (S^*) and the optimal total purchase quantity (Q^*) are dependent on the value of μ and D_0.

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