Bearing Capacity Determination by Multiple Regression

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Abstract*— *Terzaghi's equations have been widely used for the determination of bearing capacity of soil. The research focused on the specific contributions of cohesion and angle of internal friction to bearing capacity of soil, making use of Terzaghi's equations. Samples of subsoil were obtained from three different locations. In their natural states, the samples were subjected to basic index properties tests; their cohesion and angle of internal friction were also measured and natural bearing capacity determined. Thereafter, fines were separated from coarse content of the soil. The fines and the coarse fractions were then reconstituted in varying proportions. Also, the cohesion and angle of internal friction of the reconstituted samples were measured and bearing capacity determined. Using multiple linear regression statistical analysis tool, predictive models for generating the bearing capacity were developed and validated. The study revealed a low level of variance between experimental values and model values of bearing capacity.

Keywords*— *angle of internal friction; bearing capacity; cohesion; multiple linear regression; subsoil.

I. Introduction

According to Reference [1], factors that affect the bearing capacity of foundations include the soil properties, footing geometry and the interaction between them. In the past, research into the undrained bearing capacity of footings has been limited as the interaction of these factors makes the solution of this problem much more complex. However, the recent advancements in numerical methods such as finite element method and the rapid increase in computing power mean that more rigorous solutions to both two dimensional and three dimensional bearing capacity problems can be found. Specifically, it has been established that factors such as soil strength, foundation width, foundation depth, soil weight and surcharge, particle angularity, relative density, porosity, particle-size distribution and water content, all affect the bearing capacity of soil [2, 3, 4]. The determination of the bearing capacity of foundations has been developed through both experimental investigations and numerical/theoretical analyses [1, 5, 6, 7].

Over the years, a number of equations have been proposed by researchers for the determination of bearing capacity of soil [8, 9]. Reference [9] proposed bearing capacity equations for different footing geometry are as shown below.

$$Q_u = c\,N_c + \gamma D N_q + 0.5\gamma\,B N_\gamma \quad (1)$$

$$Q_u = 1.3cN_c + \gamma D N_q + 0.4\gamma B N_\gamma \quad (2)$$

$$Q_u = 1.3cN_c + \gamma D N_q + 0.3\gamma B N_\gamma \quad (3)$$

Equations (1) to (3) are Terzaghi' bearing capacity (in kN/m^2) equations for shallow strip footing, shallow square footing and shallow circular footing respectively; where: c = cohesion of soil (kN/m^2), γ = effective unit weight of soil (kN/m^3); D = depth of footing (m), B = width of footing (m). Values of bearing capacity factors N_c, N_q, and N_γ for different angles of internal friction,ϕ, as proposed by Terzaghi are shown in Table I.

TABLE I: VALUES OF TERZAGHI'S BEARING CAPACITY FACTORS

(ϕ)degrees	N_c	N_γ	N_q
0	5.14	0	1
5	6.5	0.1	1.6
10	8.4	0.5	2.5
15	11	1.4	4
20	14.8	3.5	6.4
25	20.7	8.1	10.7
30	30	18.1	18.4
35	46	41.1	33.3
40	75.3	100	64.2
45	134	254	135

Source: Adapted from Reference [10]

Based on Terzaghi's bearing capacity equation, there are three components contributing to the bearing capacity:

(a) Surcharge pressure**:** Foundations are normally not placed directly on the ground level. Instead, they are installed at a depth below the existing ground level. The soil pressure arising from the depth of soils serves as a surcharge imposing a uniform pressure at foundation level.

(b) Self-weight of soils: The self-weight of soils contribute to the bearing capacity and is represented by $0.5\gamma BN_\gamma$ (γ = density of soils).

(c)Shear strength**:** The shear strength of soils contributes to the bearing capacity and is represented by cN_c. Shear strength of a soil refers to the maximum or limiting value of shear stress induced within its matrix before yielding. Determination of shear strength parameters must take place prior to analytical and design procedures in connection with foundations, retaining walls and earth retaining structures. Shear strength within a soil matrix is due to cohesive and frictional forces between adjacent particles. Therefore, the soil shear strength is to some extent surface dependent. Any action that will hinder or promote the interlocking or welding of soil particles will invariably affect soil shear strength [11].

The parameters N are all functions of the internal friction angle ϕ. Terzaghi's theory is an extension of the analytical work of Reference [12], who provided the first two terms. The solution was later shown to be exact, as it satisfies both the upper and lower bound theorems of plasticity theory. In the vast literature on the bearing capacity of shallow foundations, numerous analytical expressions for the factors N have been proposed. Indeed, Reference [13] tabulated 15 different solutions since 1940. Of these, the solutions due to References [8], [14] and [13] are the most widely used in practice.

A great deal of laboratory testing has been performed to predict the ultimate bearing capacity of foundations. However, the investigations are typically limited in scope. Results obtained from laboratory testing are typically problem specific and are difficult to extend to field problems with different material or geometric parameters. There have been several numerical methods for bearing capacity problems so far. Each problem was solved with certain assumptions and results were compared to laboratory testing. Very few rigorous numerical studies have been undertaken to determine bearing capacity behaviour [1].

It is necessary to determine the bearing capacity of soil through the process of geotechnical investigations prior to the design of foundation. However, soil investigation is often neglected or rejected by most people on the basis of cost despite the fact that the cost of carrying out geotechnical investigations for a project is negligible compared to the total cost of the project. Reference [15[opined that the cost of carrying out geotechnical investigations for a project is less than 1.0% of the project cost. Thus in most cases, the engineer uses his experience of a particular area or deductive reasoning to predict the bearing capacity and choose a foundation type. Furthermore, existing methods for the determination of soil bearing capacity is based on its geotechnical properties. Reference [16] modeled the relationship between fines content and bearing capacity of soil, using square footing. Specific contribution of the cohesion and angle of internal friction to the bearing capacity of soil in a specified area is uncertain; hence this study. The aim of this research was to develop models for determining the bearing capacity of soil samples in order to provide a guide that will lead to a faster means of determining bearing capacity, thus reducing the incidence of collapse of structures. The specific objectives of this research were to: (i) determine the specific effects of cohesion and angle of internal friction on the bearing capacity of selected soil samples; (ii) develop multiple regression models relating the cohesion and angle of internal friction to the bearing capacity of the soil samples; and (iii) evaluate the developed regression model.

II. MATERIALS AND METHODS

A. *Collection and Preparation of Samples*

Three different locations were identified on Obafemi Awolowo University, Ile-Ife, southwestern Nigeria, and one lateritic soil sample taken from each location. The sampling depth is between 0.6m and 1.2m. The soil samples were packed in polythene bags from the sampling locations, properly sealed and labelled for easy identification and then transported to the geotechnical engineering laboratory, department of Civil Engineering, Obafemi Awolowo University, Ile-Ife, Nigeria [17].

B. *Preliminary Tests and Determination of Index Properties*

Shear strength tests (to measure the shear strength parameters and determine the bearing capacity) were conducted on the soil samples in their natural states. The natural moisture content of each sample was also determined. The bulk samples were then air-dried before subjecting them to the basic geotechnical index property tests. All the tests were conducted in accordance with the methods by Reference [18].

C. *Separation of Fines and Coarse Components of Soil Samples*

USCS and AASHTO define fines content of soil as soil particles passing through sieve No. 200 (75µm opening). Fines play critical roles in engineering properties of cohesive soils [19, 20, 21]. The soil samples were soaked in water containing 4% sodium hexametaphosphate, a dispersing agent (commercially named Calgon) in the laboratory, for 12-24 hours so that all the fines would get soaked and detached from the coarser soil samples. The soil was then washed through sieve size No. 200 with 75µm opening. The soil passing 75µm sieve size was oven dried and referred to as 100% fines. The soil sample retained on sieve 75µm opening was also oven dried (after thorough mixing) and referred to as 100% coarse [16, 17].

D. *Remoulding or Reconstitution of the Soil Samples*

The fines and coarse fractions were thoroughly puverised and added together in varying ratios of fines to coarse from 10:100 to 100:0 in 10% increment. The ratio started with 10:100 and not 0:100 because, laboratory compaction test could not be carried out on the sample containing 0% fines (i.e. 100% coarse) and thus cohesionless. This is because the process of lubrication which aids compaction is limited to soils containing fines and cohesionless soils are compacted or densified by vibration and not by impact which laboratory compaction utilizes [22].

E. Compaction and Determination of Shear Strength Parameters of the Soil Samples

The purpose of shear strength testing is to establish empirical values for the shear strength parameters. For the research, triaxial apparatus was used (Fig. 1). Many variations of test specimen are possible with triaxial apparatus but the three principal types of test are unconsolidated-undrained, consolidate-undrained and drained shear strength tests. Furthermore, close monitoring and a considerable length of time are required for the drained test which can be economically justified only for large projects [11]. Consequently, the shear strength parameters for the reconstituted soil samples were determined under unconsolidated-undrained condition. The apparatus was set up with all the needed kits. The reconstituted soil samples were allowed to homogenise and then compacted in the laboratory using standard proctor test to determine the optimum moisture content (OMC) and the maximum dry density (MDD) of each sample. The values of the OMC were used in subsequent unconsolidated-undrained triaxial tests, which were carried out in accordance with Reference [18]. The Mohr-Coulomb circles were plotted and shear strength parameters were determined therefrom.

Fig. 1*: Triaxial test on soil samples*

F. Determination of Bearing Capacity

After determining the shear strength parameters, Terzaghi's bearing capacity equations (1) to (3) were used to determine the bearing capacity of the soil samples at different footing geometry, assuming typical footings of unit depth and unit width.

G. Development of Predictive Models

In order to investigate the specific (qualitative and quantitative) relationship between the bearing capacity and shear strength parameters, predictive models were developed. Multiple linear regression statistical package was used to model the relationship between the soil bearing capacity (dependent variables) and cohesion and angle of internal friction (independent variables). The general regression model is usually expressed by an equation

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + .+ \beta_n x_n \quad (4)$$

where:

y = dependent variable

x_1, x_2, x_3, ………x_n = independent variables of order n

β_1, β_2, β_3, ………, β_n = regression coefficients

β_0 = value of y when independent variables are zero, or the intercept on y-axis.

Values of bearing capacity deduced from experimental tests are compared with values derived from multiple regression analysis.

III. RESULTS AND DISCUSSION

A. General Description of Soil Samples from Selected Locations

Soil samples were obtained from selected locations within Obafemi Awolowo University (OAU) campus, Ile-Ife, Nigeria. Ile –Ife lies between latitude 7° and 28°N and longitude 4° and 34°E. Sample collected from New Market area is referred to as NM; sample collected from Opa Dam is referred to as OD, while sample collected from Tonkere Road is referred to as TR.

B. Results of Preliminary Analysis of the Soil Samples in their Natural States

Results of classification and index properties determination for the soil samples are as shown in Table II. The Table shows that sample OD has the highest fines content of 55.00%, natural moisture content (NMC) of 16.90%, liquid limit (LL) of 41.00% and plastic limit (PL) of 30.73%. Sample TR on the other hand, has the lowest fines content of 41.22%, LL of 39.87% and PL of 31.01%. Samples NM and TR contain less clay than sample OD. The particle size curves (Fig. 2) for the three samples have some measure of resemblance, which implies that the grading of these samples, which are from the same area, are not too different. This is also as reported by References [16] and [17]. The activities of the soil samples as presented in Table II reveal that none of the soil samples is active due to the fact that they all have activity values less than 1.25 [23]. It is expected that the void ratio should increase as the maximum dry density increases, so the porosity will decrease with the increase in the maximum dry density of the soils. However, sample OD with the highest MDD value

(1.91) has the lowest void ratio, but the difference between the values of void ratio for the three soil samples is almost negligible. This could be attributable to the fact that the soil could generally be described as having the same geotechnical properties, with only slight differences. Table III displays the compaction parameters of the soil samples in their natural states, and Fig. 3 shows compaction characteristics of the soil samples in their natural states. Table IV summarizes the shear strength parameters and the bearing capacity of the soil samples, with different footing geometry, in their natural states.

The shear strength parameters and bearing capacity of the soil samples in their natural states clearly shows that expectedly, each sample has its own peculiarity and thus should be treated differently. The Sample (NM) with the lowest cohesion and lowest angle of internal friction has the highest bearing capacity; while sample OD with the highest cohesion and highest angle of internal friction does not have the lowest bearing capacity. Thus generalization cannot be made at this stage.

TABLE II: INDEX PROPERTIES OF THE SOIL SAMPLES

Property	NM	OD	TR
Natural Moisture Content (%)	19.74	16.90	17.05
Specific Gravity (Gs)	2.66	2.86	2.69
Liquid Limit, LL (%)	45.29	41.00	39.87
Plastic Limit, PL (%)	32.68	30.73	31.01
Plasticity Index, PI (%)	12.61	10.27	8.86
Percentage passing sieve No. 200 (Fines content)	32.70	55.00	41.07
Percentage clay sized particles	14.51	27.48	24.74
Percentage silt sized particles	18.19	27.52	16.33
Activity	0.87	0.37	0.36
Porosity, n	0.333	0.329	0.363
Void ratio, e	0.50	0.49	0.57

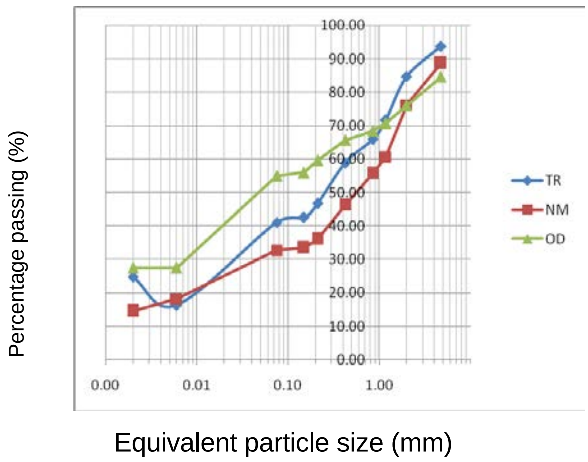

Fig. 2. *Particle size distribution curves for the soil samples [16]*

TABLE III: COMPACTION PARAMETERS OF THE SOIL SAMPLES IN THEIR NATURAL STATES

Compaction Parameters	NM	OD	TR
Optimum moisture content, OMC (%)	17.39	19.42	16.38
Maximum dry density, MDD (Mg/m^3)	1.77	1.91	1.71
Maximum bulk density, MBD (Mg/m^3)	2.08	2.29	1.99

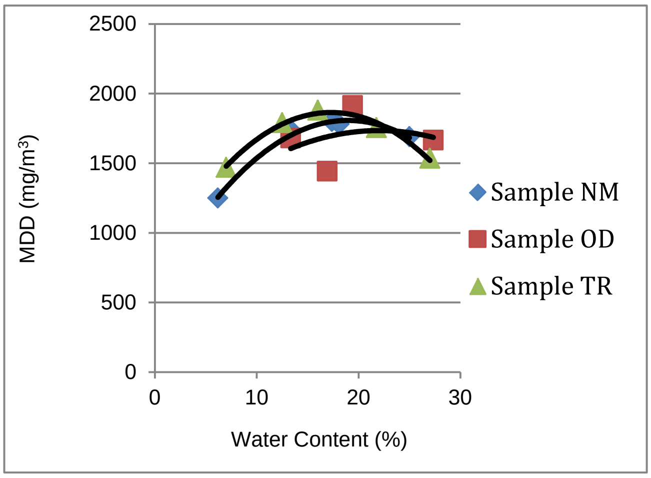

Fig.3. *Compaction characteristics of the soil samples in their natural states*

C. Results of Compaction and Determination of Bearing Capacity of Soil Samples

Table V is a display of the variation of optimum moisture content (OMC) and shear strength parameters with the varying fines content. The OMC of all the soil samples increases with increasing fines content, which agrees with the findings of Reference [24]. For all the soil samples, cohesion increases with increasing fines content, while angle of internal friction reduces with increasing fines content. Furthermore, the bearing capacity of all the soil samples increases with increasing cohesion, while the bearing capacity reduces with increase in angle of internal friction. In other word, there is a direct relationship between bearing content and cohesion of the soil samples, while there is an inverse relationship between the bearing capacity and angle of internal friction.

D. Evaluation of Developed Models

The models developed yield predictive equations (which represent the relationship between cohesion, angle of internal friction and bearing capacity of the soil samples) displayed in Table VI. The models were developed for each soil sample at different footing configurations. Based on R^2 values, the models generated could be said to give representations between the cohesion, angle of internal friction and bearing capacity of the selected soil samples. Table Vii also displays a comparison between the measured and model values of bearing capacity of the soil samples with different footing configurations. As observed, the level of variance is minimal for all the soil samples and all the footing configurations, with circular footing having the minimum variance and strip footing having the highest variance for all the selected soil samples. Generally speaking, sample TR has the lowest value of variance. The variance, though generally minimal, could be attributable to some other factors which affect the bearing capacity of soil.

TABLE IV: BEARING CAPACITY OF THE SOIL SAMPLES IN THEIR NATURAL STATES

Sample Identification	Cohesion, c (kN/m^2)	Angle of internal friction, ϕ (degrees)	Footing Configuration	Bearing Capacity (kN/m^2)
NM	21	13	Strip footing	2074.23
			Square footing	2279.45
			Circular footing	2194.87
OD	36	17	Strip footing	1077.67
			Square footing	1282.97
			Circular footing	1264.71
TR	35	15	Strip footing	1012.15
			Square footing	1213.71
			Circular footing	1197.91

IV. CONCLUSION

From the findings of this research work, the following conclusions are made in relation to the stated objectives of the research: (i) The bearing capacity of the studied soil samples generally increased with increase in cohesion and reduced with increase in angle of internal friction; (ii) multiple regression models have been generated between cohesion, angle of internal friction and bearing capacity; (iii) the developed regression models have been evaluated and found to be valid for the selected soil samples and described procedures and conditions. It is recommended that the research be carried out in some other geographical locations before generalisation could be made. It is also recommended that deep foundations and other bearing capacity equations, apart from Terzaghi's be considered in further studies.

TABLE V: OPTIMUM MOISTURE CONTENT AND SHEAR STRENGTH PARAMETERS OF SOIL SAMPLES

Sample Identification	% Fines	OMC (%)	Cohesion, c (kN/m^2)	Angle of internal friction, ϕ (degrees)	Bearing Capacity (kN/m^2)
NM	10	9.37	8	42	113.32
	20	11.91	10	39	207.85
	30	13.11	20	31	384.09
	40	16.99	26	28	693.51
	50	18.92	41	27	1310.90
	60	22.04	49	20	1857.63
	70	23.98	52	12	2129.16
	80	27.57	57	5	3138.96
	90	29.32	59	2	3107.96
	100	32.12	63	0	5075.37
OD	10	10.11	5	41	96.34
	20	10.98	9	40	167.09
	30	13.02	13	32	440.39
	40	15.13	19	30	781.73
	50	18.09	28	29	1242.62
	60	19.75	34	24	1361.58
	70	22.11	51	19	2230.83
	80	25.86	59	9	3045.21
	90	28.97	62	3	3612.85
	100	33.06	67	0	5279.11
TR	10	9.92	10	36	126.69
	20	11.01	14	29	151.09
	30	12.88	19	27	400.93
	40	14.98	31	22	764.21
	50	17.85	38	16	1031.15
	60	20.88	46	13	1737.41
	70	23.03	49	8	1907.13
	80	26.92	62	5	2921.99
	90	28.35	64	2	3200.03
	100	32.84	66	0	5852.30

TABLE VI: REGRESSION MODELS EQUATIONS FOR SOIL SAMPLES

Sample Identification	Footing Configuration	Model Equation	R^2
NM	Strip footing	b.c. = 3895.842 – 62.165c + 13.429ϕ	0.841
	Square footing	b.c. = 3206.788 – 48.794c + 28.250ϕ	0.865
	Circular footing	b.c. = 2751.846 – 40.914c + 31.278ϕ	0.875
OD	Strip footing	b.c. = 7272.618 – 115.517c - 48.096ϕ	0.832
	Square footing	b.c. = 6908.519 – 106.772c - 43.316ϕ	0.850
	Circular footing	b.c. = 6292.336 – 96.209c - 38.415ϕ	0.859
TR	Strip footing	b.c. = 3561.977 – 54.560c - 17.400ϕ	0.822
	Square footing	b.c. =3539.778 – 52.253c - 14.867ϕ	0.849
	Circular footing	b.c. = 3298.227 – 48.064c - 13.056ϕ	0.860

Note:
b.c. = bearing capacity in kN/m^2
c = cohesion in kN/m^2
ϕ = angle of internal friction, in degrees

TABLE VII: VALUES OF THE MODEL AND MEASURED RESULTS OF BEARING CAPACITY

Sample Identification	Footing Configuration	Bearing Capacity (kN/m^2)		% difference
		Measured values	Model values	
NM	Strip footing	2074.23	2764.954	6.91
	Square footing	2279.45	2549.364	2.70
	Circular footing	2194.87	2299.266	1.04
OD	Strip footing	1077.67	2296.374	12.19
	Square footing	1282.97	2328.355	10.46
	Circular footing	1264.71	2175.757	9.11
TR	Strip footing	1012.15	1391.377	3.79
	Square footing	1213.71	1487.918	2.74
	Circular footing	1197.91	1420.147	2.22

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