The impact of unsteady of waves in cylindrical bodies that are in a deformable the medium

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Annotation. The article elucidates issues of the development of techniques for solving the problem, the impact of non-stationary of waves at the N- layer cylindrical bodies (shell) are in an infinite linear elastic the medium as well as her algorithms. A closed system of differential equations and the corresponding initial and boundary conditions. The analytical results obtained are of theoretical and practical importance. The technique is universal in nature, it is valid for any of the rheological properties of the media.

Keywords: cylindrical body wave, shell, Fourier transform, the elastic medium.

Introduction. In case of sufficient extent and impact of the cavity, directed perpendicular of longitudinal axis of the surrounding environments and cavity lining are plane strain, and the problem of determining the stress state of the array and lining are reduced to the plane problem of the dynamic theory of elasticity [1,2,3,4]. In [5,6,7] solved the problem of tensions deformed state of cylindrical bodies (shell) that are in infinite linear elastic medium in the propagation of longitudinal and transverse waves. Unlike of other works, this study being developed technique and algorithm of solutions of the problem of unsteady interaction of waves in layered cylindrical bodies. It is valid for any of the rheological properties of the media.

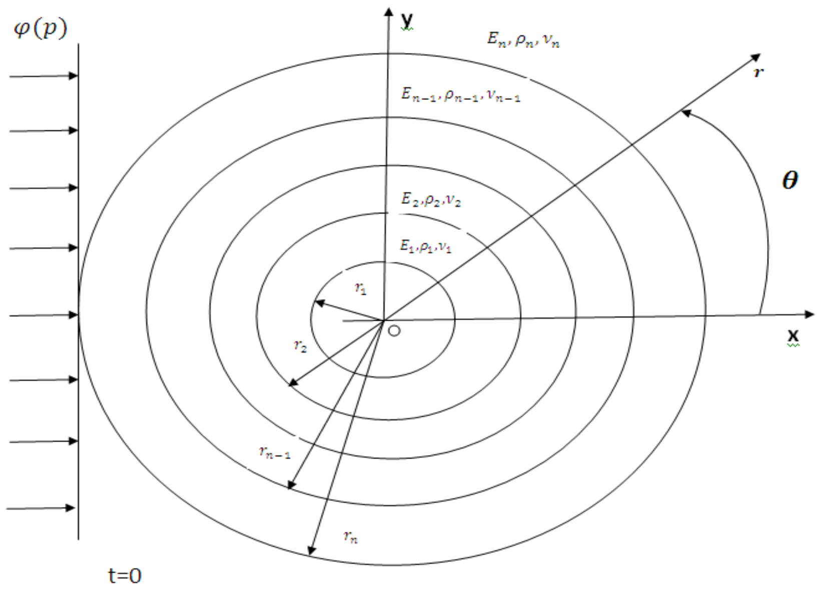


Fig.1. The analytical model of the layered cylindrical bodies in an elastic medium.

Statement of the problem. At the N- layer cylinder are falling transient of the stress wave $\sigma_{xx}^{(i)}$ and $\sigma_{xy}^{(i)}$, front of which is parallel to the longitudinal axis of the cylinder [1] (Figure 1). Required to determine the dynamic stress-strain state of the cylinder and the environment caused by the incident voltage pulse. Assume that time t is counted from when the incident pulse touches external surfaces (N-1) -th of the cylinder at point $r = r_N$, $\theta = 0$. Up to this point is retained at rest. In accordance with the above, the problem of finding the fields of the diffracted waves and the stress-strain state caused by the incident pulse [1]

$$\sigma_{xxn}^{(p)} = \sigma_0 H(t), \quad \sigma_{xyn}^{(p)} = \sigma_0 \frac{\nu_n}{1-\nu_n} H(t), \; t = t - (x + r_n)/C_{Pn}, \tag{1}$$

where σ_0 - the amplitude of the incident wave; - The unit Heaviside function. Please find a solution for a planar waves of private stage. Stress tensor in a general form

$$\sigma_{ijn} = \sigma_{ijn}^{(p)} + \sigma_{ijn}^{(s)}$$

where $\sigma_{ijn}^{(p)}$ - voltage when the incident wave, $\sigma_{ijn}^{(s)}$ - the voltage of the reflected wave. In the a polar coordinate system associated with the cylinder, and displacements voltage in the incident wave $r = r_n$ are of the form:

$$\sigma_{rrn}^{0} = \sigma_0[(\varepsilon_n + 1)\cos 2\theta] H_0(z)/2$$

$$\sigma_{r\theta n}^{0} = \sigma_0(\varepsilon_n - 1)\sin 2\theta H_0(z)/2;$$

$$\sigma_{\theta\theta n}^{0} = \sigma_0\left[\varepsilon_n - (\varepsilon_n + 1)\cos 2\theta\right] H_0(z)/2; \; z = C_{\rho n} t - r_n + r_n \cos\theta, \quad \varepsilon_n = -\nu_n/(1-\nu_n)$$

where $H_0(z)$ -is the unit Heaviside function; σ_0 - voltages on the wave front, is propagating x_1; r_j - radius the layered of bodies (j = 1, ...n); Cpj- of expansion velocity of the wave, ν_j - Poisson's ratios, ρ_j - densities environments. In the absence of static of mass forces, the displacement vector $\vec{u}_j = \left[u_{rj}, u_{\theta j}, u_{zj}\right]^T$ in an elastic medium is defined by equation

$$(\lambda_j + 2\mu_j) grad\, dir\vec{u}_j - \mu_j \; rot\, rot\vec{u}_j = \rho_j \left(\partial^2 \vec{u}_j / \partial t^2\right) \tag{2}$$

Displacement vector (u_j) is expressed by scalar (φ_j) and vectorial ($\vec{\psi}_j$) potentials of[2]

$$\vec{u}_j = grad\varphi_j + rot\vec{\psi}_j$$

and the equation (2) takes the form

$$\nabla^2\varphi_j - c_{j1}^{-2}\left(\partial^2\varphi_j / \partial t^2\right) = 0,$$

$$\nabla^2\psi_j - c_{j2}^{-2}\left(\partial^2\vec{\psi}_j / \partial t^2\right) = 0. \tag{3}$$

Where $\nabla^2 = \dfrac{\partial^2}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial}{\partial r} + \dfrac{1}{r^2} + \dfrac{\partial^2}{\partial\theta^2} -$ differential operators in cylindrical coordinates.

At infinity $r \to \infty$ the potentials of longitudinal and transverse waves at $j = n$ satisfy the Summerfield radiation condition [1]:

$$\lim_{r\to\infty} \varphi_n = 0,$$

$$\lim_{r\to\infty} (\sqrt{r})^{\kappa} \left(\frac{\partial \varphi_n}{\partial r} + i\alpha_n \; \varphi_n \right) = 0, \tag{4}$$

$$\lim_{r\to\infty}\psi_n = 0,$$

$$\lim_{r\to\infty}(\sqrt{r})^{\kappa}\left(\frac{\partial\psi_n}{\partial r} + i\beta_n\,\psi_n\right) = 0\,.$$

On pin of two bodies $r = r_j$, the equality displacement and stress (the condition hard contact)

$$u_{rj} = u_{r(j+1)};\quad \sigma_{rrj} = \sigma_{rr(j+1)}$$

$$u_{\theta j} = u_{\theta(j+1)};\ \sigma_{r\theta j} = \sigma_{r\theta(j+1)}\quad ,$$

(5)

$$\sigma_{rr1} = 0,\ \sigma_{r\theta 1} = 0. \quad (6)$$

Task is solved under the following initial conditions:

and on the free surface ($r = r_1$):

$$\left.\frac{\partial\varphi_j}{\partial r} + \frac{1}{r}\frac{\partial\psi_j}{\partial\theta}\right|_{t=0} = \left.\frac{\partial}{\partial t}\left(\frac{\partial\varphi_j}{\partial r} + \frac{1}{r}\frac{\partial\psi_j}{\partial\theta}\right)\right|_{t=0} = 0, \quad (7)$$

$$\frac{1}{r}\frac{\partial\psi_j}{\partial\theta} - \frac{\partial\varphi_j}{\partial r} = \left.\frac{\partial}{\partial t}\left(\frac{1}{r}\frac{\partial\psi_j}{\partial\theta} - \frac{\partial\varphi_j}{\partial r}\right)\right|_{t=0} = 0,$$

Fourier integral. The stress field induced forces (1) satisfies the wave equation (3), i.e. it satisfies each cylindrical layer. To address formulated above problem of the variable is time t- integral Fourier transform with respect to time

$$\varphi_j{}^F\ (\Omega) = \int_{-\infty}^{\infty}\varphi_j(\tau)\,\exp(-i\Omega\tau)\,d\tau;\ \varphi_j(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\varphi_j{}^F\ (\Omega)\,\exp(i\Omega\tau)\,d\omega; \quad (8)$$

$$\psi_j{}^F\ (\Omega) = \int_{-\infty}^{\infty}\psi_j(\tau)\,\exp(-i\Omega\tau)\,d\tau;\ \psi_j(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\psi_j{}^F\ (\Omega)\,\exp(i\Omega\tau)\,d\omega,$$

where Ω - parameter of the Fourier integral; φ^F_j ψ^F_j The image of the Fourier transform of functions $\varphi_j(t)$ and $\psi_j(t)$, respectively. Using the zero initial conditions (7) to get an image problem (3). Then the solution of the problem would have shown the form

$$\begin{pmatrix}\varphi_j^F(r,\theta,\Omega)\\ \psi_j^F(r,\theta,\Omega)\end{pmatrix} = \begin{pmatrix}\overline{\varphi}_j^F(r,\Omega)\\ \overline{\psi}_j^F(r,\Omega)\end{pmatrix}\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix};$$

(9).

Here

$$\overline{\varphi}_j^F(r,\Omega)=\begin{cases} A_m H_m^{(1)}(\Omega r/C_{Pn}) \quad npu \quad r\ge r_n, \\ A_{mj}H_m^{(1)}(\Omega r/C_{Pj})+B_{mj}H_m^{(2)}(\Omega r/C_{Pj}) \ npu \\ r_1\le 2\le r_n \quad (j=1,2,...n+1), \\ A_{m1}I_n(\Omega r/C_{Sn}) \quad npu \quad 0\le r\le r_1; \end{cases}$$
$$\psi_j^F(r,\Omega)=\begin{cases} C_{mj}\ H_m^{(1)}(\Omega r/C_{Sj})+L_{mj}H_m^{(2)}(\Omega r/C_{Sj}) \\ npu \quad r_1\le r\le r_n, \\ C_m H_m^{(1)}(\Omega r/C_{Sn}) \quad npu \quad r\ge r_n, \\ C_{m1}I_n(\Omega r/C_{S1}) \quad npu \quad 0\prec r\le r_1. \end{cases} \tag{10}$$

Coefficients is A_{m1}, A_{mj}, A_{mN}, B_{mj}, C_{mj}, C_{mn} – determined from the of boundary conditions (4) - (6). The boundary conditions are taking into account incident wave (1) take the form:

$$\begin{aligned} &a)\quad \sigma_{rrn}^F+\sigma_{rrn}^{(i)F}=\sigma_{rr(n-1)}^F, \quad б)\quad \sigma_{r\theta n}^F+\sigma_{r\theta n}^F=\sigma_{r\theta(n-1)}^F, \\ &в)\quad u_{rn}^F+u_{rn}^{(i)F}=u_{r(n-1)}^F, \qquad г)\quad u_{\theta n}^F+u_{\theta n}^{(i)F}=u_{\theta(n-1)}^F, \end{aligned} \tag{11}$$

here

$$\begin{aligned} &a)\quad \sigma_{rrn}^{(i)}(\Omega)=\sigma_{01}^{(P)}\sum_{k=0}^{\infty}(-1)^k \in_k I_k(\Omega r/C_{Pn})\cos k\theta; \\ &б)\quad \sigma_{rrn}^{(F)}(\Omega)=\overline{\sigma}_{rrn}^F(\cos^2\theta+\in_n\sin^2\theta); \\ &в)\quad \sigma_{r\theta n}^{(F)}=-\overline{\sigma}_{rrn}^F\left[(1-E_n)|2\right]\sin 2\theta; \\ &г)\quad u_{rm}^F=\overline{u}_{rm}^F\cos\theta;\ д)\quad u_{\theta m}^F=\overline{u}_{\theta m}^F\sin\theta; \quad \sigma_{01}^{(P)}=\sigma_0 e^{-n\Omega/C_{Pn}} \end{aligned} \tag{12}$$

Having substituted (9) and (10) into the boundary conditions (4), (5) and (6), we obtain the system of complex algebraic of equations with (4n + 3) in the form of of an unknown

$$[Z]\{g\}=\{P\}, \tag{13}$$

$$[Z]=\left(\begin{array}{cc|cc} [Z_1] & & & 0 \\ & [Z_2] & & \\ \hline & & [Z_{(N-1)}] & \\ 0 & & & [Z_N] \end{array}\right)$$

where Z- the block matrix; $\lfloor Z_j \rfloor$ matrix of dimension 4x4, elements of which are functions of Bessel and Hankel m- th order of first and second kind; {g}- column vector of the unknown coefficients; *{P}*- *{0,0.....0,P₁ₙ, P₂ₙ , Pₐₙ , P₄ₙ }ᵀ*- column vector, characterizing the load falling. Let the waves interact with with the stepped cylindrical opening at $r = r_1$ and the opening of voltagefree from ($\sigma_{rr1}=0$, $\sigma_{r\theta 1}=0$). The only voltage which does not vanish at $r = r_1$, is the hoop stress $\sigma_{\theta\theta n}/\sigma_0$. Applying the Fourier transform to the equation of motion and granichnm conditions [9], we obtain an expression for the hoop stress at

$$\sigma_{rrn}=\sigma_0 H(t)\cos nt, \quad \sigma_{r\theta n}=\tau_0 H(t)\sin\theta$$

$$\sigma^*_{\theta\theta n} = \frac{\sigma_{\theta\theta n}(r_{01}\theta,t)}{\sigma} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\Delta_n(r_0\Omega)e^{i\Omega t}}{\Omega_1[\Delta_1\Delta_2+\Delta_3\Delta_4]}d\Omega,$$
$$\Delta_n(r_{01}\Omega) = (\Delta_3+\tau_0 E_n)\lfloor 2\Omega H^{(1)}_{n-1}(\Omega) - ((2n^2+2n)+\Omega^2)H^{(1)}_n(\Omega)\rfloor + \quad (14)$$
$$+[\tau_0\Delta_2-\Delta_4]\left[2n(n+1)H^{(1)}_n((C_{Pn}/C_{Sn})\Omega) + \frac{2C_P n\Omega}{C_{S1}}H^{(1)}_{n-1}\left(\frac{C_{pn}}{C_{Sn}}\Omega\right)\right].$$

Expression $(\Delta_k(k=1,2,3,4,5)$ is given in [10]. Improper integral (14) solved numerically using the developed algorithms [10]. Practical calculations (4) on the computer can be made as follows. Because the numerical integration in between infinite limits is unthinkable, then the integral (14) is replaced by

$$\sigma^*_{\theta\theta n} = \frac{1}{2\pi}\int_{\omega_a}^{\omega_b}\frac{\Delta_1(r_{01}\Omega_1)}{\Omega_1[\Delta_2\Delta_3+\Delta_4\Delta_5]}e^{-i\Omega t}d\Omega.$$

(15)

The limits of integration ω_a, ω_b selected depending on the kind of the incident pulse. The numerical values of the spectral density $\sigma^{(i)F}_{rr}(\Omega)$ (12) end of the incident pulse, only a small frequency range Ω differs substantially from zero. The limits of integration ω_a, ω_b should be selected in accordance with this range, taking into account the required accuracy. Thus it remains an open question as to what will make the error of neglecting the contribution of the integrals of the type (14) within the limits of integration - ∞ before ω_a and from ω_b before ∞. The numerical summation of of an infinite sum (12) is of course also possible. However, in [10] it is shown that for sufficiently large n (n-order Bessel and Hankel functions) can be constructed the asymptotic representation of the general term of this sum. The result becomes possible or error estimate of the transition from finite to of an infinite sum or approximate summation of of an infinite sum. In view the above it will keep in (12) is an infinite sum. Calculation of the subject method reduces to the construction of two algorithms for computing: the coefficients $Z_{ke}(\Omega)(k,e=1,2)$ (13) and the integral (15). The first and second algorithm does not depend on the form of mathematical model of the object.

Algorithm for computing. The solution of equations (13) is performed using the method Gauss. Set all numerical parameters necessary for the calculations. Magnitude $\sigma^F_{\theta\theta n}/\sigma_0$ of (15) is calculated on a computer following way

$$\chi_1(r_0\Omega_1) = (\Delta_1(r_0,\Omega_1)/\Omega_1(\Delta_2\Delta_3+\Delta_4\Delta_5))e^{i\Omega t} \quad (16)$$

possible to numerically integrate by writing it in the form

$$\chi(r_0\Omega_1) = x_1(r_0,\Omega_1) - ix_1(r_0,\Omega_1).$$

Incident pulse $\sigma^{(i)F}_{xx}(\Omega)$ [15] described by the expression

$$\sigma^{(i)F}_{xx}(\Omega) = f_1(\Omega) - if_2(\Omega), \quad (17)$$

where $f_1(\Omega)$, $f_2(\Omega)$- real functions. Use the formula for of Euler $lxp(i\Omega t)$, dividing (16) on the real and imaginary (17) parts, after some transformations

$$\sigma^F_{\theta\theta} = \frac{1}{2\pi}\int_{-\infty}^{\infty}[x_1(\Omega) - ix_2]d\Omega \quad (18)$$

Dividing the integral (18) into two terms

$$\sigma^F_{\theta\theta n} = \frac{1}{2\pi}\int_{-\infty}^{0}[x_1(\Omega) - ix_2(\Omega)]d\Omega + \frac{1}{2\pi}\int_{0}^{\infty}[x_1(\Omega) - ix_2(\Omega)]d\Omega$$

and Replacing with in the first integral variable Ω on - Ω , we will have

$$\sigma^F_{\theta\theta n} = \frac{1}{2\pi}\int_0^\infty [x_1(\Omega_1) - x_1(-\Omega_1)] - i[x_2(\Omega_1) - x_2(-\Omega_1)]d\Omega. \qquad (19)$$

Since (19) is an inverse Fourier transform, and the left part contains the real magnitude [15], the following relation holds: $x_1(\Omega_1) = -x_1(-\Omega_1);\quad x_2(\Omega_1) = -x_2(-\Omega_1).$ Given his, from (16) finally obtain

$$\sigma^F_{\theta\theta n} = \frac{1}{\pi}\int_0^\infty x_1(\Omega)d\Omega,\quad \sigma^F_{\theta\theta n} = \frac{1}{\pi}\int_{\omega_a}^{\omega_b} x_1(\Omega)d\Omega. \qquad (20)$$

The magnitude of the integral (20) is found numerically by using Romberg method [15]. In the calculation of the integral method Romberg falls many times calculate integrand. The inverse Fourier transform for a certain images, the original of which is known in advance, showed that when the length of of the integration step of 0.01 does not exceed the the error of of procedure 0,3-0,5%.

Discussion of the numerical results. The improper integral (20) is calculated by the method of Romberg. To do this improper integral is replaced by the integral [15**]**

$$1(\tau_1) = \int_{W_a}^{W_b} f(x,\tau)dx \qquad (21)$$

finite limits which are picked up according to the nature of the spectral function.

The integral (21) is calculated by the method of Romberg, and the null (n = 0) series of approximate values $T_0 / K = 0,1,2,3$. K - integral trapezoid in fission the interval $[W_a \; W_b]$ into two equal parts

$$T_0^{(k)} = \frac{W_a - W_b}{2^{k+1}}[f(\xi_0,\tau) + 2f(\xi_1,\tau) + ... + f(\xi_k,\tau)],$$

$$\xi_i = x_a + i\frac{W_a - W_b}{2^{km}},\quad i = 0,1,2...2^k$$

Using $T_0^{(\kappa)}$ are computed n=1,2,...K. Series of approximate values of the integral T(τ_1):

$$T_n^{(k)} = (4^n T_{n-1}^{k=1} - T_{n-1}^k)/(4^n - 1) \qquad k = 0,1...k -$$

Presented of schemes constants according to computing $f(\tau_1) = T_k^0$ step $I_{\tau_1} = 0,02.$- For each fixed value calculations $f(\tau_1)$ carried out on the interval $\sigma_1 \le x \le \sigma_2$ by the formula (21) with к=11.

In some large values $f(\tau_1)$ repeated the calculations with high accuracy corrections were given less 3%.

Of unsteady of waves Diffractions on a cylindrical body. Let the the inner boundary $(r = r_1)$ free from tension, and at the contact with the environment, the condition of equality of a stirred and voltage (5) [1,2,4,10]. After Fourier transformation obtain cylindrical of the Bessel equation which decision is expressed (9) and (10). In our problem will be six of the arbitrary constants, which are determined from the boundary conditions (6) and (11). Here are some of them:

$$\left.\begin{aligned}
\sigma_{rr2} &= 2\mu_2 r^{-2}\sum_{k=1}^{2}\sum_{n=0}^{\infty}\int_{-\infty}^{+\infty}\left[C_{nk}\varepsilon_{1n}^{(k)} + D_{nk}\varepsilon_{2n}^{(k)}\right]e^{i\Omega r}d\Omega,\\
\sigma_{\theta\theta_2} &= 2\mu_2 r^{-2}\sum_{k=1}^{2}\sum_{n=0}^{\infty}\int_{-\infty}^{+\infty}\left[C_{nk}\varepsilon_{3n}^{(k)} + D_{nk}\varepsilon_{4n}^{(k)}\right]e^{i\Omega r}d\Omega,\\
\sigma_{r\theta_2} &= 2\mu_2 r^{-2}\sum_{k=1}^{2}\sum_{n=0}^{\infty}\int_{-\infty}^{+\infty}\left[C_{nk}\varepsilon_{5n}^{(k)} + D_{nk}\varepsilon_{6n}^{(k)}\right]e^{i\Omega r}d\Omega,\\
\sigma_{rr1} &= 2\mu_2 r^{-2}\sum_{k=1}^{2}\sum_{n=0}^{\infty}\int_{-\infty}^{+\infty}\left[A_n\delta_n^{(1)} + B_n\delta_n^{(2)}\right]e^{i\Omega r}d\Omega,
\end{aligned}\right\} \quad (22)$$

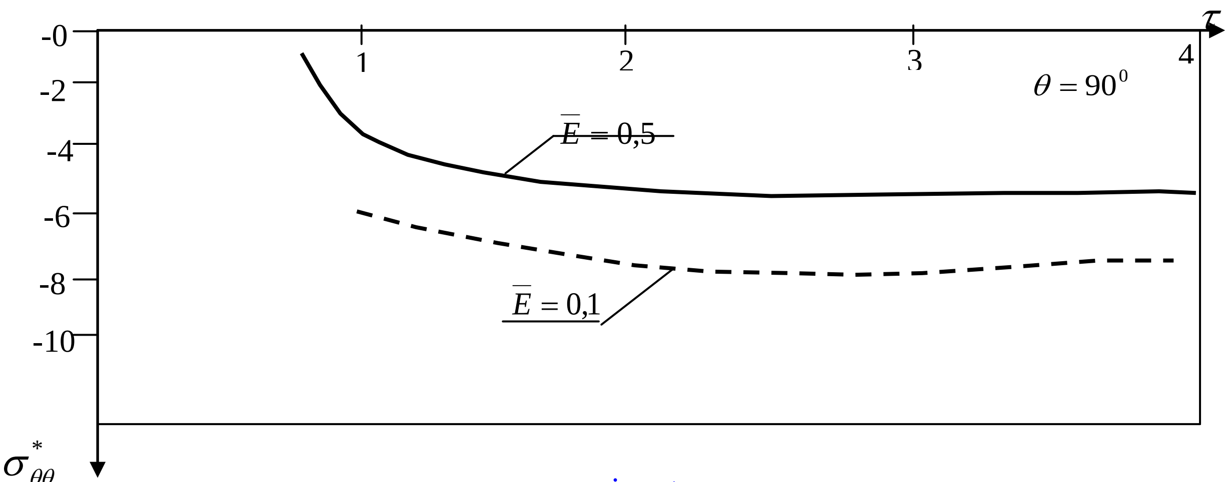
where C_{nk}, D_{nk}, A_n, B_n - arbitrary constants:
$C_{nk} = \delta_{kn}^{(c)}/\Delta_n$, $D_{kn} = \delta_{kn}^{(D)}/\Delta_n$, $A_n = \delta_n^{(A)}/\Delta_n$, $B_n = \delta_n^{(B)}/\Delta_n$; $\delta_{kn}^{(k)}$ and Δ_n square complex matrices (6x6). The remaining elements of the stress tensor are recorded as in (22).

$$\begin{aligned}
&C_{nk} = \operatorname{Re} C_{nk} + i\operatorname{Im} C_{nk}, \quad D_{nk} = \operatorname{Re} D_{nk} + i\operatorname{Im} D_{nk},\\
&A_n = \operatorname{Re} A_n + i\operatorname{Im} A_n, \quad B_n = \operatorname{Re} B_n + i\operatorname{Im} B_n,\\
&\delta_n^{(e)}\operatorname{Re}\delta_n^{(e)} + i\operatorname{Im}\delta_n^{(e)}, \quad e = 1,2, \ \varepsilon_{mn}^{(k)} = \operatorname{Re}\varepsilon_{mn}^{(k)} + i\operatorname{Im}\varepsilon_{mn}^{(k)},\\
&e^{i\Omega t} = \cos\Omega t + i\sin\Omega t, \ m = 1,2,....5,
\end{aligned} \quad (23)$$

Having substituted (23) into (22), after some transformations we obtain the stress tensor

$$\sigma_{ji} = \sum_{k=1}^{2}\sum_{n=0}^{\infty}\int_{\omega_a}^{\omega_b}\operatorname{Re}\sigma_{ij}^{1}d\Omega. \quad (24)$$

All of these procedures laid down in machine memory. Designed the universal algorithm for computing the intervals of type (24). The results of of calculations are given at $\theta = 90^0$ ($\nu_1 = 0{,}2;\ \nu_2 = 0{,}25;\ r_0/r_1 = 0{,}5;\ \eta = 0{,}1$). In the integration limit adopted the following value $\omega_a = 10^{-4}, \omega_b = 4, h = 10^{-2}$.
Changing the hoop stress $\sigma_{\theta\theta}^{*}(\theta = 90^0, r = r_1)$ depending on τ shown in Fig.2.



Rice 2. Dependence of the annular of cylindrical layer of voltage times $\sigma_{\theta\theta}^{*} * 10$

Distinction between the voltages on the outer and inner surfaces reaches $\approx 15-20\%$, and the distinction between the voltages on the middle and inner surface $\approx 10\% (r_0 / r_1 = 0,5)$.

Cconclusions:

- We propose a mathematical formulation of the problem of the dynamic interaction of cylindrical bodies being in an elastic medium. The displacement vector represented by an auxiliary vectors. Auxiliary vector cylindrical bodies satisfy the Helmholtz equation and solutions are expressed in terms of Bessel functions and Henkel. Also proposed a method solution of problem.

- Comparison of the results for shear waves with longitudinal by waves shows that longitudinal waves occur larger voltage that of the shear wave;

- Calculations show that for $\tau = 12\alpha / C_{P_1}$ The results of this research are close to the exact static value $\alpha_{\theta\theta}^{*} = 8,13$. It is seen that the maximum stress and displacement depend essentially on $\bar{\eta}$ and $\bar{\eta}$.

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