Analytical Design Of A Two-Dimensional Dynamic Chain

Victor Kravets
Department of Automobiles and Automobile Economy
Dnipro University of Technology
Dnipro, Ukraine
prof.w.kravets@gmail.com

Valeriy Domanskyi
Department of electric transport
O. M. Beketov National University of Urban Economy in Kharkiv
Kharkiv, Ukraine

Illia Domanskyi
Department of Power Engineering
Ukrainian State University of Science and Technologies
Dnipro, Ukraine
i.v.domanskyi@ust.edu.ua

Tamila Kravets
Karlsruhe,
Baden-Wurttemberg, Germany
ktamila940@gmail.com

***Abstract*— A mechanical system consisting of a sequence of two material points in one-dimensional, rectilinear, free motion and interacting with each other by means of elastic and damping elements (two-dimensional dynamic chain) is considered. The free motion of the dynamic chain is determined by the kinetic energy of the material points, the potential energy of the elastic element, the dissipation energy of the damping element and is described by Lagrange equations of the second kind, where the deviations of the point masses from the natural configuration are taken as generalized coordinates. The compiled mathematical model of the two-dimensional dynamic chain is reduced to a canonical matrix form of the fourth order in order to unify the algorithm for the analytical solution of a system of differential equations, based on the roots of the characteristic equation of the fourth degree. Formulas are given for the four roots of the characteristic equation, depending on the variable parameters of the dynamic chain. The dynamic quality of the chain is determined by the distribution of the roots in the complex plane. The distribution of the roots in the complex plane is ensured by the choice of variable parameters, which constitutes the analytical design procedure two-dimensional dynamic chain.**

Keywords—dynamic chain; mathematical model; characteristic equation; root distribution; dynamic design

I. Introduction

Analytical design is the most important stage in the development of transport systems, preceding the computational and natural experiment. Analytical design determines the main dynamic qualities of the technical system. Calculation schemes in the form of a dynamic chain [1] are widely used in the design of railway [2, 3], automobile [4, 5], rocket and space [6], and vibration [7] transport. The theoretical basis for analytical design is classical mathematical methods, residue theory, and modal control theory [8-11]. A mathematical model of the free motion of a dynamic chain is constructed in the traditional form of Lagrange equations of motion of the second kind and is reduced to canonical matrix form in order to unify analytical algorithms [11]. The number of degrees of freedom of the dynamic target determines the order of the system of differential equations and the degree of the corresponding characteristic equation. The distribution of the roots of the characteristic equation in the complex plane determines the dynamic qualities of a technical system, including stability, stability margin, oscillation frequencies, resonant frequencies, beats, oscillation, etc. Expressing the roots of the characteristic equation in variable parameters of a technical system and controlling the distribution of roots in the complex plane is possible only in certain special cases [12-19]. The objective of this work is to apply the modal control method to the analytical design of a two-dimensional dynamic chain in free motion.

II. Statement of the Problem

A two-dimensional dynamic chain is considered, the calculation scheme of which is shown in Fig. 1.

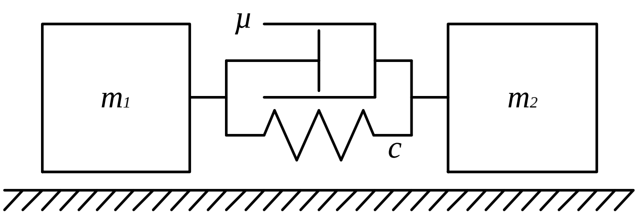

Fig. 1. Calculation scheme of a two-dimensional dynamic circuit. Here m_1, m_2 - are the mass of the chain links; c - is the elasticity coefficient; μ - damping coefficient.

Deviations of m chain links from the natural configuration $q_1(t)$, $q_2(t)$ are taken as generalized coordinates.

It is necessary to construct a mathematical model of the free motion of a two-dimensional dynamic chain in canonical form, to compile the corresponding characteristic equation and find its particular solutions, to establish the types of distribution of roots in the complex plane, technically implemented by choosing the design parameters of the system.

III. Mathematical model

The free motion of a two-dimensional dynamic chain is determined by a system of linear Lagrange differential equations:

$$m_1\ddot{q}_1 = -\mu\dot{q}_1 - cq_1 + \mu\dot{q}_2 + cq_2,$$
$$m_2\ddot{q}_2 = -\mu\dot{q}_1 + cq_1 - \mu\dot{q}_2 - cq_2.$$

New variables (phase coordinates) are introduced:

$$\dot{q}_1 = x_1,\ q_1 = x_2,\ \dot{x}_2 = x_1,\ \dot{x}_1 = \ddot{q}_1,$$
$$\dot{q}_2 = x_3,\ q_2 = x_4,\ \dot{x}_4 = x_3,\ \dot{x}_3 = \ddot{q}_2.$$

The mathematical model of a two-dimensional dynamic chain takes the following canonical matrix form:

$$\frac{d}{dt}\left[X_i(t)\right] = D\cdot\left[X_i(t)\right] \qquad (i=1,2,3,4).$$

where $[X_i(t)]$ is the column matrix of phase coordinates:

$$X(t) = \begin{Vmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Vmatrix},$$

D - is a dynamic square matrix of the fourth order:

$$D = \begin{Vmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{Vmatrix}.$$

Here

$$d_{11} = \begin{Vmatrix} -\dfrac{\mu}{m_1} & -\dfrac{c}{m_1} \\ 1 & 0 \end{Vmatrix},\ d_{22} = \begin{Vmatrix} -\dfrac{\mu}{m_2} & -\dfrac{c}{m_2} \\ 1 & 0 \end{Vmatrix},$$
$$d_{12} = \begin{Vmatrix} \dfrac{\mu}{m_1} & \dfrac{c}{m_1} \\ 0 & 0 \end{Vmatrix},\ d_{21} = \begin{Vmatrix} \dfrac{\mu}{m_2} & \dfrac{c}{m_2} \\ 0 & 0 \end{Vmatrix}.$$

the corresponding characteristic determinant of the fourth order is formed:

$$\Delta(\lambda) = \det(D - \lambda E),$$

where E - is the identity matrix of the fourth order.

Characteristic equation

$$\Delta(\lambda) = 0$$

is transformed into an algebraic equation of the fourth degree:

$$a_4\lambda^4 + a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0$$

The roots of this equation are found using the obvious formula [16]:

$$\lambda_{1,2,3,4} = -\frac{a_3}{4} \pm$$
$$\pm\frac{1}{\sqrt{2}}\sqrt{\left[\frac{3}{2}\left(\frac{a_3}{2}\right)^2 - a_2\right] \pm \sqrt{\left[\left(\frac{a_3}{2}\right)^2 - a_2\right]^2 - 4a_0}},$$

if the coefficients satisfy the condition:

$$a_1 + \frac{a_3}{2}\left[\left(\frac{a_3}{2}\right)^2 - a_2\right] = 0.$$

Here

$$a_4 = \det(-E),\ a_0 = \det(D)$$

or the determinants are represented by an equivalent, compact notation:

$$a_4 = \begin{vmatrix}0 & 0 & 0 & 0\end{vmatrix},\ a_0 = \begin{vmatrix}1 & 2 & 3 & 4\end{vmatrix},$$

where 0 - means the corresponding column of the matrix - E; 1, 2, 3, 4 is the column number of the matrix D.

Then the coefficients a_3, a_2, a_1, are found using symmetric determinants of the following structure:

$$a_3 = \begin{vmatrix}1 & 0 & 0 & 0\end{vmatrix} + \begin{vmatrix}0 & 1 & 0 & 0\end{vmatrix} + \begin{vmatrix}0 & 0 & 1 & 0\end{vmatrix} + \begin{vmatrix}0 & 0 & 0 & 1\end{vmatrix};$$

$$a_1 = \begin{vmatrix}0 & 2 & 3 & 4\end{vmatrix} + \begin{vmatrix}1 & 0 & 3 & 4\end{vmatrix} + \begin{vmatrix}1 & 2 & 0 & 4\end{vmatrix} + \begin{vmatrix}1 & 2 & 3 & 0\end{vmatrix};$$

$$a_2 = \begin{vmatrix}1 & 2 & 0 & 0\end{vmatrix} + \begin{vmatrix}1 & 0 & 3 & 0\end{vmatrix} + \begin{vmatrix}1 & 0 & 0 & 4\end{vmatrix} + \begin{vmatrix}0 & 2 & 3 & 0\end{vmatrix} + \begin{vmatrix}0 & 2 & 0 & 4\end{vmatrix} + \begin{vmatrix}0 & 0 & 3 & 4\end{vmatrix};$$

or

$$a_2 = \begin{vmatrix}0 & 0 & 3 & 4\end{vmatrix} + \begin{vmatrix}0 & 2 & 0 & 4\end{vmatrix} + \begin{vmatrix}0 & 2 & 3 & 0\end{vmatrix} + \begin{vmatrix}1 & 0 & 0 & 4\end{vmatrix} + \begin{vmatrix}1 & 0 & 3 & 0\end{vmatrix} + \begin{vmatrix}1 & 2 & 0 & 0\end{vmatrix}.$$

Due to the structural features of the two-dimensional dynamic chain under consideration, the coefficients of the characteristic equation in the expanded form take the form:

$$a_4 = 1,\ a_3 = \mu\left(\frac{1}{m_1} + \frac{1}{m_2}\right),\ a_2 = c\left(\frac{1}{m_1} + \frac{1}{m_2}\right),\ a_1 = 0,\ a_0 = 0,$$

i.e. the dynamic matrix *D* is singular (singular):

$$\det(D) = 0$$

For a homogeneous two-dimensional dynamic chain: $m_1 = m_2 = m$, introducing the reduced coefficients of elasticity and damping:

$$\bar{c} = \frac{c}{m},\ \bar{\mu} = \frac{\mu}{m},$$

we get:

$$a_2 = 2\bar{c},\ a_3 = 2\bar{\mu}.$$

Then the roots are found using the formula:

$$\lambda_{1,2,3,4} = -\frac{\bar{\mu}}{2} \pm \frac{1}{\sqrt{2}}\sqrt{\left[\frac{3}{2}\bar{\mu}^2 - 2\bar{c}\right] \pm \sqrt{\left[\bar{\mu}^2 - 2\bar{c}\right]^2}},$$

given that:

$$\bar{\mu}\left[\bar{\mu}^2 - 2\bar{c}\right] = 0.$$

From which it follows:

$$\lambda_{1,2} = -\frac{\bar{\mu}}{2} \pm \frac{\bar{\mu}}{2};\ \lambda_{3,4} = -\frac{\bar{\mu}}{2} \pm \frac{\bar{\mu}}{2}$$

and the corresponding distribution of roots in the complex plane is shown in Fig. 2.

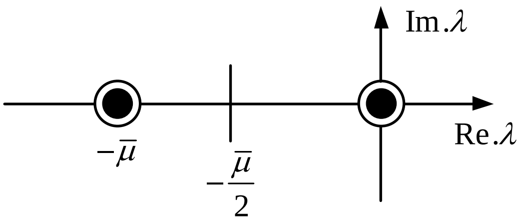

Fig. 2. A double, negative, real root is conjugate with a double, zero root relative to the center of symmetry.

Due to the structural feature of the dynamic chain under consideration $a_0 = 0$, $a_1 = 0$ the characteristic equation of the fourth degree is reduced to a quadratic one:

$$\lambda^2\left(\lambda^2 + 2\bar{\mu}\lambda + 2\bar{c}\right) = 0$$

and allows for other analytical solutions:

$$\lambda_{1,2} = 0,\ \lambda_{3,4} = -\bar{\mu} \pm \sqrt{d},$$

Where $d = \bar{\mu}^2 - 2\bar{c}$.

In particular, for $d=0$ we obtain $\lambda_{1,2} = 0$, $\lambda_{3,4} = -\bar{\mu}$, i.e. the distribution of roots shown in Fig. 2.

For $\bar{c} = 0$: $\lambda_{1,2,3} = 0$, $\lambda_4 = -2\bar{\mu}$ and the corresponding distribution of roots is shown in Fig. 3.

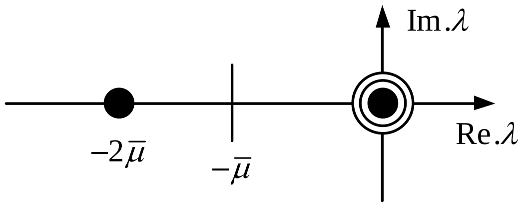

Fig. 3. Negative, real root and triple zero

At $\bar{\mu} = 0$: $\lambda_{1,2} = 0$, $\lambda_{3,4} = \pm i\sqrt{2\bar{c}}$. The distribution of roots is shown in Fig. 4.

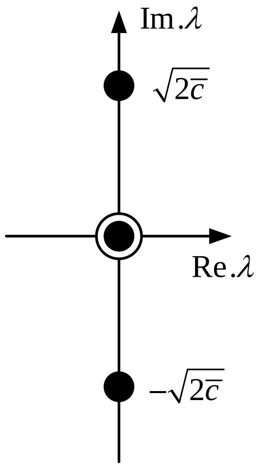

Fig. 4. Conjugate imaginary roots and double zero

At $d > 0$: $\lambda_{1,2} = 0$, $\lambda_{3,4} = -\bar{\mu} \pm \sqrt{d}$. The distribution of roots is shown in Fig. 5.

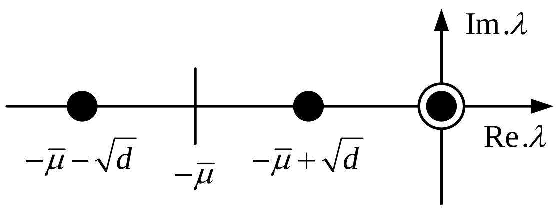

Fig. 5. Conjugate, negative real roots and double zero.

At $d < 0$: $\lambda_{1,2} = 0$, $\lambda_{3,4} = -\bar{\mu} \pm i\sqrt{d}$. The distribution of roots is shown in Fig. 6.

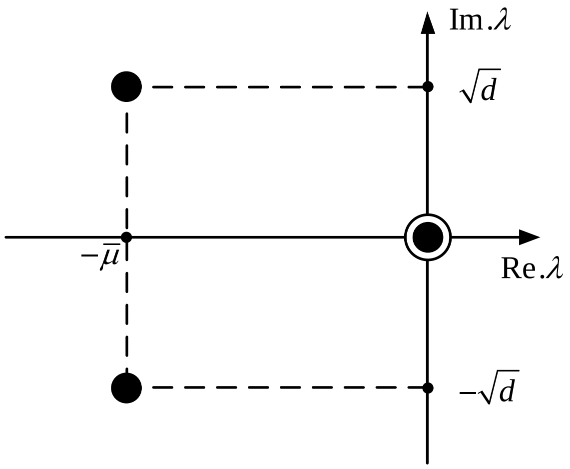

Fig. 6. Complex conjugate roots with negative real part and double zero

Analysis of the dynamic properties of a two-dimensional homogeneous chain:

- under external disturbance, the movement of a two-dimensional, homogeneous dynamic chain as a whole is unstable due to the presence of a multiple zero root (Fig. 2, 3, 4, 5, 6);

- the transient process is aperiodic and damping in nature due to the presence of negative real roots (Fig. 2, 3, 5);

- the transient process has an undamped, oscillatory nature with a frequency $\sqrt{2\bar{c}}$ subject $\bar{\mu}=0$ to the presence of conjugate imaginary roots (Fig. 4);

- the transient process has a damped, oscillatory nature with a frequency $\sqrt{2\bar{c}-\mu^2}$ due to $2\bar{c}>\bar{\mu}^2$ the presence of complex conjugate roots with a negative real part (Fig. 6).

Thus, by selecting variable design parameters, a distribution of roots is ensured that corresponds to the required dynamic quality of a two-dimensional, homogeneous chain.

References

[1] Pavlovskij, M.A. Teoretichna mexanika [Theoretical Mechanics], Kyiv, Technika Publ., 2002, 512 p.

[2] Lazaryan, V.A., Dinamyka transportnyh sredstv [Vchicle dynamics], Kyiv, Naukova dumka, 1985, 528 p.

[3] Blokhin, E.P., Manashkin, L.A., Dinamyka poezda [The dynamics of the train], Moscow, Transport, 1982, 222 p.

[4] Martunjuk, A.A., Lobas, L.G., Nikitina, N.V., Dinamika i ustoychivost' dvizhenia kolesnyh transportnyh sredsv [Dynamics and stability of motion wheelsets transport vehicles], Kyiv, Tekhnika Publ., 1981, 223 p.

[5] Kravets, V.V., Bass, K.M., Kravets T.V., Tokar, L.M., Dynamic design of ground transport with the help of computation experiment, MMSE Journal, 2015, 1. p.p. 105-111. ISSN 2412-5954, Open access www.mmse.xyz, DOI 10.13140/RG.2.1.2466.6643.

[6] Igdalov, I.M., Kuchma, L.D., Polyakov, N.V., Sheptun, Yu.D., Dinamicheskoe proektirovanie raket. Zadachi dinamiki raket i ih kosmicheskih stupeney [Dynamic design of missiles. The tasks of the dynamics rockers and their cosmic stages], Dnipro, Dnipro national university publ., 2010, 264 p.

[7] Vibrations in Engineering: Handbook. In 6 volumes /Editorial Board: V. N. Chelomey (president). - Moscow: Mashinostroenie, 1981. - Vol. 4. Vibration Processes and Machines / Ed. by E. E. Lavendel . 1981.-509 p.

[8] Ovchinnikov P.P. Vishha matematika [Higher mathematics], part 2, Kyiv, Tehnika Publ., 2000, 797 p. [in Ukrainian].

[9] Smirnov, V.I. 1974. A course of higher mathematics [in Russian]. - Moscow: Nauka V.2,656 P.

[10] Kuzovkov, N.T. 1976. Modal control and monitoring facilities [in Russian]. - Moscow: Mechanical Engineering, P. 184.

[11] Korn, G., Korn, T. Spravochnik po matematike dlya nauchnyh rabotnikov i inzhenerov [Mathematical Handbook for Scientists and Engincers], Moscow, Nauka Publ. 1984, 832 p. [in Russian].

[12] Kravets, V.V., Bas K.M., Kravets, Vl.V. 2012. Dynamic design of the simplest vehicle component [in Russian]. - Sevastopol: Messenger of SebNTU, Issue 135, P. 188-191

[13] Kravets, V.V., Bas K.M., Kravets, Vl.V., Burov, V.S. 2014. Analytical method of the simplest vehicle component dynamic design in terms of external effect [in Russian]. - Sevastopol: Scientific Messenger of the First Ukrainian Marine Institute, Issue 1, P. 79-82.

[14] Kravets Victor V., Bass Konstantin M., Kravets Tamila V. & Tokar Lyudmila A. (2016). Analytical Modeling of Transient Process In Terms of One-Dimensional Problem of Dynamics With Kinematic Action. Mechanics, Materials Science & Engineering, Vol 2. doi:10.13140/RG.2.1.4017.0005

[15] Kravets Victor V., Kravets Tamila V., Fedoriachenko Serhii A. & Loginova Anastasia A. (2016). Analytical Simulation of Dynamical Process in One-Dimension Task. Mechanics, Materials Science & Engineering, Vol 6. doi: 10.13140/RG.2.2.20337.34347

[16] Kravets V, Chibushov Y. (1994) Method of Finding the Analytical Solution of the Algebraic Particular Aspect Equation. Poland, Rzeszow, Folia Sci. Univ. Tech. Resoviensis, Math. 16, 104-117.

[17] Kravets, V.V., Bass, K.M., Kravets, Vl.V., Tokar, L.A. (2014) Analytical Solution of Kolmogorov Equations for Four-Condition Homogenous Symmetric and Ergodic System. Open Journal Of Applied Sciences, 4, 497 – 500. http://dx.doi.org/10.4236/ojapps.2014.410048.

[18] Kravets V.V., Kravets Vl.,V, Burov O. V. (2021) Analytical Modeling of the Dynamis System of the Fourth Order, TMLAI, Vol.9, №3 p. 14-24. https://doi.org/10.14738/tmlai.93.9947

[19] Alpatov, A., Kravets, V., Kravets, Vl., Lapkhanov, E. (2021) Analytical modeling of the binary dynamic circuit motion. TMLAI, Vol, 9, № 5, p. 23-32. https:///doi.org/10.14738/tmlai. 95.10922